LDPC CODED ADAPTIVE MULTILEVEL MODULATION FOR SLOWLY VARYING RAYLEIGH-FADING CHANNELS

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ABSTRACT

Adaptive coded modulation is a promising idea for bandwidth-efficient transmission on time-varying wireless channels. On power limited Additive White Gaussian Noise (AWGN) channels, low-density parity-check (LDPC) codes are a class of error-control codes which have demonstrated impressive error-correcting qualities, under some conditions performing even better than turbo codes. We propose a combination of these two principles: A rate-adaptive LDPC-coded modulation scheme designed for slowly varying flat fading channels. The scheme's theoretical performance is evaluated by calculating its bandwidth efficiency for a given target bit error rate (BER). Finally we extend the scheme to an effectively zero-error hybrid FEC-ARQ system, without degrading bandwidth efficiency.

Keywords: Rate-adaptive modulation, error control coding, flat-fading channels, LDPC codes, ARQ.

1. INCREASING WIRELESS SPECTRAL EFFICIENCY BY LINK ADAPTION

Future generations of wireless communication systems will have to include some form of *link adaptation*, i.e., *rate-adaptive transmission schemes*, in order to increase the throughput of information to the levels demanded by high-quality multimedia services. This is because wireless and mobile channels exhibit time-varying channel-signal-to-noise ratio (CSNR), usually caused by multipath reflections, shadowing, and the relative movement between transmitter and receiver in a mobile system. Upcoming Wireless Local Area Network (WLAN) standards such as Hiperlan/2 are already beginning to explore link adaptation as a means towards increased spectral efficiency, by means of punctured convolutional codes.

One variant of the link adaption idea is to transmit with constant power, but to adapt channel coding and modulation in such a way that highly bandwidth efficient multilevel coding is used when the CSNR is high, and less bandwidth efficient coding (more redundancy) is used when the CSNR is low. This assumes that the transmitter has access to the *channel state information* (CSI) at any time. For slowly varying channels (those with limited mobility and thus limited Doppler shifts) the CSNR can be regarded as nearly constant for shorter periods of time (the *coherence time*), which makes it possible too feed back CSI to the transmitter using a feedback or return channel.

During a time interval of length less than the coherence time the channel can now be well approximated by an AWGN channel. An efficient transmission scheme designed for an AWGN channel with the same stationary signal-to-noise ratio will therefore be a good scheme for a flat fading channel, as long as the scheme does not require processing of blocks longer than the channel coherence time. When choosing the individual channel codes employed in an adaptive scheme, one should thus ensure that each code has satisfactory performance on an AWGN channel within some certain CSNR range. One could say that the principle of adaptive coded modulation effectively reduces a time-varying channel to a "bank" of time-multiplexed AWGN channels, each of which is to be used to represent some finite (small) interval on the CSNR axis. This is beneficial since the search for good codes on AWGN channels is a problem which has been subject to much research and large breakthroughs in the last decade.

2. THE FLAT FADING CHANNEL MODEL

The *flat-fading* channel is a much-used model for narrowband transmission over wireless and mobile communication channels, and the one to be used here. On a flat-fading channel, all signal frequencies are attenuated by the same factor. The received signal y_k in the complex baseband domain can then be written at a given time instant k as

$$y_k = a_k x_k + z_k \tag{1}$$

where a_k is the *fading amplitude*, x_k is the transmitted channel symbol (in general complex), and z_k is complex AWGN. The fading amplitude is also in general complex, but may be assumed to be real-valued if *perfect coherent detection* is assumed at the receiver end. We shall adopt this commonly used assumption throughout this paper.

