ADAPTIVE CODED MODULATION FOR WIRELESS OFDM CHANNELS: UPPER BOUNDS ON AVERAGE SPECTRAL EFFICIENCY AND INFLUENCE OF IMPERFECT CHANNEL KNOWLEDGE

K. J. Hole, *G. E. Øien,* and *H. Holm*

Norwegian University of Science and Technology (NTNU)

ABSTRACT

An adaptive coded modulation (ACM) scheme for a frequency-flat fading channel utilizes a set of channel codes with different spectral efficiencies. A feedback channel between the decoder and encoder makes it possible for the encoder to switch adaptively between these codes based on channel state information (CSI) fed back from the decoder. An orthogonal frequency division multiplexing (OFDM) system may utilize ACM on its subchannels to obtain a large average spectral efficiency (ASE), measured in transmitted information bits per second per unit bandwidth. In this paper we present a new upper bound on the ASE of OFDM/ACM systems. We also discuss how CSI estimation errors will influence the performance of an ACM system, and limit its possible region of operation with regard to channel signal-to-noise ratios and terminal velocities.

1. INTRODUCTION

Many authors have studied **adaptive coded modulation** (ACM) schemes for spectrally efficient transmission on channels with slowly varying frequency-flat fading (see Hole and Øien (1) and the references therein). An ACM scheme utilizes a set of channel codes (e.g., trellis codes) designed for additive white Gaussian noise (AWGN) channels of different qualities. A feedback channel between the decoder and encoder makes it possible for the encoder to switch adaptively between these codes based on channel state information (CSI) fed back from the decoder. In this way, the ACM scheme achieves a large **average spectral efficiency** (ASE), measured in transmitted information bits per second per unit bandwidth. The reader is referred to the WWW page http://www.tele.ntnu.no/projects/beats for a complete overview of the authors' publications and research results on ACM.

The bandwidth for which the fading is frequency-flat may be small in some communication systems, but **orthogonal frequency division multiplexing** (OFDM) can be used to decompose a broadband frequency-selective fading channel into a set of parallel narrowband frequency-flat fading channels, cf. Nee and Prasad (2). ACM may be utilized on each individual narrowband subchannel to achieve broadband OFDM transmission with large total ASE. For an ACM system, it is vital to obtain reliable CSI, which is used both at the transmitter for adapting the modulation and coding scheme, and at the receiver for symbol detection/decoding. The CSI fed back to the transmitter will typically be noisy predictive estimates of the **channel signal-to-noise ratio** (CSNR) at the times of adaptation. Such estimates can be derived e.g. by the aid of deterministic **pilot symbols** (PSs) periodically inserted into the information stream. CSI estimation errors will influence the performance of an ACM system, for a range of channel qualities and mobility constraints.

The remainder of the paper is organized as follows. Section 2 introduces a model of an ACM system on an arbitrary flat-fading channel. A previously obtained upper bound on the ASE of ACM is also presented. Section 3 extends the ACM model to obtain an OFDM/ACM model with one or more receive antennas, while Section 4 develops a new upper bound on the total ASE of this model. The obtained bound is plotted for indoor OFDM/ACM channels. Section 5 discusses how CSI estimation errors will affect the performance of an ACM system, assuming **maximum a posteriori**-optimal PS-aided channel prediction, Øien *et al.* (3).

2. FLAT-FADING CHANNELS AND ACM

The initial discrete-time system model consists of a transmitter and a receiver, each with one single-element antenna, communicating over a wireless channel with frequency-flat fading. The multipath fading is assumed to remain nearly constant over hundreds of channel symbols. PSs are sent repeatedly over the channel to ensure that the receiver is able to fully compensate for the amplitude and phase variations in the received signal, i.e., we assume ideal channel estimation and coherent detection. Hence, we may model the fading amplitude, $\alpha(t)$, as a (wide-sense stationary and ergodic) stochastic variable with *real* values.

Let the transmitted signal have complex baseband representation $x(t)$ at time index $t\in\{0,1,2,\ldots\}$. The received baseband signal is then given by $y(t)=\alpha(t)\cdot x(t)+n(t)$ where $n(t)$ denotes complex AWGN. The real and imaginary parts of the noise are statistically independent, both with variance $(N_0B)/2$ where N_0 [W/Hz] is the total one-sided noise power spectral density and B [Hz] is the one-sided channel bandwidth.

