On the Amount of Fading in MIMO Diversity Systems

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***Abstract*—In this paper, a closed-form expression is presented for the amount of fading (AF) experienced at the output of a space–time block-coded multiple-input–multiple-output (MIMO) diversity system operating on identically distributed spatially correlated Nakagami-m fading channels. For the separable correlation model (Kronecker model), the AF is presented for identically distributed Rayleigh fading channels and different types of antenna correlation models. For an MIMO diversity system based on the Kronecker model, it is shown, by capitalizing on recent results in IEEE Transactions on Communications, Vol. 51, No. 8 (August 2003), p. 1389, that the average symbol error rate (SER) at high signal-to-noise ratio (SNR) may be directly expressed in terms of the AF, when a constant correlation model is assumed.**

***Index Terms*—Amount of fading, coding gain, diversity, fading channels, multiple-input–multiple-output (MIMO).**

I. Introduction

SEVERAL performance measures may be employed to characterize the behavior of wireless communication systems operating on fading channels. For systems utilizing spatial diversity techniques, it is of interest to employ measures that can capture and quantify the improvement on system performance caused by reducing the fading-induced fluctuations of the received signal. Commonly encountered measures in this respect are the average symbol error rate (SER) or the average bit error rate (BER). The diversity order of a spatial diversity system is usually determined by the slope of the average SER curve at high signal-to-noise ratios (SNRs), whereas different levels of correlation between the diversity branches are visible as shifted versions of the SER curve, relative to a benchmark SER curve. However, as noted in [2], the average error rate may, in some cases, be difficult to evaluate analytically, since it requires statistical averaging of the conditional error rate over the statistics of the fading. A more simple, yet effective, way of quantifying the severity of fading (and the effect of correlation) can be obtained by using a measure directly related to the moments of the fading distribution itself.

In [3], Charash introduced the notion of amount of fading (AF) to quantify the severity of fading experienced for a particular channel model. In terms of the probability density function (pdf) of the instantaneous fading amplitude $\alpha = |h|$ of a single complex fading channel h, the AF is defined by [3, eq. (2)], [4, eq. (2.5)]

$$\mathrm{AF} = \frac{\mathrm{Var}\{\alpha^2\}}{\left(\mathcal{E}\{\alpha^2\}\right)^2} \tag{1}$$

with $\mathcal{E}\{\cdot\}$ and $\mathrm{Var}\{\cdot\}$ denoting the statistical average and variance, respectively. For a single Nakagami-m fading channel [5], $\mathrm{AF} = 1/m$. Hence, for large values of m [line-of-sight (LOS)], the fading channel will approach a nonfading additive white Gaussian noise (AWGN) channel. For a single Rayleigh fading channel ($m = 1$), $\mathrm{AF} = 1$.

Although the AF originally was defined and applied to quantify the severity of fading experienced at the output of a single fading channel, it is, in this paper, employed to quantify the degree of fading experienced at the output of a multiple-input multiple-output (MIMO) system. The benefit of transmitting from multiple antennas in a wireless system may either be utilized to improve the diversity order (high-reliability solution) or to improve the capacity (high-rate solution). These two transmission strategies are commonly denoted as MIMO diversity and spatial multiplexing, respectively [6]. In this study, an MIMO-diversity system is considered, where transmit diversity is realized by utilizing space–time block coding (STBC)[1] at the transmitter [7], [8].

There are some examples in the literature of how the AF can be applied as a performance measure in communication systems employing spatial diversity. In [2], closed-form expressions for the AF are obtained for three dual-branch diversity-combining techniques in the presence of log-normal fading, namely, maximum-ratio combining (MRC) [4, Sec. 9.2], selection combining (SC) [4, Sec. 9.7], and switch-and-stay combining (SSC) [4, Sec. 9.8]. In [9], the square root of the AF was applied to the combiner output to assess the effectiveness of a hybrid-selection/MRC diversity-combining scheme in the presence of Rayleigh fading.[2]

In this paper, the AF is presented for an MIMO diversity system operating on identically distributed spatially correlated Nakagami-m fading channels. The necessary moments are derived by utilizing a recent and compact representation of the characteristic function (CF) of the (instantaneous) combined fading power at the output of an MIMO diversity system [10].

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[1] Space-time block codes are designed to achieve the maximum diversity order for a given number of transmit and receive antennas, subject to the constraint of having a simple decoding algorithm.

[2] The square root of the AF is equal to a statistical measure known as the coefficient of variation.

Subsequently, it is assumed that the correlation properties at the transmitter is independent of the correlation properties at the receiver. Under this assumption, the channel correlation matrix can be written as a Kronecker product [11, Ch. 9] of a transmit correlation matrix and a receive correlation matrix [12], [13]. In the literature, this model is frequently denoted as the Kronecker model. The Kronecker model has been validated for non-LOS (NLOS) scenarios [14], [15], but the accuracy of the model has recently been questioned, at least for antennas with high spatial resolution (large antenna arrays) [16]–[18]. Since all the measurement campaigns referenced in this paper (both supporting and questioning the Kronecker model) have been conducted in NLOS scenarios (typically modeled with Rayleigh fading channels), the AF expressions based on the Kronecker model are, in this paper, limited to systems operating on Rayleigh fading channels.

The remainder of this paper is organized as follows. Section II presents statistical information of the combined fading power in an MIMO diversity system. In Section III, this information is used to obtain a compact closed-form expression for the AF. The general result (valid for identically distributed Nakagami-m fading channels) is then simplified by introducing the Kronecker model and by incorporating different types of antenna correlation models. In Section IV, it is shown that by capitalizing on recent results in [1], the average SER at high SNR may be expressed in terms of the AF, when a constant correlation model is assumed at both ends of the MIMO link. Numerical results are presented in Section V, and the main results of the paper are summarized in Section VI.

II. Statistics of the Combined Fading Power

To obtain an expression for the AF, the statistics of either the combined SNR or combined fading power must be known. For a flat-fading MIMO diversity system with n_{T} transmit antennas and n_{R} receive antennas, the combined SNR (per symbol) may be written as [19]

$$\gamma_{\mathrm{c}} = \frac{P_{\mathrm{T}}\|\mathbf{H}\|_{\mathrm{F}}^2}{\sigma^2 n_{\mathrm{T}}} = \frac{P_{\mathrm{T}}}{\sigma^2 n_{\mathrm{T}}}\sum_{i=1}^{n_{\mathrm{R}}}\sum_{j=1}^{n_{\mathrm{T}}}\alpha_{ij}^2 \tag{2}$$

where $\|\mathbf{H}\|_{\mathrm{F}}^2$ represents the combined fading power (squared Frobenius norm of the channel matrix $\mathbf{H}$ [11]), and $\alpha_{ij}^2 = |h_{ij}|^2$ represents the fading power of a single narrowband channel between the jth transmitter and the ith receiver. From (2), it can be seen that the statistics of γ_{c} is governed by the statistics of $\|\mathbf{H}\|_{\mathrm{F}}^2$. For simplicity, just a single subscript n will be used to distinguish between the entries of the channel matrix,[3] and the squared Frobenius norm may then be expressed as $\|\mathbf{H}\|_{\mathrm{F}}^2 = \sum_{n=1}^{N}\alpha_n^2$, where $N = n_{\mathrm{T}}\cdot n_{\mathrm{R}}$ denotes the maximum number of channels in the MIMO channel. Viewing all the fading amplitudes in the set $\{\alpha_n\}_{n=1}^N$ as identically distributed Nakagami-m random variables (RVs) [5], the pdf of a single fading amplitude α_n is equal to [5]

$$p(\alpha_n) = \frac{2m^m\alpha_n^{2m-1}}{\Omega^m\Gamma(m)}\cdot \mathrm{e}^{-\frac{m\alpha_n^2}{\Omega}} \tag{3}$$

where $\Gamma(\cdot)$ denotes the gamma function,[4] $\Omega = \mathcal{E}\{\alpha_n^2\}$ denotes the average fading power, and m is the Nakagami fading parameter.

