Analysis of Adaptive Coded Modulation with Antenna Diversity and Feedback Delay

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Overview

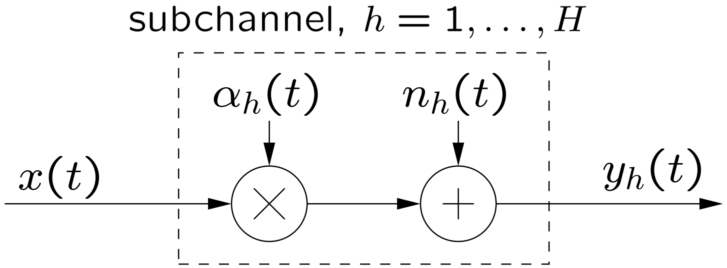
- Motivation
- Channel Model
- Adaptive Coded Modulation
- BER Degradation Due to Delay
- Example
- Conclusions

Motivation

- Multipath fading: Channel signal-to-noise ratio (CNR) varies with time.
- Adaptive coded modulation: splitting the CNR range.
  - When CNR is low: transmitting with low rate code.
  - Take advantage of favorable conditions by using high rate code.
- Requires transmitter knowledge about channel state information (CSI).
  $\Rightarrow$ Need feedback channel!
- Delay in feedback channel will affect BER performance.
- Increasing the no. of receive antennas might make the system more robust.

Channel Model

- Use H receive antennas; each branch is modeled as a flat Rayleigh-fading channel.



- Transmit power $E[x^2(t)] = S$,
  noise power $E[n_h^2(t)] = N_0 B$.
  $\Rightarrow$ Instantaneous received CNR is

$$\gamma_h(t) = \frac{\alpha_h^2(t) \cdot S}{N_0 B}$$

- RMS value of fading envelope assumed independent of h
  - $E[\alpha_h^2(t)] = \Omega$
  - thus, $E[\gamma_h(t)] = \overline{\gamma}_h = \frac{\Omega \cdot S}{N_0 B}$

Maximal Ratio Combining

- Receiver implements maximal ratio combining (MRC), assuming statistically independent branch signals.

- CNR of output of MRC:

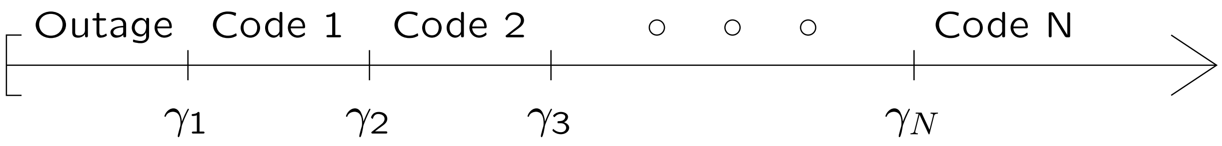
$$\gamma = \sum_{h=1}^{H} \gamma_h, \qquad \overline{\gamma} = H\overline{\gamma}_h,$$

governed by following PDF:

$$p_\gamma(\gamma) = \left(\frac{H}{\overline{\gamma}}\right)^H \frac{\gamma^{H-1}}{(H-1)!} \exp\left(-H\frac{\gamma}{\overline{\gamma}}\right), \quad \gamma \geq 0\,, \qquad (1)$$

- An Adaptive Coding Scheme for fading channels consists of N codes, each designed to combat AWGN.

- CNR range divided into $N+1$ fading regions:



- Thresholds determined such that BER for each code is lower than predetermined BER_0 under idealized assumptions.

Constructing the Adaptive Coding Scheme

- $M_1 < M_2 < \cdots < M_N$ denotes the number of symbols in N QAM constellation.

- Code n accepts $L \cdot \log_2(M_n) - 1$ info bits, generates $L \cdot \log_2(M_n)$ coded bits that specify L symbols.

- L 2-dimensional symbols $\Longleftrightarrow$ one $2L$-dimensional symbol.

- Model for BER-CNR-relationship for code n (on AWGN channel):

$$\text{BER}_n = \begin{cases} a_n \cdot \exp\left(-\frac{b_n \gamma}{M_n}\right), & \gamma \geq \gamma_n^\star \\ 1/2, & \gamma < \gamma_n^\star \end{cases}$$

($\gamma_n^\star = \frac{\ln(2a_n)M_n}{b_n}$ is smallest CNR ensuring that BER is at most 1/2.)

- Thresholds are calculated from target BER_0,

$$\gamma_n = \frac{M_n K_n}{b_n}, \quad n = 1, 2, \cdots, N$$

$K_n = -\ln(\text{BER}_0/a_n)$.

BER Degradation Due to Delay

- CSI estimated at receiver at time t.
- In practical system: CSI available to transmitter will be delayed so that it arrives at time $t + \tau$.
- Thus, transmitter uses code corresponding to (outdated) CNR estimate γ, while actual CNR is γ_τ.
- BER as function of γ_τ:

$$\mathrm{BER}_n^\tau(\gamma_\tau \mid \gamma) = \begin{cases} a_n \cdot \exp\left(-\frac{b_n \gamma_\tau}{M_n(\gamma)}\right), & \gamma_\tau \geq \gamma_n^\star \\ 1/2\,, & \gamma_\tau < \gamma_n^\star \end{cases}$$

BER Degradation Continued . . .