Denote the average transmit power by S [W]. The instantaneous received CSNR is represented by $\gamma(t)=\alpha(t)^2S/(N_0B)$. (In the sequel we omit the time reference t and refer to α and γ.). We shall assume that the probability density function of γ is a known and *continuous* function on $[0,\infty)$. The CSNR has expectation $E[\gamma] = \gamma_{av} = \Omega S/(N_0 B)$, where $E[\alpha^2] = \Omega$ is the average received power gain. The model contains a noiseless and zero-delay *feedback channel* from the receiver to the transmitter, such that both the transmitter and receiver have perfect knowledge of the instantaneous received CSNR γ at all times.

We now let N ($<\infty$) quantization levels (or **fading regions**) represent the time-varying received CSNR γ. When ACM is used, one trellis code designed to combat AWGN is assigned to each fading region. The N fading regions are defined by the thresholds $0<\gamma_1<\gamma_2<\ldots<\gamma_{N+1}=\infty$. Code n, $n=1,2,\ldots,N$, is utilized every time the instantaneous received CSNR γ falls in region n, i.e., when $\gamma_n\leq\gamma<\gamma_{n+1}$. The system is designed such that the BER never exceeds a given target maximum, denoted BER_0, for any CSNR γ. Fading region $n=1$ represents the smallest values of γ for which information is transmitted. When $\gamma<\gamma_1$ the channel quality is too bad to successfully transmit any information with the available codes. Details on how to choose thresholds $\{\gamma_n\}$ to achieve $BER \leq BER_0$ for a given set of N codes can be found in Hole *et al.* (4).

2.1 Known Upper Bound on ASE of ACM

Let $4\leq M_1<M_2<\ldots<M_N$ denote the number of symbols in N digital modulation (e.g., quadrature amplitude modulation (QAM)) constellations of growing size, and let code n be based on a constellation with M_n symbols. For some small fixed $G\in\{1,2,\ldots\}$, the encoder for code n accepts $G\cdot \log_2(M_n)-c$ information bits at each time index $k=G\cdot t\in\{0,G,2G,\ldots\}$ and generates $G\cdot\log_2(M_n)$ coded bits, $1\leq c<G\cdot\log_2(M_n)$. The coded bits specify G modulation symbols in the nth constellation. These symbols are transmitted at time indices $k,k+1,\ldots,k+G-1$. The G two-dimensional symbols can be viewed as one $2G$-dimensional symbol, and for this reason the code is said to be a $2G$-dimensional trellis code.

Assuming Nyquist signaling, the time used to transmit one modulation symbol is $T_s=1/B$ [s]. Since the number of information bits per modulation symbol is $\log_2(M_n)-c/G$, the information rate of code n is $R_n=(\log_2(M_n)-c/G)/T_s$ [bits/s], and the spectral efficiency is $R_n/B=\log_2(M_n)-c/G$ [bits/s/Hz]. Consequently, the ASE of all N codes is

$$\begin{aligned}\mathrm{ASE}(\{\gamma_n\}) &= \textstyle\sum_{n=1,2,\ldots,N} R_n/B\cdot \mathrm{P}(\gamma_n,\gamma_{n+1}) \\ &=\textstyle\sum_{n=1,2,\ldots,N} (\log_2(M_n)-c/G)\cdot \mathrm{P}(\gamma_n,\gamma_{n+1}) \quad \text{[bits/s/Hz]}, \quad (1)\end{aligned}$$

where $\mathrm{P}(\gamma_n,\gamma_{n+1})$ is the probability of the instantaneous CSNR γ falling in fading region n.

In principle, infinitely many sets of $2G$-dimensional codes are available. We restrict ourselves to code sets which contain N codes and consider all such sets having the same parameters c, G, and $\{M_n\}$. Hole (5) has proved the following result, which yields an upper bound on the ASE when inserted in Equation (1).

Theorem 1: For given c, G, and $\{M_n\}$, the ASE defined by Equation (1) is maximized for the minimum possible thresholds

$$\gamma_n^* = M_n/2^{c/G} - 1, \quad n=1,2,\ldots,N.$$

We remark that Theorem 1 is valid for all targets BER_0.

3. BROADBAND OFDM/ACM TRANSMISSION

Frequency division multiplexing (FDM) can be used to split a broadband channel with intersymbol interference (ISI) into many parallel narrowband channels, each having little or no ISI. The OFDM technique uses FDM with subchannels (subcarriers) whose frequency spectra overlap. To make it possible for the receiver to recover each subchannel signal from the received OFDM signal, the generated subchannel signals must be mutually orthogonal over a symbol period. This is obtained by letting the spacing between any two adjacent subcarriers be $1/T_s$ where T_s is the symbol period associated with each subcarrier (see Nee and Prasad (2) for details).