When the instantaneous fading amplitude α_n is distributed according to (3), the instantaneous fading power α_n^2 will follow a gamma distribution[5] with shape parameter m and scale parameter Ω/m. In short, $\alpha_n^2 \sim \mathcal{G}(m,(\Omega/m))$. Hence, $\|\mathbf{H}\|_{\mathrm{F}}^2$ represents a sum of N identically distributed possibly correlated gamma variates.

III. Amount of Fading

The CF of $\|\mathbf{H}\|_{\mathrm{F}}^2 \triangleq \alpha_{\mathrm{c}}^2$, representing a sum of N identically distributed correlated gamma variates, may be written compactly as [10]

$$\begin{aligned}\Phi_{\alpha_{\mathrm{c}}^2}(\jmath w) &= |\mathbf{I}_{N\times N} - \jmath w\mathbf{R}_{\mathrm{H}}|^{-m}\\ &= |\mathbf{I}_{N\times N} - \jmath w\Lambda|^{-m}\\ &= \prod_{n=1}^{N}(1-\jmath w\lambda_n)^{-m}\end{aligned} \tag{4}$$

where w is the variable of the transform domain, $|\cdot|$ denotes the determinant operator, $\mathbf{I}_{N\times N}$ is the identity matrix of size $N\times N$, and Λ is the diagonal eigenvalue matrix of the (complex) channel correlation matrix $\mathbf{R}_{\mathrm{H}} = \mathcal{E}\{\mathrm{vec}(\mathbf{H})\mathrm{vec}(\mathbf{H})^{\mathrm{H}}\}$, containing the set of eigenvalues $\{\lambda_n\}_{n=1}^N$.[6] Since $\mathbf{R}_{\mathrm{H}}$ is a Hermitian matrix, the existence of Λ is guaranteed by the spectral theorem [11, Th. 6.2].[7] The moments of an RV can be determined from its CF, according to [21, eq. (2-1-74)]

$$\mathcal{E}\left\{\alpha_{\mathrm{c}}^{2q}\right\} = \frac{1}{\jmath^q}\cdot\left.\frac{d^q\Phi_{\alpha_{\mathrm{c}}^2}(\jmath w)}{dw^q}\right|_{w=0}. \tag{5}$$

[3] $n = n(i,j) = n_{\mathrm{T}}\cdot(i-1)+j$ for $i\in[1,2,\ldots,n_{\mathrm{R}}]$ and $j\in[1,2,\ldots,n_{\mathrm{T}}]$.

[4] $\Gamma(z) = \int_0^\infty t^{z-1}\mathrm{e}^{-t}\mathrm{d}t, \mathbb{R}(z) > 0$ [20].

[5] $Y = \alpha_n^2$ follows a gamma distribution with shape parameter $a > 0$ and scale parameter $b > 0$, when the pdf of Y is given by $p_Y(y) = [y^{a-1}\mathrm{e}^{-y/b}]/[b^a\Gamma(a)]$. The short-hand notation $Y\sim\mathcal{G}(a,b)$ is used to denote that Y follows a gamma distribution with shape parameter a and scale parameter b.

[6] The superscript $(\cdot)^{\mathrm{H}}$ denotes the Hermitian transpose operator, and the $\mathrm{vec}(\cdot)$ operator stacks the individual columns of the argument matrix on top of each other, i.e., represents the matrix as a single-column vector [11, Sec. 9.3].

[7] In addition, this means that the algebraic multiplicity of eigenvalues in $\mathbf{R}_{\mathrm{H}}$ (the number of eigenvalues) is equal to the geometric multiplicity of eigenvalues (the size of the nullspace spanned by the eigenvectors). Thus, if $\mathbf{R}_{\mathrm{H}}$ is an $N\times N$ Hermitian matrix of rank $r < N$ (rank deficient), exactly $N-r$ of the eigenvalues of $\mathbf{R}_{\mathrm{H}}$ are equal to zero. This means that the effective number of terms contributing to the product in (4) (i.e., product terms different from 1), in general, is equal to r. For full-rank matrices, $r = N$. In this paper, it is assumed that $\mathbf{R}_{\mathrm{H}}$ is full rank.

Using logarithmic derivation [22], the first- and second-order derivatives of (4) may be expressed as

$$\Phi'_{\alpha_c^2}(jw) = \left[\sum_{n=1}^{N} \frac{jm\lambda_n}{1-jw\lambda_n}\right] \cdot \Phi_{\alpha_c^2}(jw) \quad (6)$$

$$\Phi''_{\alpha_c^2}(jw) = \left[\left(\sum_{n=1}^{N} \frac{jm\lambda_n}{1-jw\lambda_n}\right)^2 - \sum_{n=1}^{N} \frac{jm\lambda_n^2}{(1-jw\lambda_n)^2}\right] \cdot \Phi_{\alpha_c^2}(jw). \quad (7)$$

Using (5), the mean and variance of α_c^2 may be identified as

$$\mathcal{E}\left\{\alpha_c^2\right\} = m \cdot \sum_{n=1}^{N} \lambda_n \quad (8)$$

$$\mathrm{Var}\left\{\alpha_c^2\right\} = m \cdot \sum_{n=1}^{N} \lambda_n^2. \quad (9)$$

According to (1), the AF may now be expressed as $\mathrm{AF} = (\sum_{n=1}^{N} \lambda_n^2 / m \cdot (\sum_{n=1}^{N} \lambda_n)^2)$. For normalized average power on all channels, i.e., $\mathcal{E}\{\alpha_n^2\} = \Omega = 1$ for all $n \in [1, 2, \ldots, N]$, the channel correlation matrix $\mathbf{R}_{\mathrm{H}}$ will contain just 1s on the main diagonal. Since the sum of eigenvalues equals the sum of diagonal entries in a matrix [23, Ch. 5], the AF for identically distributed Nakagami-m fading channels may be written as

$$\mathrm{AF} = \frac{\mathrm{tr}(\Lambda^2)}{m \cdot N^2} \quad (10)$$

where $\mathrm{tr}(\cdot)$ is the matrix trace operator [11].

In the following, it is assumed that the correlation between the antennas at the transmitter is independent of the correlation between the antennas at the receiver. This separability assumption (Kronecker model) has been validated by some authors [14], [15], and has become quite popular, due to its analytical tractability. However, note that some authors recently have questioned its accuracy [16]–[18].

All the measurement campaigns on the Kronecker model referenced in this paper (both supporting and questioning the Kronecker model) have been conducted in NLOS scenarios. As a result, we have chosen to limit the analysis based on the Kronecker model to identically distributed Rayleigh fading channels. According to Appendix A, the AF may then be written as

$$\mathrm{AF} = \frac{\sum_{j=1}^{n_{\mathrm{T}}} \|\mathbf{t}_j\|^2 \sum_{i=1}^{n_{\mathrm{R}}} \|\mathbf{r}_i\|^2}{N^2} \quad (11)$$

where $\|\cdot\|^2$ denotes the squared Euclidean vector norm, and the vectors $\mathbf{t}_j$ and $\mathbf{r}_i$ denote rows j and i of the transmit and receive correlation matrices, respectively. Next, it is shown that the result in (11) may be simplified even further, when specific types of antenna correlation models are taken into account. For simplicity, results are presented for identical correlation models at the transmitter and the receiver. Results for other scenarios (i.e., nonidentical correlation models at the transmitter and the receiver) may also easily be obtained, although not presented in this paper.

