Maximum Average Spectral Efficiency for Slowly Varying Rayleigh Fading Channels with Pilot-Symbol-Assisted Channel Estimation

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Abstract

In this paper we find a closed-form approximation to the maximum average spectral efficiency (MASE) of a slowly varying Rayleigh fading channel. Optimal pilot-symbol-assisted channel estimation and partial channel knowledge available at the transmitter is assumed. The approximation is a function of the average channel-signal-to-noise-ratio, the pilot symbol period, and the fading bandwidth. This expression is used to derive the pilot symbol period maximizing the approximated MASE. We show results for varying fading bandwidths, and compare our results to previous work on MASE where the ideal case of perfect channel knowledge was assumed, as well as to previous work on optimal channel estimation where the MASE was numerically evaluated. The results may be used to provide a benchmark when evaluating the performance of practical rate-adaptive coded modulation schemes. Such schemes have been shown to be candidates for spectrally efficient, low-delay information transmission on fading channels.

1 Introduction

If mobile multimedia communications is to be successful in the future, available channel bandwidth must be efficiently exploited. It is therefore of vital importance to find transmission schemes with high *spectral efficiency*, preferably as close to the *maximum average spectral efficiency* (MASE) of mobile channels as possible. The MASE is a measure of a channel's capacity in the information theoretic sense, i.e. the upper information rate limit (measured in information bits/s/Hz) for error-free transmission [1]. Here, we shall consider *narrowband* mobile channels with multipath transmission, meaning that the transmitted signals are exposed to stochastically varying, *frequency-independent* attenuation. This may be modelled by stochastic *flat-fading* channel models—such as Rayleigh, Rice, or Nakagami fading [2]. In [3] Goldsmith and Varaiya derived the MASE of a flat-fading single-user channel with arbitrary fading distribution and additive white gaussian noise (AWGN), assuming perfect *channel state information* (CSI) available both at the transmitter and the receiver at all times. Alouini *et al.* [4] later derived closed-form expressions for the channel capacity on a Nakagami multipath fading channel, of which *Rayleigh fading* is a special case.

In [3] it was also shown that the MASE can be approached using a certain *rate-adaptive* transmission scheme, in which the number of information bits per channel symbol is instantly and continuously updated according to the instantaneous *channel-signal-to-noise-ratio* (CSNR).[1] The transmission rate of the capacity-achieving scheme is high when the CSNR is high, decreasing smoothly as the CSNR decreases and going to zero below a threshold value.

In practice, the CSI has to be *estimated*, which means that the idealized assumption of perfect CSI does not hold. In this paper we shall focus on the effect of CSI estimation on the MASE when *least-mean-squares-error* (LMSE) *pilot-symbol-assisted* channel estimation is used on a slowly varying Rayleigh fading channel.

2 System model

Denoting the transmitted complex baseband symbol at time index t by c_t, the received symbol after transmission on a flat-fading channel can be written as

$$r_t = \alpha_t c_t + w_t. \qquad (1)$$

Here α_t is the *complex fading amplitude*, and w_t is complex-valued AWGN with statistically independent real and imaginary components. Since we assume Rayleigh fading, $\{\alpha_t\}$ is also a complex gaussian process, typically correlated in time. We assume that a constant average transmit power P [Hz] is used, and that the variance of the complex AWGN is N_0 [Hz]. The instantaneous received CSNR at a given time t is

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[1] Ideally, also the transmit power should be adapted, but this has been shown to yield only a small rate improvement in practice.

then

$$\gamma_t = \frac{|\alpha_t|^2 P}{N_0}\,, \tag{2}$$

with

$$E[\gamma_t] = \overline{\gamma} = \frac{P\sigma_\alpha^2}{N_0} \text{ where } \sigma_\alpha^2 = E[|\alpha_t|^2]. \tag{3}$$

3 Pilot-symbol-assisted channel estimation

Channel estimation is necessary both for receiver detection and for transmitter adaption. We shall assume that known pilot symbols with constant amplitude $\sqrt{P}$ (and pseudorandom phase θ as described in [5], so as to prevent the pilots from representing a deterministic tone) are periodically transmitted for channel estimation, say every Lth symbol.