A block diagram of an OFDM system using ACM is depicted in Figure 1.

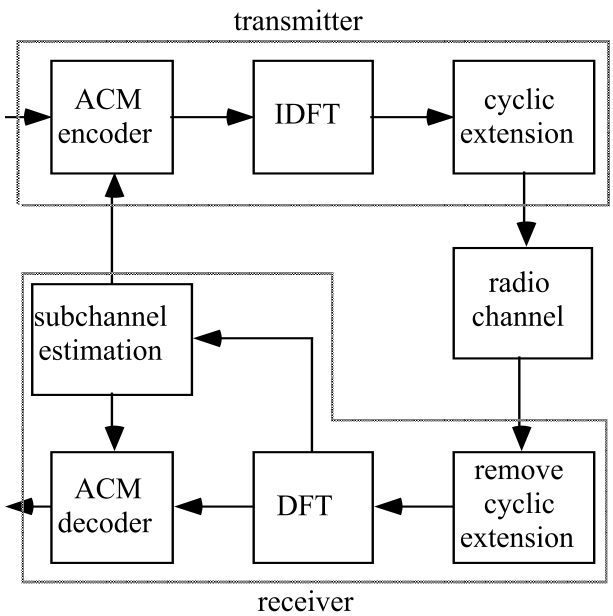

Figure 1: OFDM/ACM system model.

The ACM encoder accepts information bits from the source and generates modulation symbols which are shifted into an inverse discrete Fourier transform (IDFT) to produce OFDM symbols. Prior to channel transmission, the OFDM symbols are fed to a guard time insertion circuit to reduce the influence of intracarrier ISI and intercarrier interference (ICI). The guard interval is obtained by preceding each OFDM symbol by a cyclical extension of the signal itself.

Let T_g denote the guard interval duration, and note that the OFDM symbol time is equal to the modulation symbol time T_s on a subchannel. Since the insertion of the guard interval will reduce the data throughput, T_g is usually no more than $T_s/4$. We assume that T_g is large enough to remove all ICI and ISI.

The OFDM symbols are transmitted over a broadband radio channel, viewed as Q parallel narrowband subchannels. The receiver first removes the guard intervals from the received noisy OFDM symbols; a discrete Fourier transform (DFT) then produces noisy modulation symbols that are shifted into the ACM decoder to generate estimates of the transmitted information bits. We assume that the transmitter has only one transmit antenna, while the receiver may contain an antenna array with H (≥ 1) receive antennas (not shown). It is also assumed that the fading processes on the H antenna branches are statistically independent.

3.1 Details of ACM Codec

We now study the encoding process in more detail. Starting with subchannel q=1 and ending with subchannel q=Q, the ACM encoder accepts $i_q(p)$ information bits for each subchannel at time index $p \in \{0,1,2,\ldots\}$ (different time scale from previously used time scale), where $i_q(p)$ is determined by the instantaneous received CSNR on subchannel q. The number of bits $i_q(p)$ will vary over time and $i_q(p)$ need not be equal on all subchannels. Using $i_q(p)$ information bits, the ACM encoder generates G consecutive modulation symbols for subchannel q.

The $Q \cdot G$ modulation symbols generated by the ACM encoder for all subchannels at time index p are divided into G blocks of Q symbols, one symbol from each subchannel. Each block is thus a "frequency domain" vector. One block at a time is fed into the IDFT which transforms the block of Q modulation symbols from the frequency domain to the time domain to generate one OFDM symbol. A total of G consecutive OFDM symbols are thus generated for every $\sum_{q=1,2,\ldots,Q} i_q(p)$ input bits. Observe that while the IDFT always operates on fixed length blocks of Q modulation symbols, the total number of *information bits* $\sum_{q=1,2,\ldots,Q} i_q(p)$ represented by the G OFDM symbols varies over time, leading to variable rate transmission.

For each transmitted OFDM signal, H noisy and independently faded versions are obtained at the receive antennas. The guard intervals are first removed; one DFT for each antenna then produces Q noisy modulation symbols; one symbol per subchannel. For each of the Q subchannels, the H antenna outputs are to be combined by a **maximal ratio combining** (MRC) unit included in the receiver (not shown).The MRC unit receives a total of $H \cdot Q$ noisy modulation symbols every time an OFDM symbol is transmitted.