The fading amplitude a (for notational simplicity we suppress the time dependence from now on) can be described as a stochastic variable, whose distribution we assume to be known. One particularly interesting distribution is the *Nakagami* distribution [9]:

$$p_A(a) = \frac{2}{\Gamma(m)} \left(\frac{m}{2\sigma^2}\right)^m a^{2m-1} \mathrm{e}^{-ma^2/2\sigma^2} \qquad (2)$$

Here, $m > \frac{1}{2}$ is the Nakagami parameter, σ^2 is the fading amplitude variance, and $\Gamma(x)$ is the gamma function defined by

$$\Gamma(x) = \int_0^\infty y^{x-1} \mathrm{e}^{-y} \, \mathrm{d}y, \qquad x > 0. \qquad (3)$$

The Nakagami distribution is of such interest because it allows for modelling of a wide range of fading environments by varying m, while yielding closed-form expressions for many interesting features of the channel, such as channel capacity. For $m = 1$ the model corresponds to the *Rayleigh fading* model which is common in mobile communications when there is no line-of-sight (LOS) path between transmitter and receiver. Increasing m will model less severe fading environments.

If a is Nakagami distributed, the CSNR will be Gamma distributed as shown in Equation (4), γ and $\overline{\gamma}$ being the instantaneous and expected CSNR respectively [4].

$$p_\gamma(\gamma) = \left(\frac{m}{\overline{\gamma}}\right)^m \frac{\gamma^{m-1}}{\Gamma(m)} \exp\left(-m\frac{\gamma}{\overline{\gamma}}\right), \qquad \gamma \geq 0 \qquad (4)$$

Under the assumption of perfect knowledge of the channel state information (CSI) both on the receiver and transmitter side, the *maximum average spectral efficiency* (MASE), or channel capacity, of a Nakagami multipath fading (NMF) channel has been shown to be given by [1, Eq. 23]

$$\mathrm{MASE} = \log_2(\mathrm{e})\mathrm{e}^{m/\overline{\gamma}} \sum_{k=0}^{m-1} \left(\frac{m}{\overline{\gamma}}\right)^k \Gamma\left(-k, \frac{m}{\overline{\gamma}}\right) \qquad (5)$$

where $\Gamma(x, y)$ is the socalled *complementary incomplete Gamma function*, which is commonly available in numerical software such as MATLAB or Maple. The MASE provides an ultimate performance benchmark for the spectral efficiency of any practical communication system communicating over this channel. It has been shown that in order to efficiently exploit the channel's capacity over time without excessive delay, a transmitter should switch between different codes of different spectral efficiencies—in other words adapt itself to the changes in channel quality as done by adaptive coded modulation.

3. ACHIEVING CAPACITY ON AWGN CHANNELS: LDPC CODES

In [10, 13] it is shown that carefully designed *Low-Density Parity Check* (LDPC) codes almost attain the Shannon channel capacity, i.e. the ultimate performance limit, for AWGN channels with BPSK modulation. The reason for this record-breaking performance of LDPC codes can at least partly be explained by the construction of the codes, to be explained below. According to Shannon's Channel Coding Theorem [11] the channel capacity can be reached by using a code consisting of very (in principle infinitely) long codewords picked at random from some suitable distribution, in combination with a decoder performing an exhaustive nearest neighbour search.

Since a nearest neighbour search without any constraints imposed is an NP-complete problem, such decoding was assumed to be impossible in practice—before the LDPC codes were invented by Gallager [3]. As implied by their name, limiting the space of possible codes to LDPC codes provide an additional property in the controlled sparseness (i.e., very low density of non-zero bits in each column) of their parity check matrices $\mathbf{H}$. This reduces the number of equations to be solved by the decoder and makes practical decoding possible by means of iterative methods similar to those used in turbo decoders.

To be more precise, an LDPC code is by definition a linear error-correcting block code, which is specified by a very sparse parity check matrix $\mathbf{H}$. A *regular* LDPC code has an $(N - K) \times N$ $\mathbf{H}$ matrix with a constant low number t of 1s in each column, placed at random. An *irregular* LDPC code has a non-uniform distribution of 1s in the rows and columns.