From (1) the received symbol at time t, r_t is itself a complex gaussian when conditioned on c_t. The LMSE estimator for α_t, based on all available knowledge (which is to say, all received symbols and transmitted pilot symbols), is known from standard estimation theory [6] to be

$$\widehat{\alpha}_t = E[\alpha_t \mid \mathbf{r}, \mathbf{p}]\,, \tag{4}$$

where $\mathbf{r}$ is the vector of all received symbols and $\mathbf{p}$ is the vector of all transmitted pilot symbols. The LMSE estimator defined above is known to have the following properties:

1. The estimator minimizes the variance of the estimation error $e_t = \alpha_t - \widehat{\alpha}_t$ over all estimators utilizing the same a priori knowledge. Thus $\sigma_e^2 = E[|\alpha_t - \widehat{\alpha}_t|^2]$ is minimal.[2]

2. The estimation error is uncorrelated with the estimate (in the case of gaussian variables, this corresponds to statistical independency), i.e.

$$E[e_t \widehat{\alpha}_t^*] = 0. \tag{5}$$

3. The estimator is unbiased, so $E[\widehat{\alpha}_t] = 0$.

Since α_t is gaussian, the LMSE estimate is a linear function of the observations, and may be found by linear *Wiener filtering* [7]. We shall return to this later on; for now, observe that Equation (1) may be rewritten as

$$\begin{aligned} r_t &= \alpha_t c_t + (\widehat{\alpha}_t - \widehat{\alpha}_t) c_t + w_t \\ &= \widehat{\alpha}_t c_t + (\alpha_t - \widehat{\alpha}_t) c_t + w_t \\ &= \widehat{\alpha}_t c_t + e_t c_t + w_t. \end{aligned} \tag{6}$$

[2]This variance is time-independent only if the pilot symbol frequency $\frac{1}{L}$ fulfills the Nyquist criterion relative to the bandwidth of the fading process [7]. We shall assume that this is the case.

While seemingly trivial, this observation, made by Baltersee *et al.* [8], is the key to all of the following analysis. What Equation (6) shows is that a Rayleigh fading channel with pilot-symbol-assisted LMSE channel estimation is equivalent to a channel model with a modified additive noise term,

$$n_t = e_t c_t + w_t\,, \tag{7}$$

and a modified received signal term,

$$\tilde{s}_t = \widehat{\alpha}_t c_t\,, \tag{8}$$

instead of the original perfect-channel-knowledge Rayleigh fading model. Due to the properties of the linear LMSE estimator, $e_t c_t$ is gaussian and statistically independent of $\widehat{\alpha}_t c_t$ (and of course of the noise w_t) when conditioned on c_t. For a given c_t the effective channel noise can therefore be viewed as *additive, colored gaussian noise*—colored because of the memory in the fading process, which may be described in the frequency domain as follows:

Following Baltersee *et al.* [8] we assume that the complex gaussian fading process α_t is perfectly bandlimited to the normalized angular frequency interval $[-2\pi W, 2\pi W]$, and has variance σ_α^2. W is called the *fading bandwidth.* Note that for a mobile channel with Doppler frequency f_D [Hz] and Nyquist symbol frequency f_S [Hz],

$$W = \frac{f_D}{f_S}\,, \tag{9}$$

and that we always have $W \leq 0.5$. The assumption of slow fading can now be translated to $W \ll 0.5$. The *power spectral density* of the fading process $\{\alpha_t\}$ can now be modelled as

$$S(\omega) = \begin{cases} \frac{\sigma_\alpha^2}{2W} \text{ for } \omega \in [-2\pi W, 2\pi W] \\ 0 \text{ elsewhere} \end{cases} \tag{10}$$

It has been shown by Meyr *et al.* [7] that the actual shape of the fading spectrum is of no consequence for the LMSE estimator performance. Therefore, as is also done in [7, 8], the ideal low-pass spectrum is chosen due to ease of analysis.