Starting with subchannel q=1 and ending with subchannel q=Q, the H received, noisy and independently faded modulation symbols belonging to subchannel q are co-phased, weighted by their respective magnitudes, and then summed by the MRC unit, to produce an estimate of the transmitted modulation symbol on subchannel q. The receiver buffers a total of $G \cdot Q$ such estimates for each block of G transmitted OFDM symbols. The ACM decoder accepts G modulation symbol estimates at a time and produces $\sum_{q=1,2,\ldots,Q} i_q(p)$ estimates of the transmitted information bits.

3.2 OFDM Subchannel Model

Denote by $\alpha(h,q)$ the fading amplitude on the hth antenna branch on the qth OFDM subchannel, h=1,2,…,H, q=1,2,…,Q. Since the same average transmit power S is used on all subchannels, the instantaneous received CSNR on antenna branch h becomes $\gamma(h,q) = \alpha(h,q)^2 \mathrm{S}/(N_0 B)$. The average power gain $E[\alpha(h,q)^2] = \Omega_q$ is assumed to be the same for all H antenna branches on subchannel q.

Since the instantaneous received CSNR after MRC is equal to $\gamma(q)=\sum_{h=1,2,\ldots,H} \gamma(h,q)$, the average received CSNR is

$$\gamma_{\mathrm{av}}(q)=E[\gamma(q)]=\sum_{h=1,2,\ldots,H} E[\gamma(h,q]=H\Omega_q S/(N_0 B). \quad (2)$$

It is convenient to view the H antenna branches with MRC as a single OFDM subchannel with instantaneous received CSNR $\gamma(q)$. This channel is modeled by the flat-fading channel described in Section 2.

4. UPPER BOUND ON ASE OF OFDM/ACM

To upper bound the ASE of an OFDM system utilizing ACM on its subchannels, we first need to determine the total system bandwidth. Each subchannel has Nyquist bandwidth B=1/T_s [Hz]. Moreover, each subchannel has excess bandwidth β/T_s [Hz] for some rolloff factor $0 \leq \beta < 1$. Because of the frequency spectra overlapping, only half of the two outer subchannels' excess bandwidths contribute to the total bandwidth of the

OFDM system, giving a total information signal bandwidth of $W_{total}=(Q+\beta)/T_s$ [Hz] for Q sub-channels.

The total duration of the OFDM interval including the guard interval is $T_{total}=T_s+T_g$. The average number of information bits per interval T_{total} is obtained by summing the average number of information bits per modulation symbol for each subchannel,

$$<\#bits> = \sum_{q=1,\ldots,Q} \sum_{n=1,\ldots,N} (\log_2(M_n)-c/G)\, P_q(\gamma_n,\gamma_{n+1}) \quad \text{[bits/interval]}, \quad (3)$$

where $P_q(\gamma_n,\gamma_{n+1})$ is the probability that the instantaneous CSNR γ_q on subchannel q falls in fading region n. In general, this probability depends on the average received CSNR $\gamma_{av}(q)$ defined by Equation (2). The total ASE of the OFDM/ACM system is now given by

$$ASE_{OFDM}(\{\gamma_n\}) = <\#bits>/(T_{total}\cdot W_{total}) \quad \text{[bits/s/Hz]}. \quad (4)$$

Theorem 2: For given c, G, and $\{M_n\}$, the total ASE defined by Equations (3) and (4) is maximized for the minimum thresholds γ_n* (given in Theorem 1).

Proof: Since the subchannels are mutually orthogonal, they do not mutually interfere and the total ASE in Equation (4) is optimized by maximizing the ASE on the individual subchannels. The result now follows from Theorem 1.

As an example, we consider indoor wireless OFDM/ACM based on N=11 two-dimensional constellations. The system parameters considered are the same as in the HIPERLAN/2 standard for transmission in the 5 GHz frequency band (Q=52, B=312.5kHz, β=.025, T_s=3.2μs, T_g=800ns, W_{total}=16.26MHz). Since the total OFDM channel bandwidth (<20 MHz) is small compared to the carrier frequency (= 5 GHz), it is reasonable to assume that the instantaneous received CSNR $\gamma(h,q)$ on all $H\cdot Q$ antenna branches have the same distribution and the same received average power gain, i.e., $\Omega_q=\Omega$ for all q.