In any case, the parity check matrix may be generated simply by running a binary random generator (in the regular case, with probability t/N for emitting 1, and probability $1 - t/N$ for emitting 0). This is in effect very similar to the code construction used by Shannon in his proof of the Channel Coding Theorem, but with the constraint of matrix sparseness added in order to be able to find viable decoding algorithms. If the matrix found does not provide a good code, the random generator is simply run once more. Theoretically, however, the probability of finding a good code by such a random construction is very close to 1 if the code length N is large.

If we assume that the parity check matrix has linearly independent rows, the rate R of the code is K/N information bits per coded bit. In order to reduce the probability of low weight codewords (i.e., provide better distance properties in the code), for regular LDPC codes it is common to impose the constraint $t \geq 3$, and to not allow any two columns in $\mathbf{H}$ to have an overlap of non-zero bits by more than 1.

By using Gauss-Jordan elimination on the parity check matrix we may obtain the $N \times K$ *generator matrix* $\mathbf{G}$, satisfying $\mathbf{HG} = 0$. A codeword $\mathbf{t}$ is generated as $\mathbf{t} = \mathbf{Gs}$ where $\mathbf{s}$ is a K-dimensional information bit vector and all algebraic operations are performed modulo 2 ($\mathbf{t}$ and $\mathbf{s}$ are defined as column vectors).

4. RATE-ADAPTIVE LDPC-CODED MODULATION SYSTEM

The impressive error-correcting performance of LDPC codes on power-limited AWGN channels, in combination with the potential of increased bandwidth efficiency in wireless applications demonstrated by adaptive coded modulation, makes us consider the following question: Are LDPC codes also good codes to use in combination with higher order modulations, and subsequently in a rate-adaptive coded modulation system designed to exhibit good performance on bandwidth-limited time-varying channels? This is the topic of the remainder of this paper.

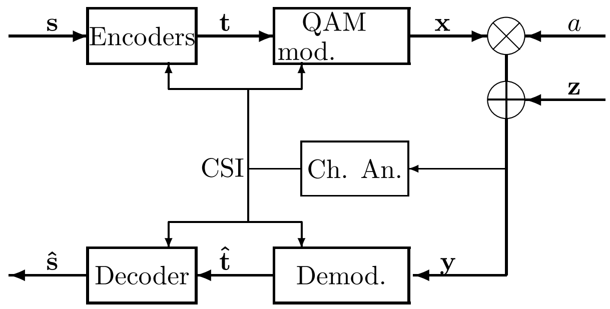

Figure 1: Adaptive LDPC-coded transmission system. The channel analyser (Ch. An.) finds the channel state information (CSI) and submit this information to the transmitter and receiver through a noiseless zero-delay return channel.

Figure 1 shows the communication system under analysis. Taking the adaptive system presented in [4] as a reference point, we have replaced the multidimensional trellis codes of that paper with I LDPC codes ("Encoders" in the figure), followed by Gray code mappings to I M-QAM constellations of varying sizes ("QAM mod." in the figure) [7]. For each M-QAM constellation we construct an LDPC code such that the lengths of all codes equal a constant number l of M-QAM symbols (the *channel vector length*) in the corresponding constellation. The encoder and decoder hence have a given set of I LDPC generator matrices $\mathbf{G}$ and parity check matrices $\mathbf{H}$, respectively. When code n is used, a source vector $\mathbf{s}$ consisting of $K = nl$ information bits is encoded to the binary codeword $\mathbf{t}$ of size $N = (n+k)l$. Here $N - K = kl$ is the number of parity check bits in $\mathbf{t}$. The number of parity check bits to be transmitted in *each channel symbol* $\mathbf{x}[j]$ $j = 1, \ldots, l$ is then equal to k, whereas the number of *information* bits in each channel symbol (which equals the code's spectral efficiency) is n. This is accomplished by letting the QAM modulator map $\mathbf{t}$ to a complex channel vector $\mathbf{x}$ of length l, by taking l distinct sets of $n+k$ bits from $\mathbf{t}$ and mapping each such set into a complex channel symbol.