1) Constant Correlation: For a constant correlation model, applicable for an array of three antennas placed on an equilateral triangle or for closely spaced antennas other than linear arrays [24], the correlation matrix $\mathbf{R}_Y$ of size $n_Y \times n_Y$ at the transmitter/receiver may be written as

$$\mathbf{R}_Y = \begin{bmatrix} 1 & y & \cdots & y \\ y^* & 1 & \cdots & y \\ \vdots & \vdots & \ddots & \vdots \\ y^* & y^* & \cdots & 1 \end{bmatrix} \quad (12)$$

where the subscript $Y \in \{T, R\}$ is used to distinguish between the transmit correlation matrix $\mathbf{R}_{\mathrm{T}}$ and the receive correlation matrix $\mathbf{R}_{\mathrm{R}}$, $y \in \{t, r\}$ denotes the complex transmit/receive correlation coefficient, and y^* denotes the complex conjugate of y. For constant correlation matrices at each end of the MIMO link, the AF (denoted $\mathrm{AF}_{\mathrm{con}}$) is expressible as

$$\mathrm{AF}_{\mathrm{con}} = \frac{\left[1 + |t|^2(n_{\mathrm{T}} - 1)\right]}{n_{\mathrm{T}}} \cdot \frac{\left[1 + |r|^2(n_{\mathrm{R}} - 1)\right]}{n_{\mathrm{R}}}. \quad (13)$$

An alternative expression to (13) is obtained by noting that the fading amplitude (envelope) correlation coefficient ρ^{env} and the fading power correlation coefficient ρ^{pow} may be assumed equal for all practical purposes [5], [25]. The correlation between fading envelopes can also be approximated by the squared amplitude of the complex correlations in $\mathbf{R}_{\mathrm{H}}$, referred to as power correlation coefficients [25], [26, eq. (35)]. Hence, $\rho_{ij}^{\mathrm{env}} \approx |\mathbf{R}_{\mathrm{H}}(i,j)|^2 = \rho_{ij}^{\mathrm{pow}}$, where $\mathbf{R}_{\mathrm{H}}(i,j)$ represents a single entry of the matrix at the ith row and jth column. As a result, (13) may also be written as

$$\mathrm{AF}_{\mathrm{con}} = \mathrm{AF}_{\mathrm{Tx}} \cdot \mathrm{AF}_{\mathrm{Rx}} = \frac{[1 + \rho_{\mathrm{t}}(n_{\mathrm{T}} - 1)]}{n_{\mathrm{T}}} \cdot \frac{[1 + \rho_{\mathrm{r}}(n_{\mathrm{R}} - 1)]}{n_{\mathrm{R}}} \quad (14)$$

where $\rho_{\mathrm{t}} = |t|^2$ and $\rho_{\mathrm{r}} = |r|^2$ represent the transmit and receive power correlation coefficients, respectively.

2) Circular Correlation: A circular correlation model may apply to antennas lying on a circle, or four antennas placed on a square [24]. The correlation matrix $\mathbf{R}_Y$ can then be written as

$$\mathbf{R}_Y = \begin{bmatrix} 1 & y_2 & y_3 & \cdots & y_{n_Y} \\ y_{n_Y}^* & 1 & y_2 & \cdots & y_{n_Y-1} \\ y_{n_Y-1}^* & y_{n_Y}^* & 1 & \cdots & y_{n_Y-2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ y_2^* & y_3^* & y_4^* & \cdots & 1 \end{bmatrix} \quad (15)$$

where $y_1 = 1$. Since every correlation matrix is Hermitian, it implies that $y_2 = y_{n_Y}^*$, $y_3 = y_{n_Y-1}^*, \ldots$. The AF with circular correlation at each end of the MIMO link (denoted $\mathrm{AF}_{\mathrm{cir}}$) can be expressed as

$$\mathrm{AF}_{\mathrm{cir}} = \frac{\sum_{j=1}^{n_{\mathrm{T}}} |t_j|^2}{n_{\mathrm{T}}} \cdot \frac{\sum_{i=1}^{n_{\mathrm{R}}} |r_i|^2}{n_{\mathrm{R}}}. \quad (16)$$

3) Exponential Correlation: An exponential correlation model may apply to an equispaced linear array of antenna elements [4]. The correlation matrix $\mathbf{R}_Y$ can be written as

$$\mathbf{R}_Y = \begin{bmatrix} 1 & y & \cdots & y^{n_Y-1} \\ y^* & 1 & \cdots & y^{n_Y-2} \\ \vdots & \vdots & \ddots & \vdots \\ (y^*)^{n_Y-1} & (y^*)^{n_Y-2} & \cdots & 1 \end{bmatrix}. \quad (17)$$

For exponential correlation on each side of the MIMO link, the AF (denoted $\mathrm{AF}_{\mathrm{exp}}$) can be expressed as

$$\mathrm{AF}_{\mathrm{exp}} = \frac{\left[1+2\sum_{j=1}^{n_\mathrm{T}-1}\left(1-\frac{j}{n_\mathrm{T}}\right)|t|^{2j}\right]}{n_\mathrm{T}} \times \frac{\left[1+2\sum_{i=1}^{n_\mathrm{R}-1}\left(1-\frac{i}{n_\mathrm{R}}\right)|r|^{2i}\right]}{n_\mathrm{R}}. \quad (18)$$

In [4], numerical results of the average SER and outage probability in an MRC system show that the constant correlation model suffers only a minor performance degradation when compared to the exponential correlation model, but the performance difference is more noticeable for a large number of diversity paths and for high correlation between the paths. As a result, the constant correlation model may be employed as a worst case correlation scenario, since the impact of correlation on system performance for other correlation models, typically, will be less severe. Hence, among the correlation profiles used in this paper, the impact of fading correlation is most severe (highest AF) for the constant correlation model.

IV. $\mathrm{AF}_{\mathrm{con}}$ AND ITS RELATION TO THE AVERAGE SER AT HIGH SNR

In this section, it is shown that for a constant correlation matrix at either side of an MIMO diversity system operating on identically distributed spatially correlated Rayleigh fading channels, the average SER at high SNR may be expressed in terms of $\mathrm{AF}_{\mathrm{con}}$. To this end, we will be invoking some recent results by Wang and Giannakis [1].

1) Approximate SER: In [1], the average SER P_E of an uncoded (or coded) system at high SNR is approximated by the expression

$$P_\mathrm{E} \approx (G_\mathrm{c}\cdot\overline{\gamma})^{-G_\mathrm{d}} \quad (19)$$

where G_c represents a coding gain, and G_d represents the diversity order. The diversity order determines the slope of the average SER curve versus the received average SNR $\overline{\gamma}$ at high SNR in a log–log scale, whereas the coding gain (in decibel) determines the shift of the curve in SNR relative to a benchmark SER curve given by $(\overline{\gamma}^{-G_\mathrm{d}})$.