The modified channel noise n_t in (7) consists of two statistically independent noise components, so the total effective noise variance is the sum of their variances:

$$\sigma_n^2(c_t) = |c_t|^2 \sigma_e^2 + N_0. \tag{11}$$

If conditioned on c_t the noise is stationary gaussian and statistically independent of the signal. The *unconditional* noise probability density function may be written

$$p_N(n) = E_C[p_N(n|c)] = \int p_C(c) \cdot \frac{1}{\pi\sigma_n^2(c)} e^{-\frac{|n|^2}{\sigma_n^2(c)}}\, dc\,, \tag{12}$$

where $p_C(c)$ is the distribution on c. In addition to the fact that the noise is colored, the effective noise variance averaged over all possible transmitted symbols is increased, while the effective received signal power is reduced. Averaged over all possible transmitted symbols the noise variance is

$$\sigma_n^2 = \int_{\text{all } n} |n|^2 p_N(n)\, dn = P\sigma_e^2 + N_0. \tag{13}$$

The average received signal power for a given c_t, on the other hand, is reduced from $|c_t|^2\sigma_\alpha^2$ (with perfect channel knowledge) to

$$|c_t|^2\sigma_{\hat{\alpha}}^2 = |c_t|^2(\sigma_\alpha^2 - \sigma_e^2) \tag{14}$$

with expected value $P(\sigma_\alpha^2 - \sigma_e^2)$.

4 Maximum spectral efficiency

Goldsmith and Varaiya [3] showed that any flat-fading channel with perfect CSI at the transmitter and constant transmit power has a MASE which can be stated as

$$\text{MASE} = \int_0^\infty p(\gamma)C(\gamma)\, d\gamma \text{ [bits/s/Hz]}, \tag{15}$$

where $p(\gamma)$ is the CSNR distribution and $C(\gamma)$ is the capacity that would be available for a constant CSNR γ.

Alouini and Goldsmith [4] used Equation (15) to derive a closed expression for the MASE of a stationary Rayleigh fading channel with AWGN (more generally, a *Nakagami* multipath fading channel, of which the Rayleigh fading channel is a special case) when perfect CSI is assumed at the transmitter side. The MASE expression for a flat Rayleigh fading channel with constant transmit power P, AWGN, perfect CSI at the transmitter, and expected CSNR $\overline{\gamma}(P)$ is:

$$\text{MASE}_w(\overline{\gamma}) = \frac{\exp\left(\frac{1}{\overline{\gamma}}\right)\Gamma\left(0, \frac{1}{\overline{\gamma}}\right)}{\ln(2)} \text{ [bits/s/Hz]}, \tag{16}$$

where $\Gamma(x, y)$ is the *complementary incomplete Gamma function*, defined by the integral [9]

$$\Gamma(x, y) = \int_y^\infty t^{x-1}e^{-t}dt. \tag{17}$$

As justified in Appendix A we may now use (16) to find a closed-form approximation of the MASE for a Rayleigh fading channel with LMSE channel estimation:

$$\text{MASE} \approx \frac{L-1}{L}\text{MASE}_w(\overline{\gamma}_{\text{LMSE}}) \tag{18}$$

where L is the pilot symbol period and

$$\overline{\gamma}_{\text{LMSE}} = \frac{P(\sigma_\alpha^2 - \sigma_e^2)}{P\sigma_e^2 + N_0} \tag{19}$$

is the effective average CSNR, which should be contrasted with (3) for a channel with perfect CSI.