Let the fading amplitudes on all antenna branches have a *Nakagami* distribution with parameter m=1, Stüber (6). The instantaneous antenna branch CSNR γ_q then has a gamma distribution, see Alouini and Goldsmith (7), and we can calculate the probability $P_q(\gamma_n,\gamma_{n+1})$. The total ASE defined by Equations (3) and (4) is plotted in Figure 2 for $H\in\{1,2,4\}$ receive antennas and G=c=1, using the minimum thresholds $\{\gamma_n*\}$. The constellation sizes assumed are $M_n = 2^{n+1}$; n=1,…,11.

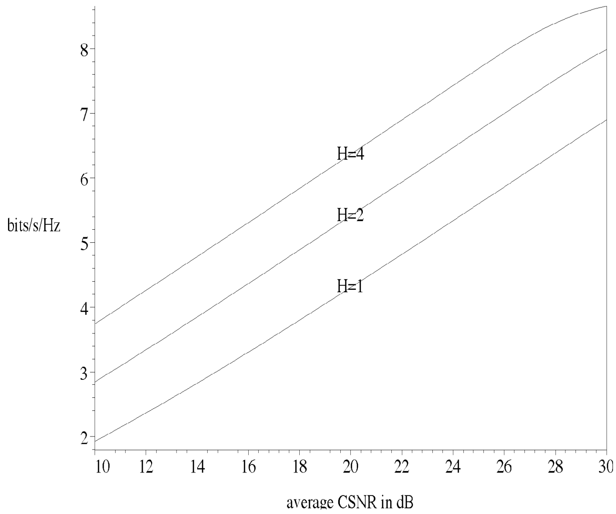

Figure 2: Total ASE of OFDM/ACM/QAM system for thresholds $\{\gamma_n*\}$ and $H\in\{1,2,4\}$ receive antennas. The subchannels all have the same average CSNR.

5. INFLUENCE OF CSI ESTIMATION ERRORS

Unfortunately, the idealized assumption of *perfect channel knowledge at the transmitter side* may not hold exactly in real situations. In practice, the time-varying fading amplitude must be estimated at the receiver—e.g., deduced from the channel's influence on deterministically known and periodically transmitted *pilot symbols*—and sent back to the transmitter via a feedback return channel. Three mechanisms may contribute to the information thus received at the transmitter being incorrect: i) Nonzero feedback delay in the return channel, implying that the forward channel may have changed by the time the transmitter adapts its rate according to the received estimate; ii) Additive channel noise, making for a noisy estimate regardless of the feedback delay; and iii) Bit errors occurring in the return channel. Here, we focus on i) and ii), still assuming that the return channel is noiseless.

In a recent doctoral thesis by Holm (8), the effect of imperfect channel knowledge on an ACM scheme was analyzed on mobile flat-fading channels with MRC receiver diversity. Among the performance measures analyzed were average BER and average spectral efficiency (ASE). Pilot symbol assisted *maximum a posteriori*- (MAP)-optimal channel prediction was used to periodically estimate the CSNR, which was then fed back to the transmitter. A closed-form BER approximation was derived, expressible as a function of the predictor coefficients, the expected CSNR, and the feedback delay normalized with respect to the Doppler period. Since the Doppler period is inversely proportional to the velocity of relative transmitter-receiver movement, this problem formulation also provides insight into ACM mobility constraints.

Figure 3 shows contours of the closed-form BER expression at the target BER, as a function of the expected CSNR and the normalized feedback delay, for the QAM-based example ACM system of (4). The contours are depicted for various choices of the pilot symbol period L and the number H of receive antennas. The acceptable operational region of the system (where

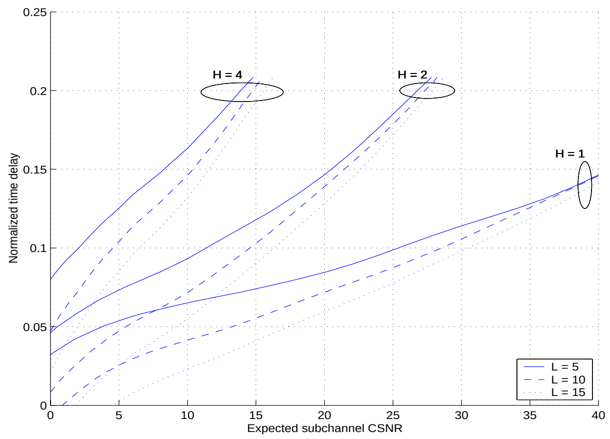

Figure 3: BER contour lines at $BER_0 = 10^{-4}$ as a function of expected CSNR and normalized feedback delay, for the QAM-based example ACM system of (4).

