Adaptive coded modulation with imperfect channel state information: System design and performance analysis aspects

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Abstract—**An adaptive coded modulation system designed for perfect channel knowledge, but operating based on noisy channel prediction, is described and analyzed with respect to error robustness and spectral efficiency. A way to improve the system design to obtain a flexible trade-off between spectral efficiency and error robustness is then suggested.**

I. Introduction

An adaptive coded modulation (ACM) [1–3] system utilizes a set of codes with different rates and thus different error-protecting properties. Each code is optimized to guarantee a certain bit error rate (BER) within a range of signal-to-noise ratios (SNRs). At specific time instants, an estimate of the instantaneous SNR is utilized to decide the highest-rate code that can be used while still fulfilling the target BER requirement. The system thus compensates for periods with low SNR by transmitting at a low rate, while transmitting at a high rate when the SNR is favorable. This way, a significant overall gain in *average spectral efficiency* (ASE), measured in bits/s/Hz, can be achieved compared to fixed-rate systems. In ACM systems, the transmitter needs accurate and updated information on the instantaneous channel quality, or SNR. In practical implementations, the channel state information (CSI) must be estimated at the receiver and provided to the transmitter through a feedback channel.

There are several potential sources of error in this approach: errors in the CSI estimation due to noise; delay in the feedback channel; and errors in the feedback channel. Errors in the feedback channel will not be considered here. We assume that the channel estimation is done by inserting deterministic *pilot symbols* periodically into the transmitted stream, similar to Cavers' pilot symbol assisted modulation (PSAM) scheme [4, 5]. The estimation process, the feedback channel, and the transmission scheme will result in some time delay before the estimated CSI can be utilized; we will refer to this as *feedback delay*. Since the CSI is delayed in the feedback channel, it is advantageous to make it an estimate of the future SNR at the time the SNR will be utilized, i.e., a *prediction* of the SNR.

II. System Model

We consider a wireless system perturbed by flat Rayleigh fading, see Fig. 1. At the receiver, *maximum ratio combining* (MRC) of H independent antenna elements is used to form the received signal. Perfect coherent detection is assumed. Denoting the transmitted complex baseband signal at time k by $x(k)$,

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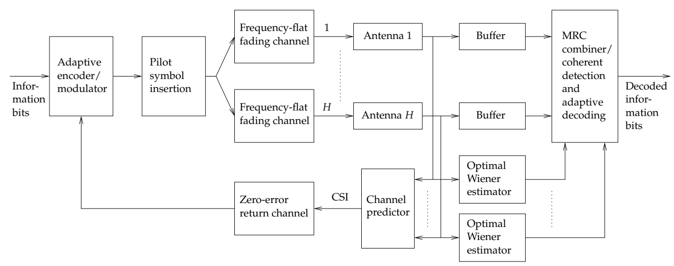


Fig. 1. ACM system with pilot symbol assisted channel estimation (for coherent detection purposes) and prediction (for transmitter adaptation purposes).

the received signal after transmission on the hth subchannel is $y_h(k) = z_h(k) \cdot x(k) + n_h(k)$, where $z_h(k)$ is the *fading amplitude*, a complex-valued Gaussian random variable with zero mean, and $n_h(k)$ represents complex-valued additive white Gaussian noise (AWGN). $x(k)$ is the information signal, except for at every L'th time instant, where a pilot symbol is transmitted. The magnitude of L must be decided as a trade-off between adequate sampling of the fading process (small L) and high spectral efficiency (large L). We assume that a fixed average transmit power P is used both for information and pilot symbols.

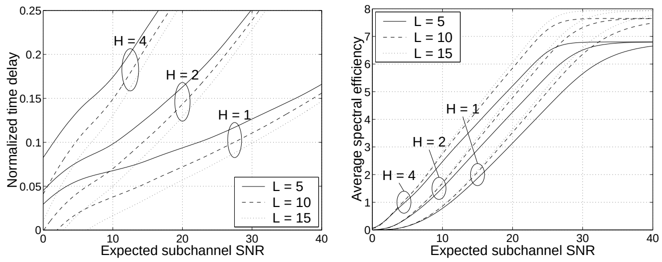
The system is designed to obtain a target BER, denoted BER_0. Code n $(n = 1, \ldots, N)$ is analyzed in order to find the lowest SNR γ_n at which it can guarantee a BER $\leq \mathrm{BER}_0$. Selecting the proper code for transmission is equivalent to deciding which region $[\gamma_n, \gamma_{n+1}\rangle$ the instantaneous SNR falls into.

III. Channel Prediction

At the pilot symbol time instants $i - kL$, a memoryless maximum-likelihood (ML) estimate of $z_h(i - kL)$ is first obtained: $\tilde{z}_h(i - kL) = z_h(i - kL) + \frac{n_h(i-kL)}{\sqrt{P}}$, where $\sqrt{P}$ is the magnitude of the pilot symbol. Using vector notation, the predicted fading amplitude can then be written as $\widehat{z}_h = \mathbf{f}^{\mathrm{H}}_{j,h}\tilde{\mathbf{z}}_{i,h}$, where $\mathbf{f}_{j,h}$ is a vector containing the predictor coefficients optimized for predicting the SNR at time j in the future, and $\tilde{\mathbf{z}}_{i,h}$ is a vector containing ML estimates up until time i. In our analysis, we have used a maximum a posteriori-optimal (MAP-optimal) predictor to benchmark performance [3]. Now, the SNR on subchannel h $(h = 1, \ldots, H)$ can be estimated, e.g. as $\widehat{\gamma}_h = \frac{|\widehat{z}_h| P}{N_0 B}$, where N_0 is the one-sided power spectral density of the AWGN and B is the transmission bandwidth. The *overall* predicted SNR after MRC is $\widehat{\gamma} = \sum_{h=1}^{H} \widehat{\gamma}_h$.