Note that the assumption of perfect CSI which was used to obtain (16) is equivalent to assuming that the CSI is perfectly estimated at the receiver end, and that the feedback channel from receiver to transmitter is error free and have zero delay. The first assumption is valid in this case since the effective CSNR degradation which results from LMSE estimation has already been taken into account in the channel model through the degradation of the average CSNR. The zero delay assumption is valid as long as the the Nyquist period of the fading process, $\frac{1}{2W}$, is much larger than the actual feedback delay. This is again the same as saying that the fading is slowly varying. Also, since the feedback channel will then typically be transmitting channel state information at a very low rate it can be heavily error protected, making it for practical purposes error free.

We need an expression for σ_e^2 in order to be able to find a closed form expression for the MASE approximation in (18). For this purpose, Wiener filter theory is used [7, 8]. The appropriate Wiener filter here is an ideal lowpass filter with frequency response

$$W(\omega) = \begin{cases} \frac{\sqrt{P}\sigma_\alpha^2}{P\sigma_\alpha^2 + 2WLN_0} \text{for } \omega \in [-2\pi W, 2\pi W] \\ 0 \text{ elsewhere} \end{cases} \tag{20}$$

This gives the estimation error variance

$$\sigma_e^2 = \frac{\sigma_\alpha^2 \cdot 2WLN_0}{P\sigma_\alpha^2 + 2WLN_0}, \tag{21}$$

which is seen to be dependent both on the pilot symbol frequency as well as on the channel dynamics through the fading bandwidth. The estimation error e_t has power spectral density

$$S_e(w) = \begin{cases} \frac{\sigma_\alpha^2 LN_0}{P\sigma_\alpha^2 + 2WLN_0} \text{ for } |\omega| \leq 2\pi W \\ 0 \text{ elsewhere} \end{cases} \tag{22}$$

so the power spectral density of the effective channel noise is

$$S_n(w) = \begin{cases} N_0 + \frac{P\sigma_\alpha^2 LN_0}{P\sigma_\alpha^2 + 2WLN_0} \text{ for } |\omega| \leq 2\pi W \\ N_0 \text{ elsewhere} \end{cases} \tag{23}$$

.

Inserting Equations (19) and (21) in (16), and taking into account the fact that every Lth symbol is a pilot symbol, the approximate MASE for a slow Rayleigh fading channel with LMSE estimated CSI and "true" expected CSNR equal to $\overline{\gamma}$ is given as:

$$
\begin{aligned}
\text{MASE} &\approx \max_L \left\{ \frac{L-1}{L\ln(2)} \exp\left(\frac{1}{\overline{\gamma}_{\text{LMSE}}}\right) \Gamma\left(0, \frac{1}{\overline{\gamma}_{\text{LMSE}}}\right) \right\} \\
&= \max_L \left\{ \frac{L-1}{L\ln(2)} \cdot \right. \\
&\quad \exp\left(\frac{1+(1+1/\overline{\gamma})2WL}{\overline{\gamma}}\right) \cdot \\
&\quad \left. \Gamma\left(0, \frac{1+(1+1/\overline{\gamma})2WL}{\overline{\gamma}}\right) \right\} \\
&\stackrel{\text{def}}{=} \max_L C_{\text{LMSE}}(L) \text{ [bits/s/Hz]}
\end{aligned} \tag{24}
$$

We see that the channel dynamics (through W) and the channel estimation overhead and accuracy (through L) enters the expression. In principle it is now possible to maximize C_{LMSE} with respect to the pilot symbol period L by setting the derivative of C_{LMSE} with respect to L to zero. However, this results in a nonlinear equation which must be solved by an iterative algorithm. A perhaps more elegant solution is to find a very tight and functionally simple upper bound $\widetilde{C}(L)$ on C_{LMSE}, and to perform maximization of this bound with respect to L instead. For x larger than approximately 0.1, $\Gamma(0,x)$ is tightly upper bounded by

$$
g(x) = \frac{1}{\sqrt{2x}} e^{-x}. \tag{25}
$$

This is easily shown using the Cauchy-Schwarz inequality [11]. For our purposes, $x = \frac{1}{\overline{\gamma}}$, so the bound is tight for $\overline{\gamma}$ below approximately 10 dB. In this region, C_{LMSE} is tightly upper bounded by

$$
\widetilde{C}(L) = \frac{L-1}{L\ln(2)} \sqrt{\frac{\overline{\gamma}}{2(1+(1+\frac{1}{\overline{\gamma}})2WL)}}. \tag{26}
$$