Capitalizing on results in [24], the moment-generating function (MGF) $\mathcal{M}_{\gamma_\mathrm{c}}(s)$ of the combined SNR γ_c of an MIMO diversity system with identically distributed channels may be expressed as

$$\mathcal{M}_{\gamma_\mathrm{c}}(s) = \prod_{n=1}^{N}\left(1-\frac{s\overline{\gamma}}{n_\mathrm{T}}\lambda_n\right)^{-1} \quad (20)$$

where s is the variable of the transform domain, the set $\{\lambda_n\}_{n=1}^N$ denotes the eigenvalues of an $N\times N$ power correlation matrix $\mathbf{C}$ defined by

$$\mathbf{C} = \begin{bmatrix} 1 & \sqrt{\rho_{12}} & \cdots & \sqrt{\rho_{1N}} \\ \sqrt{\rho_{21}} & 1 & \cdots & \sqrt{\rho_{2N}} \\ \vdots & \vdots & \ddots & \vdots \\ \sqrt{\rho_{N1}} & \sqrt{\rho_{N2}} & \cdots & 1 \end{bmatrix} \quad (21)$$

and ρ_{ij} denotes the power correlation coefficient between the instantaneous SNR received on channels i and j, respectively.[8] When decoupled correlation properties are assumed, the power correlation matrix $\mathbf{C}$ may be written as a Kronecker product $\mathbf{C}=\mathbf{C}_\mathrm{Tx}\otimes\mathbf{C}_\mathrm{Rx}$ of the transmit and receive power correlation matrices, respectively.

Applying [1, Prop. 3] to the MGF in (20), the approximate (yet accurate) expression for the average SER P_E at high SNR may be written as[9]

$$P_\mathrm{E} \approx \frac{2^{N-1}b\Gamma\left(N+\frac{1}{2}\right)}{\sqrt{\pi}\Gamma(N+1)}\cdot\left(\frac{1}{k}\right)^N \quad (22)$$

where $b=[\det(\mathbf{C}_\mathrm{Tx}\otimes\mathbf{C}_\mathrm{Rx})]^{-1}(n_\mathrm{T}/\overline{\gamma})^N$, and k is a fixed code-dependent positive constant [1]. Using (19), the MIMO diversity system may then be characterized by the following diversity order and coding gain at high SNR

$$G_\mathrm{d} = N \quad (23)$$

$$G_\mathrm{c} = k\left(\frac{2^{N-1}p\Gamma\left(N+\frac{1}{2}\right)}{\sqrt{\pi}\Gamma(N+1)}\right)^{-\frac{1}{N}} \quad (24)$$

where $p=[\det(\mathbf{C}_\mathrm{Tx})^{n_\mathrm{R}}\cdot\det(\mathbf{C}_\mathrm{Rx})^{n_\mathrm{T}}]^{-1}\cdot n_\mathrm{T}^N$. Here, the following matrix identity has been utilized [11, eq. (9.8)]:

$$\det(\mathbf{C}_\mathrm{Tx}\otimes\mathbf{C}_\mathrm{Rx}) = \det(\mathbf{C}_\mathrm{Tx})^{n_\mathrm{R}}\cdot\det(\mathbf{C}_\mathrm{Rx})^{n_\mathrm{T}}. \quad (25)$$

For uncorrelated channels, the determinant of the Kronecker product in (25) is equal to 1, and thus, as pointed out in [1], correlation increases P_E by a factor $[\det(\mathbf{C}_\mathrm{Tx})^{n_\mathrm{R}}\,\cdot$

[8]The parameter ρ_{ij} represents a power correlation coefficient since $\rho_{ij} = \mathrm{Cov}(\gamma_i,\gamma_j)/\sqrt{\mathrm{Var}(\gamma_i)\mathrm{Var}(\gamma_j)} = \mathrm{Cov}(\alpha_i^2,\alpha_j^2)/\sqrt{\mathrm{Var}(\alpha_i^2)\mathrm{Var}(\alpha_j^2)}$, where α_i^2 denotes the instantaneous fading power received on channel i, α_j^2 denotes the instantaneous fading power received on channel j, and the expressions $\mathrm{Cov}(\cdot,\cdot)$ and $\mathrm{Var}(\cdot)$ denote the covariance and variance of its arguments, respectively.

[9]This result differs from the result presented in [1, eq. 10], since the MGF of the combined SNR has been utilized and not the MGF of the fading power as used in [1]. Hence, the average SNR $\overline{\gamma}$ is, in this paper, included in the factor b, whereas, in [1], it is not.

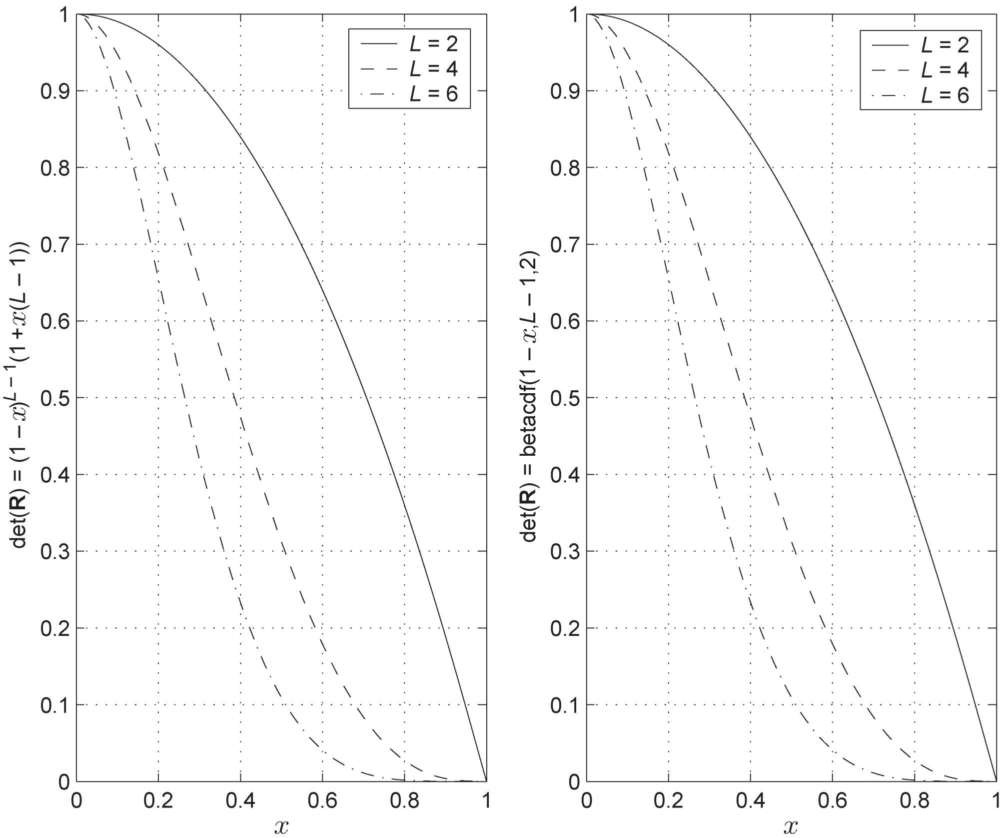

Fig. 1. Numerical comparison of closed-form expressions for the determinant of a constant correlation matrix $\mathbf{R}$. Left panel: $\det(\mathbf{R}) = (1-x)^{L-1}(1+x(L-1))$. Right panel: $\det(\mathbf{R}) = \mathrm{betacdf}(1-x, L-1, 2)$.

$\det(\mathbf{C}_{\mathrm{Rx}})^{n_{\mathrm{T}}}]^{-1} \geq 1$. A general increase in the number of transmit antennas will also increase P_{E} for power-limited systems, as long as the channel matrix is assumed unknown at the transmitter, since the available power at the transmitter must be divided between the transmit antennas. As a consequence, a penalty in error performance will be incurred [7]. If the channel matrix is known prior to transmission, beam forming could be utilized, and no error-performance penalty would be incurred.