The subsequent communication of the M-QAM symbol vectors is assumed done by means of a slowly varying flat fading channel, which may be approximated as an AWGN channel for a period of time at least as long as the transmission time of one channel vector $\mathbf{x}$ of length l. Such a vector will then be subjected to (almost) constant fading and the added channel noise $\mathbf{z}$, which is a complex AWGN vector with variance N_0.

The demodulator uses maximum likelihood soft decision detection on $\mathbf{y}$ [7], while the subsequent decoder uses an iterative belief propagation decoding algorithm, explained in [2, 5], to make an estimate $\hat{\mathbf{s}}$ of $\mathbf{s}$.

5. SYSTEM OPTIMIZATION AND PERFORMANCE ANALYSIS

5.1. Forward error correction

Having briefly described the component codes, the method of mapping from code bits to channel symbols, and the method of demodulation/decoding, we now present the parameter optimization and performance analysis of the overall rate-adaptive system. By finding each code's bit-error rate as a function of CSNR it is possible to tell for which CSNR range each code is acceptable, i.e., for which CSNRs it will perform better than a specified target bit error rate BER_0. This is done by simulating each relevant combination of LDPC code and M-QAM modulation on an AWGN channel [7]. The lower threshold CSNR for code n providing acceptable BER performance on an AWGN channel is denoted γ_n. This value is found by solving the equation $\text{BER}(\gamma_n) = \text{BER}_0$ for each code, $\text{BER}(\gamma)$ being the bit error rate for code n as a function of the CSNR γ. By always selecting the acceptable code producing the highest spectral efficiency for a given CSNR value, the spectral efficiency of the overall system will be maximized, while still maintaining the BER requirements.

For the particular code construction we use, each modulation symbol will carry n information bits and k parity bits, where n is the code index. The *average spectral efficiency* (ASE), $\langle R\rangle/W$, is a function of the expected CSNR $\overline{\gamma}$:

$$\text{ASE} = \sum_{n=1}^{I} \frac{R_n}{W} \cdot P_n = \sum_{n=1}^{I} n \cdot P_n \quad \text{[bits/s/Hz]} \qquad (6)$$

where P_n is the probability for γ being in the interval $[\gamma_n, \gamma_{n+1}]$, given by

$$P_n = \int_{\gamma_n}^{\gamma_{n+1}} p_\gamma(\gamma)\,\mathrm{d}\gamma = \frac{\Gamma(m, \frac{m\gamma_n}{\overline{\gamma}}) - \Gamma(m, \frac{m\gamma_{n+1}}{\overline{\gamma}})}{(m-1)!} \qquad (7)$$

with $p_\gamma(\gamma)$ being defined by Equation (4).

5.2. Combining FEC and ARQ protocols

Several papers report that LDPC codes under certain conditions have never made undetected errors [5, 6, 12]. This property is probably related to the *minimum distance* properties of LDPC codes. However, finding this minimum distance is a problem which is known to be NP complete [14]. Here, we simply assume—based on reported empirical results—that our LDPC codes have very good minimum distance properties, and that the decoder therefore always is able to detect whether a block is correctly decoded or not.

Under this assumption we have calculated the ASE using a suggested hybrid *Automatic Repeat reQuest* (ARQ) protocol, potentially corresponding to zero-error output. Codeword retransmission in this system is performed each time the LDPC decoder reports an undetected error, otherwise the system relies on FEC as before. A hybrid FEC-ARQ system will thus be very similar to the pure FEC system described earlier, with the exception of the threshold CSNRs $\{\gamma_n\}$ and the inherent use of retransmission.