Performing differentiation of this expression with respect to L and setting the result to zero, we obtain extremal points in $\widetilde{C}(L)$ for

$$
L_1 = \infty \text{ and } L_2 = \frac{3}{2} + \sqrt{\frac{9}{4} + \frac{1}{W(1+\frac{1}{\overline{\gamma}})}}, \tag{27}
$$

where $L_1 = \infty$ obviously corresponds to a minimum (if no pilot symbols are sent, the channel is completely unknown, and the capacity is zero). L_2 is the pilot symbol period leading to the MASE. In practice L_2 must of course be approximated by the closest integer, and must also conform to the Nyquist condition $2WL \leq 1$ if the preceding theory is to be valid, that is

$$
L \leq \frac{1}{2W}. \tag{28}
$$

We thus conclude that for $\overline{\gamma}$ below approximately 10 dB, the MASE may be approximated as

$$
C_{\text{LMSE}}\left(\min\left\{\lfloor\frac{1}{2W}\rfloor, \text{int}\left\{\frac{3}{2} + \sqrt{\frac{9}{4} + \frac{1}{W(1+\frac{1}{\overline{\gamma}})}}\right\}\right\}\right).
$$

5 Results and discussion

We shall compare our results to those of Baltersee, Fock, and Meyr [8, 10]. They investigate the MASE of a system which is similar to ours, but resort to Monte Carlo simulations instead of analytical approximations. In Figure 1 we have plotted results from their simulations, together with our MASE approximation, the perfect-channel-knowledge MASE of Alouini and Goldsmith [4], and the MASE for an AWGN channel with the same average CSNR. We observe a close resemblance between the numerical results of [8] and our closed-form approximation.[3]

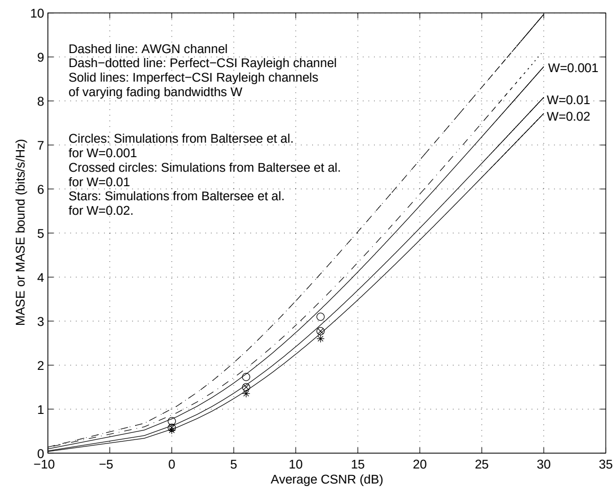

Figure 1: Exact MASE or MASE approximation as a function of $\overline{\gamma}$ for various fading bandwidths.

The most important thing to note in Figure 1 is the qualitative behaviour of the approximate MASE expression as a function of the fading bandwidth. Clearly, the perfect-CSI result of [4] is a too optimistic result when the fading bandwidth is increased much beyond, say, $W = 0.001$. On a mobile channel, this of course corresponds to a large Doppler frequency, i.e. fast terminal movement. However, for many practical situations the perfect-CSI bound is quite good; as an example a mobile communications system with a carrier frequency of 2 GHz, a symbol frequency of 200 kHz and a terminal speed of 30 m/s ($\approx$ 108 km/h) will correspond to $W = 0.001$.