IV. System Analysis with MAP-Optimal Prediction

An ACM transmission system exhibits several important performance merits: The BER is the probability that a transmitted



(a) (b)

Fig. 2. (a): Regions for which system performance is acceptable, plotted for pilot symbol spacing $L = 5$, 10, 15, and for $H = 1$, 2, and 4 receive antennas. The curves indicate the largest delay that is allowed in order to achieve BER $\leq$ BER_0 for a given expected SNR. Thus, acceptable performance results when the point specified by a SNR/delay combination is below and to the right of the curve for system parameters L and H. (b): ASE as a function of expected subchannel SNR, plotted for various L and H, for a normalized delay of 0.25.

bit is received in error. The ASE is an indicator of the attainable information rate of the system per unit bandwidth. Lastly, the probability that reliable transmission is not feasible, the *probability of no transmission*, is denoted P_{out}. If the instantaneous SNR falls below the range where the lowest-rate code can guarantee BER $\leq$ BER_0, the information sequence is buffered instead of transmitted. In [3, 6], it is demonstrated how closed-form expressions for BER, ASE, and P_{out} can be obtained. These expressions are too involved to state here; hence we will concentrate on visualizing the results. The trellis code set of [3] is used as an example. Fig. 2 (a) shows BER contours at BER $=$ BER_0 as a function of expected subchannel SNR $\overline{\gamma}_h = \frac{E[|z_h|^2]P}{N_0 B}$ and normalized feedback delay $f_D T$, where f_D is the Doppler shift [Hz] and T is delay in seconds. The results are shown for various H and L. Corresponding results for the ASE are shown in Fig. 2 (b).

The assumption of MAP-optimal prediction indicates that the channel model parameters are known exactly. In practice they must be estimated from noisy observations. However, initial analysis of this phenomenon indicates that the impact on system performance is not dramatic [7].

V. ACM Re-Design for Improved Trade-Off Between Error Robustness and Spectral Efficiency

A weakness of the ACM design methodology used so far is that the system parameters $\gamma_1, \ldots, \gamma_N$ are still designed as if the available CSI is perfect. We then analyze the effects of using such a system when this assumption does not hold. It would certainly be preferable, from an error robustness point of view, to be able to directly design the SNR interval limits to take the imperfect CSI into account. We will therefore introduce an idea which enables the system designer to do exactly this. For the class of channels under discussion, it allows us to choose any desired compromise between the "perfect CSI" system described so far, and the "almost perfect error robustness" system obtained if we demand that the system should conform to the target BER requirements for almost every expected SNR–normalized delay combination.

The idea is as follows: An unacceptably high instantaneous BER will occur if the true SNR (γ) happens to lie in a lower-indexed interval than the predicted SNR ($\widehat{\gamma}$). Thus, in essence, what we need to control is the probability $P(\gamma < \gamma^*|\widehat{\gamma})$, i.e., we need to be able to specify the $\widehat{\gamma}$ that gives any desired value of this probability for a given γ^*. Say, for example, that we want to find the smallest value $\widehat{\gamma}_n$ of $\widehat{\gamma}$ securing that the probability of γ going below γ_n is equal to ϵ for any $n = 1, \ldots, N$. We then need, for every $n = 1, \ldots, N$, to solve the following equation with respect to $\widehat{\gamma}_n$: $P(\gamma > \gamma_n|\widehat{\gamma} = \widehat{\gamma}_n) = 1 - \epsilon$.

Some manipulations yield the following result:

$$Q_H\left(\frac{\widehat{\gamma}_n H}{\overline{\gamma}(1-\rho)}, \frac{\gamma_n H}{\overline{\gamma}(1-\rho)}\right) = 1 - \epsilon, \quad (1)$$

where Q_H is the *generalized Marcum-Q function* [8], and where ρ is the correlation between the true and the predicted SNR, γ and $\widehat{\gamma}$.

In other words, the interval border $\widehat{\gamma}_n$ for the predicted SNR, which is to be used as a decision boundary for the choice between codes $n-1$ and n, must be chosen such that Equation 1 is fulfilled. This must be done for all n, if we are to ensure that the probability of choosing a code with too high rate is ϵ. We can do this for any $\epsilon \geq 0$. As ϵ is chosen smaller, though, the more emphasis is put on error robustness, and less on spectral efficiency, as the value of $\widehat{\gamma}_n$ will be increased as ϵ is decreased. The chosen ϵ should be a trade-off between the desire for a high ASE and the ability of the system to successfully provide a satisfactory BER performance under typical operating conditions.

VI. Conclusions

For the example ACM system under study, on a mobile Rayleigh fading channel, imperfect CSI lead to limited acceptable operation region in the expected SNR–Doppler shift domain. However, it is still possible to obtain large ASE gains and a reliable BER performance from an ACM system, for a relatively wide range of SNRs and Doppler shifts. Also, the use of receive diversity increases system robustness. The system parameters can also be re-designed to provide a flexible trade-off between ASE and operation region limitation. This is currently a topic for more research, as is the study of channel predictor mismatch and limited predictor complexity.

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