If the correlation level is increased, a corresponding increase in the AF will be experienced at the diversity combiner output. Our goal now is to express P_{E} in (22) in terms of AF, to directly relate this performance measure to the effect of correlation on diversity system performance. In the following, an alternative expression for the determinant of a constant correlation matrix is utilized to express (22) in terms of $\mathrm{AF}_{\mathrm{con}}$.

When both $\mathbf{C}_{\mathrm{Rx}}$ and $\mathbf{C}_{\mathrm{Tx}}$ are constant correlation matrices, their determinants may be written in closed form as [27]

$$\det(\mathbf{C}_{\mathrm{Rx}}) = (1-\sqrt{\rho_r})^{n_{\mathrm{R}}-1}\left(1+\sqrt{\rho_r}(n_{\mathrm{R}}-1)\right) \quad (26)$$

$$\det(\mathbf{C}_{\mathrm{Tx}}) = (1-\sqrt{\rho_t})^{n_{\mathrm{T}}-1}\left(1+\sqrt{\rho_t}(n_{\mathrm{T}}-1)\right) \quad (27)$$

where both ρ_{r} and ρ_{t} are real numbers taking on values between zero and one. Since normalized average power has been assumed on all channels, we have observed (see Fig. 1) that the cumulative distribution function (cdf) of a beta distributed RV [28] can be used as an alternative closed-form expression for the determinants in (26) and (27). Hence, we may view $\sqrt{\rho_{\mathrm{r}}}$ and $\sqrt{\rho_{\mathrm{t}}}$ as beta distributed RVs (see Appendix B), which implies

$$\begin{aligned}\det(\mathbf{C}_{\mathrm{Rx}}) &= I\left(1-\sqrt{\frac{n_{\mathrm{R}}\cdot\mathrm{AF}_{\mathrm{Rx}}-1}{n_{\mathrm{R}}-1}}; n_{\mathrm{R}}-1, 2\right)\\ &\triangleq \mathrm{betacdf}(\mathrm{AF}_{\mathrm{Rx}}) \qquad (28)\\ \det(\mathbf{C}_{\mathrm{Tx}}) &= I\left(1-\sqrt{\frac{n_{\mathrm{T}}\cdot\mathrm{AF}_{\mathrm{Tx}}-1}{n_{\mathrm{T}}-1}}; n_{\mathrm{T}}-1, 2\right)\\ &\triangleq \mathrm{betacdf}(\mathrm{AF}_{\mathrm{Tx}}) \qquad (29)\end{aligned}$$

where $I(\cdot;\cdot,\cdot)$ denotes the regularized beta function.[10] Upon inserting (28) and (29) into (22), the average SER may now be expressed as

$$\begin{aligned}P_{\mathrm{E}} \approx{}& \frac{2^{N-1}\Gamma\left(N+\frac{1}{2}\right)}{\sqrt{\pi}\Gamma(N+1)}\cdot\left(\frac{n_{\mathrm{T}}}{k\overline{\gamma}}\right)^{N}\\ &\cdot[\mathrm{betacdf}(\mathrm{AF}_{\mathrm{Tx}})]^{-n_{\mathrm{R}}}\cdot[\mathrm{betacdf}(\mathrm{AF}_{\mathrm{Rx}})]^{-n_{\mathrm{T}}} \qquad (30)\end{aligned}$$

valid for $n_{\mathrm{T}} \geq 2$ and $n_{\mathrm{R}} \geq 2$.

2) Exact SER: To evaluate the result in (30), it is compared to an exact SER expression. As an example, a binary phase-shift

[10] http://mathworld.wolfram.com

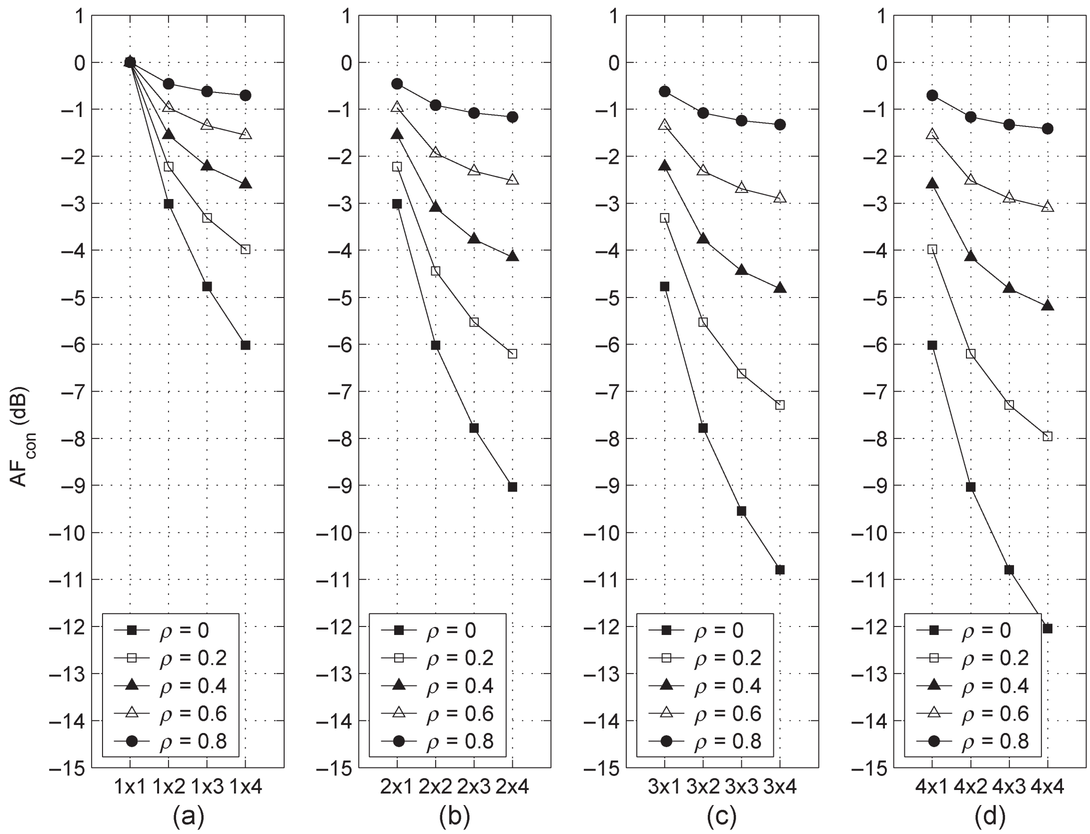

Fig. 2. $\mathrm{AF}_{\mathrm{con}}$ (in decibel) as a function of the power correlation coefficient $\rho = \rho_{\mathrm{t}} = \rho_{\mathrm{r}}$ for SIMO, MISO, and MIMO diversity systems $(n_{\mathrm{T}} \times n_{\mathrm{R}})$ operating on identically distributed Rayleigh fading channels: (a) $(1 \times n_{\mathrm{R}})$; (b) $(2 \times n_{\mathrm{R}})$; (c) $(3 \times n_{\mathrm{R}})$; (d) $(4 \times n_{\mathrm{R}})$.