Figure 2 shows the optimal pilot symbol period L given by L_2 in (27) as a function of CSNR for various fading bandwidths. As would be expected, L grows monotonously with the CSNR for all fading bandwidths, and is smaller for a given CSNR the higher W is. Note that when the L value prescribed by (27) becomes large, the Nyquist condition for the channel

[3] Unfortunately there are only a limited number of MASE simulation points in their paper, which makes closer comparison difficult.

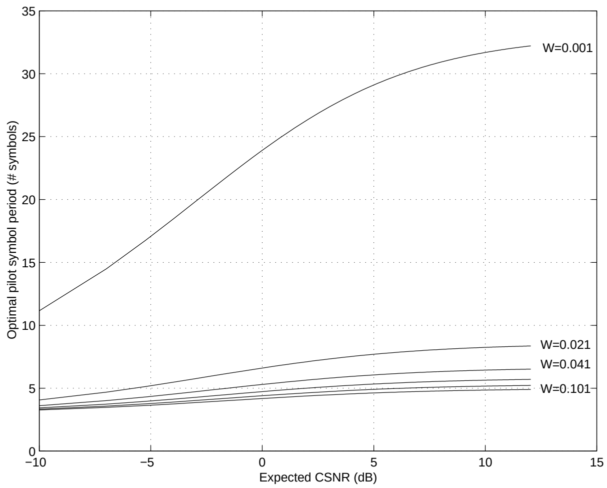

Figure 2: Optimal pilot period L as a function of $\overline{\gamma}$ for various fading bandwidths.

sampling may be violated. This happens for

$$W > \frac{\overline{\gamma}+1}{10\overline{\gamma}+6}, \qquad (29)$$

which lies in the interval $[\frac{1}{10}, \frac{1}{6}]$ for all $\overline{\gamma}$. I.e., as long as we assume $W \leq 0.1$ the Nyquist condition will never be violated with the optimal L. Finally, remember that this optimal value is only valid for CSNRs below approximately 10 dB. However, above this CSNR value we observe a flattening of the L curves which indicate that $L(\text{CSNR} > 10\text{ dB}) \approx L(\text{CSNR}{=}10\text{ dB})$, regardless of W.

It is worth noting that two factors are essentially different in Baltersee *et al.*'s system compared to ours:

- CSI at the transmitter is not assumed in [8].
- In [8], an ideal (i.e. perfectly decorrelating) interleaver is inserted after the encoder/modulator, and the corresponding deinterleaver before the demodulator/decoder.

It has been shown that transmitter CSI is not necessary to reach the MASE on a fading channel [14]. However, when the transmitter does not have CSI as in [8], it can not adapt instantaneously to changing channel conditions. Thus the same fixed channel code has to be used at all times. This makes it necessary to randomize the channel errors by means of an interleaver, since most good codes in practice are designed to correct random errors, not error bursts as would be experienced without interleaving.

However, if we allow for transmitter CSI and thus possible code adaptivity, each code in the set of allowed codes will in practice be used for an approximate AWGN channel, even if interleaving is not used [12, 13]. In fact, decorrelating interleaving would completely destroy any chances of performing rate-adaptive transmission properly, since such transmission in practice exploits exactly the fact that the fading is correlated (if not, the assumption of zero feedback delay would break down [13]). This is the main reason why we have left out the interleaver used in [8].

We believe that a rate-adaptive scheme will be able to operate closer to the MASE than would a fixed-rate scheme for a given average end-to-end transmission delay, because a MASE-achieving single code must have a large codeword length (much larger than the coherence time of the channel) [14], whereas a rate-adaptive scheme in comparison will respond to changing channel conditions more or less instantaneously, by selecting a code suitable to an AWGN channel with CSNR approximately equal to the estimated instantaneous CSNR at all times [12].