BER ≤ BER_0) is to the right of each contour, whereas to the left, the BER requirements are not fulfilled. The results indicate that the successful implementation of ACM might be feasible over a wide range of average CSNRs and terminal velocities (Doppler shifts), also when only imperfect channel information is available.

However, it must be noted that the above results do not take into account the following facts: The channel model statistics needed to optimize the predictor must also be estimated from noisy observations. This might lead to a *mismatch* between the actual statistics and the predictor, and deteriorating effects on the BER and the possible region of operation in the "average CSNR–terminal velocity" domain. Also, in practice, the system complexity must be limited by limiting the predictor filter order. The lower the order, the lower the possible prediction gain and, theoretically, the worse the achievable quality of the channel estimate.

5.1 Mismatch between Predictor and Channel Model

One way to analyze the effects of channel parameter mismatch is to assume that a MAP-optimized predictor of unlimited complexity is still used, but that it has been optimized using channel parameter estimates that have a certain percentage of error in them. This approach was taken in (9), on which the results of this section are based. In this case, the analytical tools derived in (8) can still be applied to obtain a complete system performance analysis. In particular, the dependency of the BER and ASE performance on average CSNR and terminal mobility (feedback delay normalized with respect to the Doppler period) is studied.

Again applied to the QAM-based example ACM system of (4), the analysis gives increased insight into the conditions under which an ACM system with mismatched channel prediction can operate successfully (here this again means keeping the average BER below 10^{-4}—while simultaneously providing a large ASE). One conclusion is that the example system is more robust towards estimation errors in the average CSNR than it is towards estimation errors in the Doppler period (or, equivalently, normalized feedback delay). As an example, Figure 4 shows the same contours as depicted in Figure 3, but this time with a 10 % positive error in the normalized feedback delay. Note that the use of MRC considerably decreases the sensitivity towards normalized delay estimation errors.

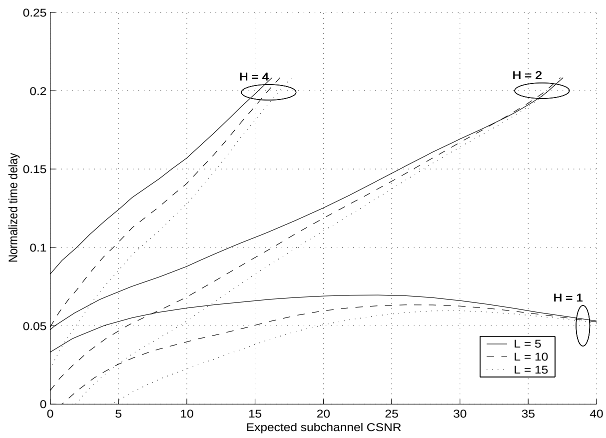

Figure 4: Corresponding contours to Figure 3, but with +10 % error in estimated normalized feedback delay.

REFERENCES

1. Hole, K. J., and Øien, G. E., 2001, IEEE Trans. Veh. Technol., 50, 205–222.
2. Nee, R., and Prasad, R., 2000, "OFDM for Wireless Multimedia Communications," Artech House, Boston, USA.
3. Øien, G. E., Holm, H., and Hole, K. J., 2002, Proc. European Signal Processing Conference (EUSIPCO), Toulouse, France.
4. Hole, K. J., Holm, H., and Øien, G. E., 2000, IEEE J. Select. Areas Commun., 18, 1153–1158.
5. Hole, K. J., 2002, Telektronikk (special issue on "Information theory and its applications"), 98, 47-52, ISSN 0085-7130, published by Telenor.
6. Stüber, G. L., 2001, "Principles of Mobile Communication," Kluwer Academic Publishers, Norwell, MA, USA.
7. Alouini, M.-S., and Goldsmith, A. J., 1997, Proc. 47th IEEE Vehicular Technology Conference (VTC'97), Phoenix, Arizona, 358-362.
8. Holm, H., Adaptive Coded Modulation Performance and Channel Estimation Tools on Flat Fading Channels, dr.ing. thesis, NTNU, March 2002.
9. Hansen, R. K., Channel Prediction for Adaptive Coded Modulation with Uncertainty in the Channel Model Parameters, siv.ing. thesis (in Norwegian), NTNU, June 2002.

IEE

Click here to return to the Session Menu