keying (BPSK) modulation scheme will be utilized. Using a general M-ary PSK (M-PSK) modulation scheme as a starting point, an expression for a BPSK modulation scheme is later obtained by letting $M = 2$. According to [4, eq. (5.67)], the average SER performance of an M-PSK modulation scheme over a fading channel is expressible as

$$P_{\mathrm{E}} = \frac{1}{\pi} \int_{0}^{\frac{(M-1)\pi}{M}} \mathcal{M}_{\gamma}\left(-\frac{g_{\mathrm{psk}}}{\sin^2\theta}\right) \mathrm{d}\theta \qquad (31)$$

where M is equal to the number of symbols in the signal constellation, $g_{\mathrm{psk}} = \sin^2(\pi/M)$, and $\mathcal{M}_{\gamma}(\cdot)$ denotes the MGF of the received SNR. For an MIMO diversity system, the MGF of the combined SNR is given by (20), but due to the multiplicative property of eigenvalues in a Kronecker product, the expression in (20) may also be written as

$$\mathcal{M}_{\gamma_{\mathrm{c}}}(s) = \prod_{i=1}^{n_{\mathrm{R}}} \prod_{j=1}^{n_{\mathrm{T}}} \left(1 - \frac{s\overline{\gamma}}{n_{\mathrm{T}}} \lambda_i \lambda_j\right)^{-1} \qquad (32)$$

where the sets $\{\lambda_j\}_{j=1}^{n_{\mathrm{T}}}$ and $\{\lambda_i\}_{i=1}^{n_{\mathrm{R}}}$ denote the eigenvalues of the transmit and receive power correlation matrices $\mathbf{C}_{\mathrm{Tx}}$ and $\mathbf{C}_{\mathrm{Rx}}$, respectively.[11] Upon inserting (32) into (31), replacing the variable s in (32) with the factor $-g_{\mathrm{psk}}/\sin^2\theta$, and assuming that both $\mathbf{C}_{\mathrm{Tx}}$ and $\mathbf{C}_{\mathrm{Rx}}$ are constant correlation matrices, the eigenvalues of the two matrices can be expressed in closed form [27]. After some manipulation, (31) reduces to

$$\begin{aligned} P_{\mathrm{E}} = \frac{1}{\pi} \int_{0}^{u} & [1 + g\lambda_{\mathrm{t},1}\lambda_{\mathrm{r},1}]^{-(n_{\mathrm{R}}-1)(n_{\mathrm{T}}-1)} \\ & \times [1 + g\lambda_{\mathrm{r},1}\lambda_{\mathrm{t},2}]^{-(n_{\mathrm{R}}-1)} \\ & \times [1 + g\lambda_{\mathrm{t},1}\lambda_{\mathrm{r},2}]^{-(n_{\mathrm{T}}-1)} \\ & \times [1 + g\lambda_{\mathrm{t},2}\lambda_{\mathrm{r},2}]^{-1} \mathrm{d}\theta \end{aligned} \qquad (33)$$

where $u = [(M-1)\pi]/M$, $\lambda_{\mathrm{t},1} = 1 - \sqrt{\rho_{\mathrm{t}}}$, $\lambda_{\mathrm{r},1} = 1 - \sqrt{\rho_{\mathrm{r}}}$, $\lambda_{\mathrm{t},2} = 1 + \sqrt{\rho_{\mathrm{t}}}(n_{\mathrm{T}} - 1)$, $\lambda_{\mathrm{r},2} = 1 + \sqrt{\rho_{\mathrm{r}}}(n_{\mathrm{R}} - 1)$, and $g = \overline{\gamma} g_{\mathrm{psk}}/n_{\mathrm{T}} \sin^2\theta$. By letting $M = 2$ (BPSK), the exact SER expression in (33) can be compared to the approximate SER expression in (30) with $k = 2$ [1].

In [1], the coding gain G_{c} was defined as the (left) shift of the average SER curve relative to the benchmark curve $(\overline{\gamma}^{-G_{\mathrm{d}}})$. To quantify the impact of increased AF in G_{c} for a fixed modulation scheme, the benchmark curve utilized in this paper is given by the approximate average SER curve at high SNR realized with uncorrelated channels. For correlated channels, the average SER curve is then visible as a (right) shifted version

[11]For $n_{\mathrm{T}} = 1$, the MGF in (32) reduces the MGF in [4, eq. (9.173)], valid for a single-input multiple-output (SIMO) system.

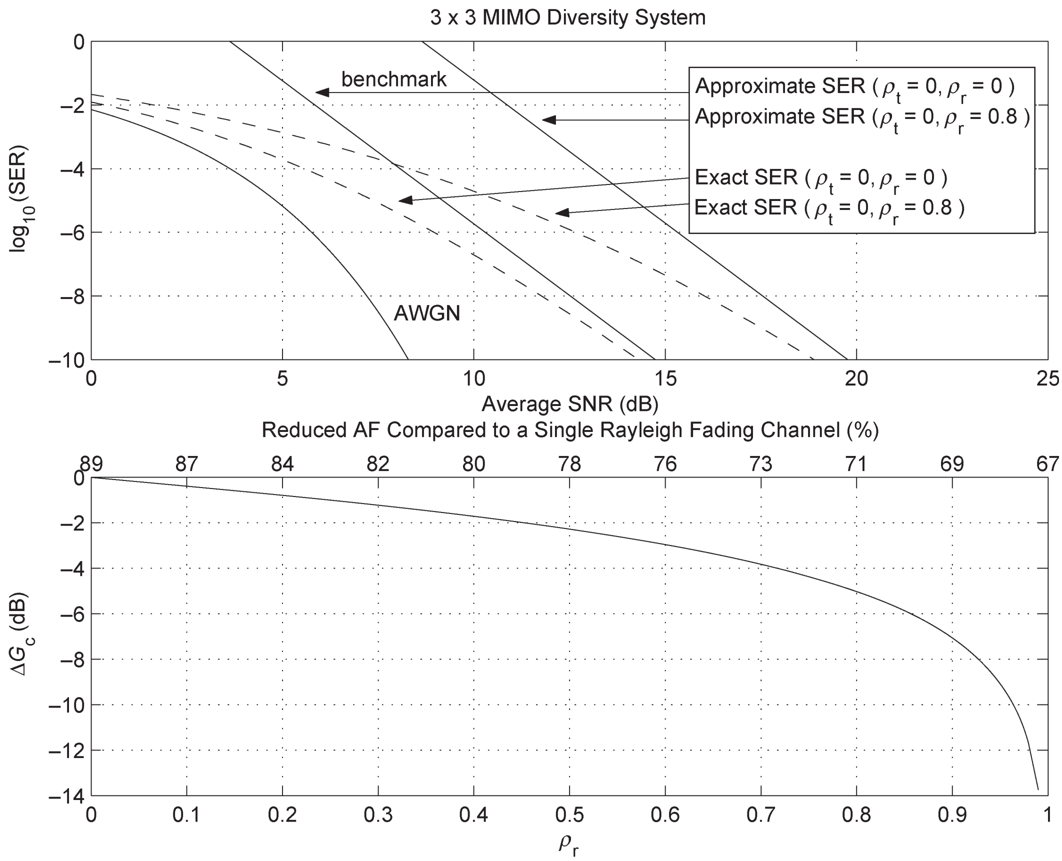

Fig. 3. Top panel: Exact average SER (dashed lines) and approximate average SER at high SNR (solid lines) for a 3 × 3 MIMO diversity system operating on identically distributed Rayleigh fading channels (constant correlation models at each end of the MIMO link). A BPSK modulation scheme is utilized. Bottom panel: Relative (right) shift (in decibel) of the SER benchmark curve presented in the top panel as a function of ρ_{r} when $\rho_{\mathrm{t}} = 0$. Using the top x axis as a reference, the AF realized by the 3 × 3 MIMO diversity system is compared (in percentage) to the AF of a single Rayleigh fading channel (100% reduction represents the nonfading AWGN channel).

of the benchmark curve, and the relative shift represents the loss in coding gain due to correlation between the diversity branches (increased AF).[12] The relative loss in coding gain ΔG_{c} (in decibel) may be expressed as

$$\Delta G_{\mathrm{c}} = 10 \cdot \left(\frac{n_{\mathrm{R}} \cdot \log\left[\det(\mathbf{C}_{\mathrm{Tx}})\right] + n_{\mathrm{T}} \cdot \log\left[\det(\mathbf{C}_{\mathrm{Rx}})\right]}{N} \right) \tag{34}$$

or equivalently, see (35) at the bottom of the next page, which is valid when $n_{\mathrm{T}} \geq 2$ and $n_{\mathrm{R}} \geq 2$.