6 Summary and conclusions

We have derived a closed-form approximation of the MASE on a slowly varying Rayleigh fading channel with pilot-symbol-assisted LMSE channel estimation and transmitter feedback. The pilot symbol period may be optimized for maximal MASE, and the optimal period is shown to increase with increasing average CSNR and decrease with increasing fading bandwidth.The analytical results show a very good correspondence with Monte Carlo simulations of the MASE performed by another research team. The most important insight is perhaps that imperfect channel estimation may severely degrade the MASE compared to the previously known perfect-knowledge bound unless the fading bandwidth is very small. However, the MASE is still significantly higher for moderate fading bandwidths than what is reflected in today's mobile systems, and can be efficiently exploited by means of rate-adaptive coded modulation schemes.

A MASE approximation

Let $\mathbf{C} = [C_1, \ldots, C_n]$ be a possible block of input symbols to the channel transmitted from time 1 to time n, let $\widehat{\alpha} = [\widehat{\alpha}_1, \ldots, \widehat{\alpha}_n]$ be the corresponding vector of estimated fading amplitudes, and let $\mathbf{N} = [N_1, \ldots, N_n]$ be a corresponding effective channel noise vector. Then,

$$\mathbf{R} = \text{diag}(\widehat{\alpha}_1, \ldots, \widehat{\alpha}_n)\mathbf{C} + \mathbf{N} \qquad (30)$$

is the corresponding received symbol vector. From [3], we have (disregarding the factor $\frac{L-1}{L}$ which for this purpose is just a scaling constant)

$$\begin{aligned} \text{MASE} \;=\; & \lim_{n\to\infty} \frac{1}{n} \left\{ \int p_{\widehat{\alpha}}(\widehat{\alpha}) \max_{p_{\mathbf{C}|\widehat{\alpha}} \in \mathcal{P}(\mathbf{c}|\widehat{\alpha})} \{h(\mathbf{R}|\widehat{\alpha}) \right. \\ & \left. -h(\mathbf{R}|\widehat{\alpha}, \mathbf{C})\} \; d\widehat{\alpha} \right\}, \qquad (31) \end{aligned}$$

where h denotes differential entropy measured in bits/s/Hz, and $\mathcal{P}$ is the set of all n-dimensional input probability density functions which fulfill the average power constraint

$$\mathrm{trace}(E[\mathbf{X}\mathbf{X}^H]) = nP. \tag{32}$$

Due to (30), we obtain

$$h(\mathbf{R}|\widehat{\alpha}, \mathbf{C}) = h(\mathbf{N}|\mathbf{C}) \tag{33}$$

where $\mathbf{N}$ is a complex, nonwhite gaussian vector where element i has variance $|C_i|^2\sigma_e^2 + N_0$. From [1, Eq. (9.94)] we then have

$$\begin{aligned} h(\mathbf{N}|\mathbf{C}) &= \log_2\left((2\pi e)^n |\Sigma_N(\mathbf{C})|\right) \\ &\leq \log_2\left((2\pi e)^n \Pi_{i=1}^n (|C_i|^2\sigma_e^2 + N_0)\right) \end{aligned} \tag{34}$$

where $\Sigma_N(\mathbf{C})$ is the autocorrelation matrix of $\mathbf{N}$ for a given $\mathbf{C}$. Using the optimal value of the pilot symbol period L given by (27) and the derived expression for the power spectral density of σ_n^2 ((23), with $|C_i|^2$ replacing P) when computing $\Sigma_N(\mathbf{C})$, it is possible to show that

$$\begin{aligned} \Sigma_N(\mathbf{C}) &= \mathrm{diag}(|C_1|^2\sigma_e^2 + N_0, \dots, |C_n|^2\sigma_e^2 + N_0) \\ &\quad + \Delta(C_1, \dots, C_n) \end{aligned} \tag{35}$$

where $\Delta(C_1, \dots, C_n)$ is a perturbation matrix whose (i, j) element is upper bounded by

$$\Delta(i, j) \leq N_0 \left(3W + \sqrt{9W^2 + \frac{W}{4}}\right) \mathrm{sinc}(|i - j|). \tag{36}$$