The spectral efficiency (SE) for code n at a CSNR γ in the hybrid FEC-ARQ system can be shown to be [8]

$$\text{SE}(\gamma, n) = [1 - \text{BLER}_n(\gamma)] \cdot n \qquad (8)$$

where $\text{BLER}_n(\gamma)$ is the probability of a block of l channel symbols being detected as incorrectly decoded when using code n at CSNR γ. Furthermore, for all code pairs $(n-1, n)$ there exists a CSNR value giving the same spectral efficiency for both codes. By solving the equation

$$\text{SE}(\gamma_n, n-1) = \text{SE}(\gamma_n, n)$$

we can thus find the ARQ-optimised CSNR thresholds, assuming that $\text{BLER}_{n-1}(\gamma) = 0$ for CSNR values larger than γ_n:

$$\begin{aligned}
\text{SE}(\gamma_n, n-1) &= \text{SE}(\gamma_n, n) \\
&\Updownarrow \\
(1 - \text{BLER}_{n-1}(\gamma_n))(n-1) &= (1 - \text{BLER}_n(\gamma_n)) \cdot n \\
&\Updownarrow \\
n - 1 &\approx (1 - \text{BLER}_n(\gamma_n)) \cdot n \\
&\Updownarrow \\
\text{BLER}_n(\gamma_n) &\approx \frac{1}{n} \qquad (9)
\end{aligned}$$

from which γ_n may be derived. The ASE when using the hybrid FEC-ARQ scheme is now given by

$$\text{ASE}_h = \sum_{n=1}^{I} n \int_{\gamma_n}^{\gamma_{n+1}} (1 - \text{BLER}_n(\gamma)) \cdot p_\gamma(\gamma)\ \mathrm{d}\gamma \qquad (10)$$

6. NUMERICAL RESULTS

6.1. System specification

We have evaluated an example system along the lines we have described, on a Rayleigh fading channel (i.e., $m = 1$). A code set has been designed, with code lengths $N = (n+k)l$ ranging from 400 to 1800 bits, and with constant l=200 [8]. We have used $I = 8$, i.e., 8 M-QAM constellations with 8 corresponding LDPC codes. The constellations range from 4-PSK to 512-QAM, offering an uncoded throughput from 2 to 9 bits/symbol. However, the corresponding binary code rates used are $n/(n+1)$, $n \in \{1\ldots, 8\}$, producing effective information rates between 1 and 8 bits/symbol. For the LDPC codes with rates 1/2, 2/3, 3/4, and 4/5 we have constructed parity check matrices with column weight $t = 3$, while the rest of the codes have column weight $t = 4$. This is done to ensure that all codes have sufficient minimum distance [6].

We have used the target bit-error rate $\text{BER}_0 = 10^{-3}$ in the FEC system. By using the same code set in both the FEC and the hybrid FEC-ARQ system, we get in total four graphs of interest; the Maximum Average Spectral Efficiency ("MASE"), the system with both FEC and ARQ ("FEC/ARQ"), the system with FEC only ("FEC") and the "baseline" system with multidimensional trellis-coded modulation ("Trellis") as described in [4].

6.2. Performance results

The bit error rates and block error rates for the 200-symbol codes are presented in Figures 2 and 3. Based on these graphs, we have found $\gamma_1, \ldots, \gamma_8$, and calculated the spectral efficiencies for the two systems (FEC and FEC-ARQ hybrid) when used on AWGN channels, that is, with constant CSNR—see Figure 4.