- Still assuming perfect γ estimated at receiver, e.g. no estimation error at time t, but $\tau > 0$.

- It can then be shown that average BER in fading region n is given by:

$$\langle \text{BER} \rangle_n^{\tau} = \mathcal{I}1(n) - \mathcal{I}2(n)$$

with the term $\mathcal{I}1(n)$ given by:

$$\mathcal{I}1(n) = \frac{a_n}{(H-1)!} \left(\frac{H}{\overline{\gamma}} \right)^H \frac{\Gamma(H, \beta_n \gamma_n) - \Gamma(H, \beta_n \gamma_{n+1})}{(\omega_n)^H}$$

where $\beta_n = \frac{H}{\overline{\gamma}} + \frac{H \rho \, b_n}{H M_n + \overline{\gamma}(1-\rho) b_n}$ and $\omega_n = \frac{H}{\overline{\gamma}} + \frac{b_n}{M_n}$,

with power correlation coefficient

$$\rho = \frac{E[\alpha_\tau^2 \alpha^2] - \Omega}{\sqrt{\sigma^2 \sigma^2}} = J_0^2(2\pi f_D \tau).$$

- $\mathcal{I}2(n)$ can be viewed as a correction term due to the BER upper bound in BER-CNR relationship.

BER Degradation Continued . . . . . .

- Average total BER for delay τ is then

$$\langle \text{BER} \rangle^{\tau} = \frac{\sum_{n=1}^{N} i_n [\mathcal{I}1(n) - \mathcal{I}2(n)]}{\sum_{n=1}^{N} i_n P_n}$$

  where $i_n = \log_2(M_n) - 1/L$ is the no. of info bits per modulation symbol in code n.

- $P_n = \Pr(\gamma_n \leq \gamma < \gamma_{n+1})$

Example

- Adaptive codec, eight 4-dimensional trellis codes, $M_n \in \{4, 8, 16, \ldots, 512\}$.

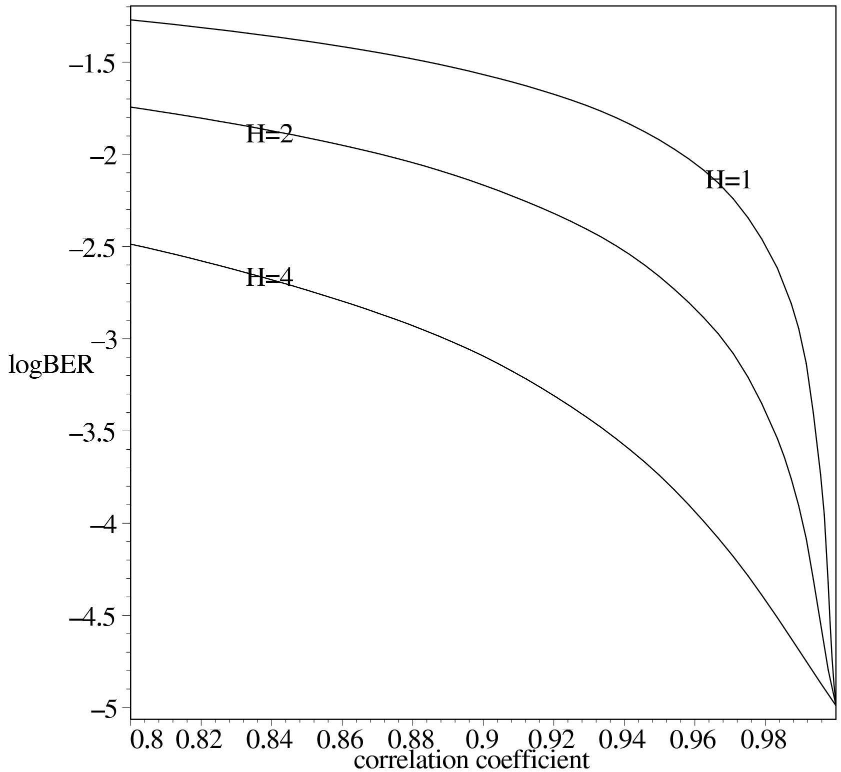
- Individual codes simulated for AWGN, obtaining a_n and b_n parameters by LMS curve fitting.

- Thresholds γ_n determined for target $\text{BER}_0 = 10^{-4}$.

- Assume $\overline{\gamma}_h = 20\,\text{dB}$.

Example . . .

- Base-10 logarithm of $\langle \text{BER} \rangle^{\tau}$ plotted as function of ρ for $H = 1, 2, 4$.



Conclusion

- Nonzero feedback delay will degrade BER performance of an adaptive coded modulation system.
- Degradation can be mitigated by using MRC.
- Still, the results suggest that adaptive (coded) modulation is best suited for moderate mobility (pedestrian speed).