V. Numerical Results

In this section, some numerical examples of the results derived in this paper are presented. Since the impact of correlation is most noticeable for the constant correlation model, the results are limited to MIMO systems operating on identically distributed Rayleigh fading channels with constant correlation models at each end of the MIMO link. Related results for other correlation models will typically be less severe.

[12]Note that with the current benchmark definition, the placement of the benchmark will differ depending on the size of the MIMO diversity system (total number of antennas).

In Fig. 2, $\mathrm{AF}_{\mathrm{con}}$ in (14) is depicted as a function of the power correlation coefficient $\rho = \rho_{\mathrm{t}} = \rho_{\mathrm{r}}$ for both SIMO, multiple-input single-output (MISO), and MIMO diversity systems. By comparing the various panels in the figure, it can be seen that $\mathrm{AF}_{\mathrm{con}}$ is progressively reduced when the number of antennas is increased either at the transmitter or the receiver. As expected, the reduction is largest for uncorrelated antennas. In Fig. 3 and Fig. 4, the approximate average SER in (30) is compared to the exact result in (33) for a 3 × 3 MIMO diversity system and a BPSK modulation scheme. In Fig. 3 (top panel), it can be seen that approximate average SER curve for correlated channels at high SNR can be obtained as a (right) shifted version of the approximate average SER curve for uncorrelated channels (benchmark). This is visualized for $\rho_{\mathrm{r}} = 0.8$. In the bottom panel, ΔG_{c} is depicted as a function of both ρ_{r} (lower x axis) and $\mathrm{AF}_{\mathrm{con}}$ (upper x axis). Using the lower x axis as reference, $\rho_{\mathrm{r}} = 0.8$ amounts to a right shift of -5 dB from the benchmark curve, which is in agreement with the observed difference of the curves in the top panel. According to the upper x axis, the AF is reduced by 71%, compared to a single Rayleigh fading channel, when $\rho_{\mathrm{r}} = 0.8$ (and $\rho_{\mathrm{t}} = 0$). For uncorrelated channels ($\rho_{\mathrm{t}} = \rho_{\mathrm{r}} = 0$), a 3 × 3 MIMO diversity system reduces the AF by 89%, compared to a single Rayleigh fading channel, which is equivalent to ninth-order diversity.

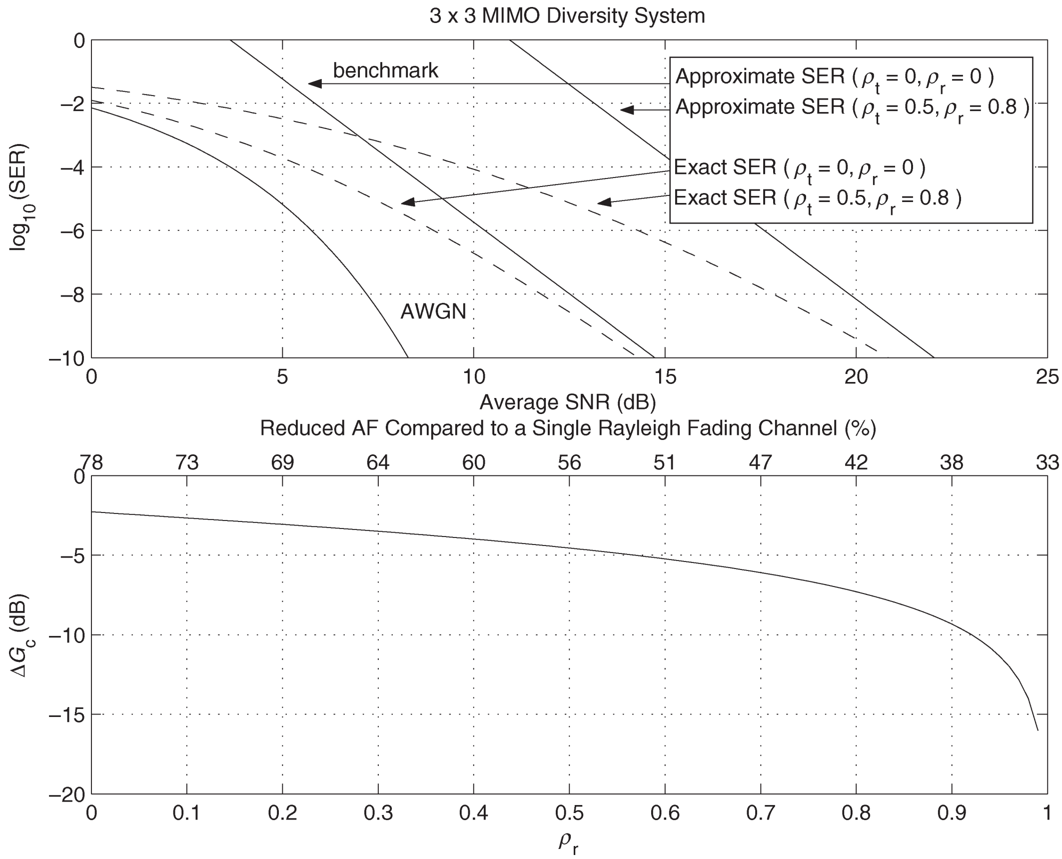

Fig. 4. Top panel: Exact average SER (dashed lines) and approximate average SER at high SNR (solid lines) for a 3 × 3 MIMO diversity system operating on identically distributed Rayleigh fading channels (constant correlation models at each end of the MIMO link). A BPSK modulation scheme is utilized. Bottom panel: Relative (right) shift (in decibel) of the SER benchmark curve presented in the top panel as a function of ρ_{r} when $\rho_{\mathrm{t}} = 0.5$. Using the top x axis as reference, the AF realized by the 3 × 3 MIMO diversity system is compared (in percentage) to the AF of a single Rayleigh fading channel (100% reduction represents the nonfading AWGN channel).

In Fig. 4 (top panel), the same set of curves as depicted in Fig. 3 (top panel) are presented, but this time, when $\rho_{\mathrm{t}} = 0.5$. Once again, excellent agreement between the exact and approximate curves at high SNR is observed. From Fig. 4 (bottom panel), it can be seen that ΔG_{c} has increased from -5 to -7.3 dB when $\rho_{\mathrm{r}} = 0.8$, due to $\rho_{\mathrm{t}} = 0.5$ at the transmitter. This corresponds to a 42% reduction in the AF compared to a single Rayleigh fading channel.