$\Delta(i, j)$ goes monotonously and fast to zero as W goes to zero. Thus, for $W \ll 0.5$, $\Delta(i, j) \ll |C_i|^2\sigma_e^2 + N_0$ for all $j \neq i$, i.e. the true noise autocorrelation matrix is an only slightly perturbed diagonal matrix. Thus its determinant is approximately equal to the product of the diagonal values, and the inequality in (34) is fulfilled with approximate equality.

Inserting this into (31) we obtain

$$\begin{aligned} \mathrm{MASE} &= \lim_{n\to\infty} \frac{1}{n} \left\{ \int p_{\widehat{\alpha}}(\widehat{\alpha}) \max_{p_{\mathbf{C}|\widehat{\alpha}}(\mathbf{c}|\widehat{\alpha}) \in \mathcal{P}} h(\mathbf{R}|\widehat{\alpha}) d\widehat{\alpha} \right. \\ &\quad \left. - \log_2\left((2\pi e)^n |\Sigma_N(\mathbf{C})|\right) \right\} \\ &\approx \lim_{n\to\infty} \frac{1}{n} \left\{ \int p_{\widehat{\alpha}}(\widehat{\alpha}) \max_{p_{\mathbf{C}|\widehat{\alpha}}(\mathbf{c}|\widehat{\alpha}) \in \mathcal{P}} h(\mathbf{R}|\widehat{\alpha}) d\widehat{\alpha} \right. \\ &\quad \left. - \max_{p_{\mathbf{C}}(\mathbf{c})} \left\{ \log_2\left((2\pi e)^n |\Sigma_N(\mathbf{C})|\right) \right\} \right\} \\ &= \lim_{n\to\infty} \frac{1}{n} \left\{ \int p_{\widehat{\alpha}}(\widehat{\alpha}) \max_{p_{\mathbf{C}|\widehat{\alpha}}(\mathbf{c}|\widehat{\alpha}) \in \mathcal{P}} h(\mathbf{R}|\widehat{\alpha}) d\widehat{\alpha} \right\} \\ &\quad - \log_2\left(2\pi e (P\sigma_e^2 + N_0)\right). \end{aligned} \tag{37}$$

$h(\mathbf{R}|\widehat{\alpha})$ will be maximal if $\mathbf{R}$ is gaussian with diagonal autocorrelation matrix for a given $\widehat{\alpha}$. Gaussianity is obtained if $\mathbf{C}$ is chosen to be gaussian for each $\widehat{\alpha}$. For a given autocorrelation matrix $\Sigma_{C|\widehat{\alpha}}$ for $\mathbf{C}$, $\mathbf{R}$ will have the distribution $\mathcal{N}(0, \Sigma_R = \mathrm{diag}(\widehat{\alpha}_1, \dots, \widehat{\alpha}_n)\Sigma_{C|\widehat{\alpha}}\mathrm{diag}(\widehat{\alpha}_1^*, \dots, \widehat{\alpha}_n^*) + \Sigma_N(\mathbf{C}))$ for a given $\widehat{\alpha}$. Since we have established that $\Sigma_N(\mathbf{C})$ is almost diagonal, for $W \ll 0.5$, Σ_R can be made approximately diagonal by choosing $\Sigma_{C|\widehat{\alpha}}$ diagonal. The solution is then approximately the same as would be the case if N were AWGN—which corresponds to a "traditional" perfect-CSI Rayleigh fading channel, with average CSNR given by (19) and corresponding MASE given by (16). We thus conclude that the MASE of such a channel (after multiplying with the factor $\frac{L-1}{L}$) is a reasonable approximation of the MASE of a Rayleigh fading channel with optimal pilot-symbol-assisted LMSE channel estimation for $W \ll 0.5$.

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