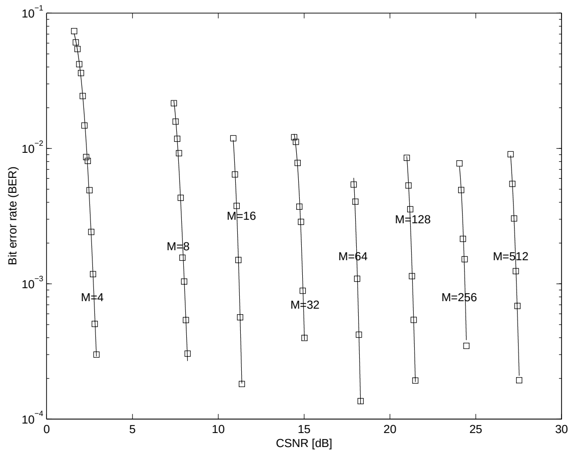

Figure 2: Bit-error-rate as a function of instantaneous CSNR

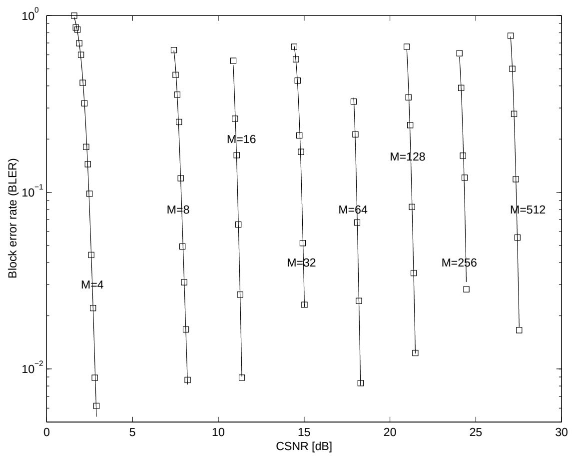

Figure 3: Block error rate as a function of instantaneous CSNR

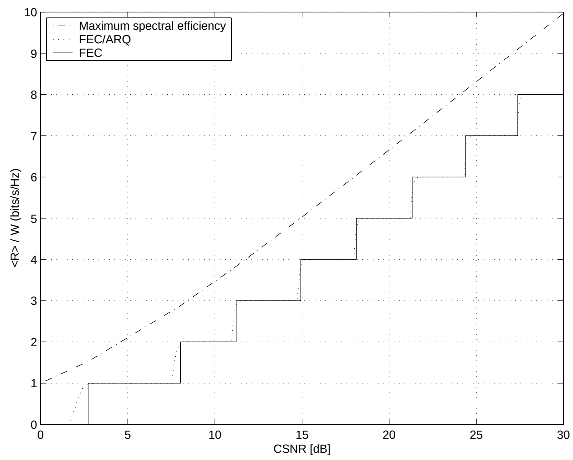

Figure 4: Spectral efficiency on an AWGN channel with constant CSNR

When combining the AWGN spectral efficiency with the probability density function for a Rayleigh fading channel (using Equations (6) and (10) with $m = 1$) we get the results as shown in Figure 5.

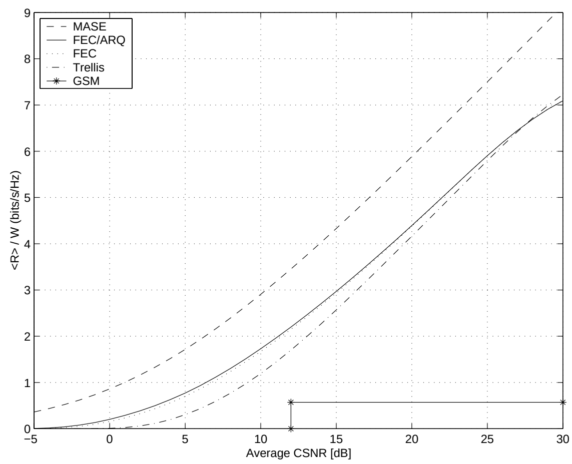

Figure 5: Average spectral efficiency as a function of average CSNR on a channel with Rayleigh fading

The spectral efficiency of GSM (0.57 bits/s/Hz until outage occurs at about 12 dB) is included as a reference. Over a wide range of CSNR values, the proposed system's bandwidth efficiency lies only about 1.3 bits/s/Hz below the channel's theoretical MASE.

It is also worth noting that for small CSNR values the hybrid FEC-ARQ system has slightly *higher* spectral efficiency than the FEC system; otherwise their performances are very similar. This rather counter-intuitive behaviour can be attributed to the fact that the ARQ system has higher spectral efficiency in the transition regions when the CSNR values are small, as witnessed by Figure 4. Stated simply, the system can transmit at a higher average rate when we allow for retransmission, and the rate reduction which comes as a result of retransmissions does not destroy all of this spectral efficiency gain. The mechanisms behind this property are not yet fully explored, but some results indicate that they may be influenced by the number of non-zero elements in the codes' parity check matrices.