VI. Conclusion

A closed-form expression for the AF in a multiple-input multiple-output (MIMO) diversity system operating on identically distributed spatially correlated Nakagami-m fading channels has been presented. For the Kronecker model, the amount of fading (AF) has been presented for identically distributed Rayleigh fading channels and different correlation models. Capitalizing on recent results in [1], it has been shown that the approximate average symbol error rate (SER) at high signal-to-noise ratio (SNR) for an MIMO diversity system based on the Kronecker model can be expressed as a function of the AF, when a constant correlation model is assumed.

Appendix A
Proof of the Amount of Fading Expression in (11)

Using a mathematical model assuming that the transmit and receive correlation properties are decoupled in an MIMO diversity system operating on identically distributed spatially correlated Nakagami-m channels, the AF may be expressed as

$$\mathrm{AF} = \frac{\sum_{j=1}^{n_{\mathrm{T}}} \|\mathbf{t}_j\|^2 \sum_{i=1}^{n_{\mathrm{R}}} \|\mathbf{r}_i\|^2}{m \cdot N^2} \tag{36}$$

where $\|\cdot\|^2$ denotes the squared Euclidean vector norm, m is the common fading parameter of all the channels, n_{T} denotes the number of transmit antennas, n_{R} denotes the number of

$$\Delta G_{\mathrm{c}} = 10 \cdot \left(\frac{n_{\mathrm{R}} \cdot \log\left[\mathrm{betacdf}(\mathrm{AF}_{\mathrm{Tx}})\right] + n_{\mathrm{T}} \cdot \log\left[\mathrm{betacdf}(\mathrm{AF}_{\mathrm{Rx}})\right]}{N} \right) \tag{35}$$

receive antennas, and $N = n_T \cdot n_R$. The vectors $\mathbf{t}_j$ and $\mathbf{r}_i$ denote rows j and i of the transmit and receive correlation matrices, respectively.

Proof: Let Λ represent the diagonal eigenvalue matrix of the complex correlation matrix $\mathbf{R}_H$. Assuming decoupled correlation properties at the receiver and transmitter, the correlation matrix $\mathbf{R}_H$ may be written as $\mathbf{R}_H = \mathbf{R}_{Tx} \otimes \mathbf{R}_{Rx}$, where the symbol $\otimes$ denotes the Kronecker product, and the matrices $\mathbf{R}_{Tx}$ and $\mathbf{R}_{Rx}$ denote the decoupled transmit and receive correlation matrices, respectively. Due to the multiplicative properties of the eigenvalues of matrices in a Kronecker product [11, Th. 9.1], the nominator in (10) may be decomposed into the product $\mathrm{tr}(\Lambda_{Tx}^2)\mathrm{tr}(\Lambda_{Rx}^2)$, where Λ_{Tx} and Λ_{Rx} are the diagonal eigenvalue matrices of $\mathbf{R}_{Tx}$ and $\mathbf{R}_{Rx}$, respectively. The following chain of equalities can then be obtained for the transmitter part

$$\mathrm{tr}\left(\Lambda_{Tx}^2\right) = \mathrm{tr}\left(\mathbf{R}_{Tx}^2\right) = \sum_{j=1}^{n_T} (\mathbf{R}_{Tx}\mathbf{R}_{Tx})_{jj} = \sum_{j=1}^{n_T}\sum_{k=1}^{n_T} t_{jk} \cdot t_{kj} \overset{(i)}{=} \sum_{j=1}^{n_T}\sum_{k=1}^{n_T} t_{jk} \cdot t_{jk}^* = \sum_{j=1}^{n_T} \|\mathbf{t}_j\|^2 \tag{37}$$

where t_{jk} denotes a single entry at the jth row and kth column of $\mathbf{R}_{Tx}$, and $\mathbf{t}_j$ denotes the jth row of $\mathbf{R}_{Tx}$. The equality (i) is true, since a correlation matrix is Hermitian symmetric. A similar chain of equalities can be obtained for the receiver part, resulting in

$$\mathrm{tr}\left(\Lambda_{Rx}^2\right) = \mathrm{tr}\left(\mathbf{R}_{Rx}^2\right) = \sum_{i=1}^{n_R} \|\mathbf{r}_i\|^2 \tag{38}$$

where $\mathbf{r}_i$ denotes the ith row of $\mathbf{R}_{Rx}$. ■

Appendix B
An Alternative Expression for the Determinant of a Constant Correlation Matrix

A constant correlation matrix $\mathbf{R}$ of size $L \times L$ is called an Lth order intraclass correlation matrix if it has the following structure [4]

$$\mathbf{R} = \begin{bmatrix} a & b & b & \cdots & b \\ b & a & b & \cdots & b \\ b & b & a & \cdots & b \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ b & b & b & \cdots & a \end{bmatrix} \tag{39}$$

with $b \geq -a/(L-1)$. By normalizing this matrix, 1s are obtained on the main diagonal and the factor b/a, off the main diagonal. Denoting $x = b/a$, and assuming that $x \in [0, 1]$, a closed-form expression for the determinant of $\mathbf{R}$ may be written as [27]

$$\det(\mathbf{R}) = (1-x)^{L-1}\left(1 + x(L-1)\right). \tag{40}$$

Due to the normalized main diagonal, and the fact that the variable x is confined to the finite interval range $x \in [0, 1]$, we have observed (depicted in Fig. 1) that the cdf of a beta-distributed RV can be used in an alternative expression for the determinant given in (40). The alternative expression may be derived as follows.

The pdf of a beta-distributed RV with free parameters $\alpha > 0$ and $\beta > 0$ is given by [28]

$$\mathrm{betapdf}(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}(1-x)^{\beta-1}x^{\alpha-1}. \tag{41}$$

The cdf can then be expressed as

$$\mathrm{betacdf}(x;\alpha,\beta) = \int_0^x \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}(1-u)^{\beta-1}u^{\alpha-1}\mathrm{d}u. \tag{42}$$

Evaluating the function $f(x,\alpha,\beta) = 1 - \mathrm{betacdf}(x;\alpha,\beta)$ when $\alpha = 2$, the following result is obtained

$$\begin{aligned} f(x,2,\beta) &= 1 - \mathrm{betacdf}(x;2,\beta) \\ &= 1 - \int_0^x \frac{\Gamma(\beta+2)}{\Gamma(\beta)}(1-u)^{\beta-1}u\mathrm{d}u \\ &= 1 - \frac{\Gamma(\beta+2)}{\Gamma(\beta)}\left[\frac{1-(1-x)^{\beta}(1+\beta x)}{\beta(\beta+1)}\right] \\ &= (1-x)^{\beta}(1+\beta x). \end{aligned} \tag{43}$$

By comparison, (43) and (40) represent identical expressions by selecting $\beta = L - 1$. Hence, the determinant of a constant correlation matrix $\mathbf{R}$ can be written as

$$\det(\mathbf{R}) = 1 - \mathrm{betacdf}(x;2,L-1). \tag{44}$$

Since the parameter β must be larger than zero, this expression is valid only when $L \geq 2$. The cdf of a beta-distributed RV is equal to the regularized beta function $I(\cdot;\cdot,\cdot)$,[10] and the determinant can also be written as

$$\begin{aligned} \det(\mathbf{R}) &= 1 - \mathrm{betacdf}(x;2,L-1) \\ &= 1 - I(x;2,L-1) \\ &= I(1-x;L-1,2) \end{aligned} \tag{45}$$

where $I(x;\alpha,\beta) = 1 - I(1-x;\beta,\alpha)$.[13] Hence

$$\det(\mathbf{R}) = \mathrm{betacdf}(1-x;L-1,2). \tag{46}$$

[13] http://functions.wolfram.com

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