7. CONCLUSIONS

Our results indicate that adaptive block-coded modulation has the potential of offering significantly better bandwidth efficiency than what is the case in today's wireless systems, as long as the channel variations are not too rapid. We believe that the choice of LDPC codes as component block codes in such a system are among the most promising possible choices.

In addition, the observation that a zero-error system (bybrid FEC-ARQ) slightly outperforms a non-zero-error (FEC) system—regarding spectral efficiency—suggests that using LDPC codes in conjunction with ARQ may reduce the need for code concatenation (e.g., outer Reed-Solomon codes) to reduce the target BER further. More work is needed on the performance degradations experienced when the idealized assumptions of a perfect, zero-delay return channel and perfect channel estimation are relaxed.

8. ACKNOWLEDGMENTS

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9. REFERENCES

[1] M.-S. Alouini and A. Goldsmith, "Capacity of Nakagami multipath fading channels," in *IEEE Transactions on Vehicular Technology*, Phoenix, Arizona, May 1997, pp. 358–362.

[2] M. P. C. Fossorier, M. Mihaljevic, and H. Imai, "Reduced complexity iterative decoding of low-density parity check codes based on belief propagation," *IEEE Transactions on Communications*, vol. 47, no. 5, pp. 673–680, May 1999.

[3] R. G. Gallager, *Low-density parity-check codes*, M.I.T Press, Cambridge, Massachusetts, 1963.

[4] K. J. Hole, H. Holm, and G. E. Øien, "Adaptive multidimensional coded modulation over flat fading channels," *Submitted to IEEE Journal on Selected Areas in Communications: Wireless Communications Series*, February 1999.

[5] D. J. C. MacKay, "Good error-correcting codes based on very sparse matrices," *IEEE Transactions on Information Theory*, vol. 45, no. 2, pp. 399–431, March 1999.

[6] D. J. C. MacKay and M. C. Davey, "Evaluation of Gallager codes for short block length and high rate applications," *http://wol.ra.phy.cam.ac.uk/mackay*, 1999.

[7] V. Markhus, "LDPC coding combined with spectrally efficient modulation on bandlimited AWGN channels," *Department of Telecommunication, NTNU*, December 2000.

[8] B. Myhre, "LDPC-koder anvendt i rate-adaptiv transmisjon over flat-fadende kanaler," *Department of Telecommunication, NTNU*, December 2000.

[9] M. Nakagami, "The m-distribution — a general formula for intensity distribution of rapid fading," in *Statistical Methods in Radio Wave Propagation*, Hoffmann W, C, Ed., pp. 3–36. Pergamon Press, 1960.

[10] S. Chung, G. Forney Jr, T. Richardson and R. Urbanke, "On the design of low-density parity-check codes whitin 0.0051 dB of the Shannon limit," *http://truth.mit.edu/sychung*, January 2000.

[11] C. E. Shannon, "A mathematical theory of communication.," *Bell Sys. Tech. J.,27:379-423,623-656*, 1948.

[12] Hongxin Song, Richard M. Todd, and J. R. Cruz, "Applications of low-density parity-check codes to magnetic recording channels," *IEEE Journal on Selected Areas in Communications*, vol. 19, no. 5, May 2001.

[13] A. Shokrollahi T. Richardson and R. Urbanke, "Design of provably good low-density parity check codes," *submitted to IEEE Transactions on Information Theory*, April 1999.

[14] Alexander Vardy, "Algorithmic complexity in coding theory and the minimum distance problem," in *Proceedings of the Twenty-Ninth Annual ACM Symposium on Theory of Computing*, El Paso, Texas, May 1997, pp. 92–109.