Blind carrier frequency offset estimation for OFDM/OQAM systems based on subchannel signals

Gang Lin, Lars Lundheim, and Nils Holte
Department of Electronics and Telecommunications
Norwegian University of Science and Technology, N-7491 Trondheim, Norway
Email: lingang;lundheim;holte@iet.ntnu.no

Abstract—New blind carrier frequency offset (CFO) estimation methods based on the correlation function of the subchannel signals are presented for OFDM/OQAM systems. The proposed estimators are robust to multipath effects due to the narrowband property of the subchannels in OFDM systems. The performance of the estimators is evaluated by asymptotic analysis and simulation results. Our results show that methods based on estimation in subchannels have better performance than method based on the signal before demodulation.

I. INTRODUCTION

Multicarrier systems are much more sensitive to carrier frequency offset (CFO) than single carrier systems. For OFDM/QAM systems, it is reported that CFO should be less than 2% of the subchannel spacing to guarantee a signal to interference ratio higher than 30 dB [1]. OFDM/OQAM systems using pulseshaping are also sensitive to CFO [2], [3].

Bolcskei presents a blind CFO estimation algorithm for both OFDM/QAM and OFDM/OQAM systems by using the correlation function of the received signals [4]. For a multipath channel, the channel information is needed for Bolcskei’s estimator. Based on the assumption of Rayleigh multipath fading, Park et al. develop a similar algorithm that does not need the channel information for OFDM/QAM systems [5]. However, such an assumption is restricted in practice. For example, even a simple time-invariant multipath channel doesn’t satisfy such assumption. Ciblat and Serpedin [6] claim that the robustness to multipath effects and precise CFO estimation can be obtained by using the conjugate correlation function of the received signals in stead of the correlation function for OFDM/OQAM systems. Whereas the implementation complexity of Ciblat/Serpedin’s estimator is much higher than Bolcskei’s estimator since in addition to the estimation of conjugate correlation function, an FFT based coarse peak search and a steepest descent based fine peak search are needed.

For OFDM/QAM systems, pulses with low sidelobes in frequency domain can be used [7], [8]. Thus each subchannel can be approximated as flat-fading for a system with many subchannels. This motivates us to estimate CFO based on the subchannel signals. In addition, since the sampling rate of the subchannel signals is N/2 times lower than that of the received signals before demodulation, where N is the number of subchannels, lower implementation complexity can be achieved if only few subchannel signals are used for CFO estimation. In [9], [10], estimation methods based on the conjugate correlation function of the subchannel signals are presented. In this paper, we present two estimators based on the correlation function of the subchannel signals. Although this seems to be a minor change, the resulting methods are quite different.

The rest of this paper is organized as follows. In section II, a time-discrete model for OFDM/OQAM systems is introduced, and the correlation function of the subchannel signals is derived. Then, in section III, two CFO estimation methods are presented. An asymptotic analysis is performed in section IV. Simulation results are shown in section V to evaluate the estimator performance and validate the theoretical analysis. At last, a short conclusion is shown in section VI.

II. SYSTEM MODEL AND CORRELATION FUNCTION

We consider a critically sampled time-discrete model for OFDM/OQAM systems with N subchannels. The subchannels are weighted by factors \(\{w_k\}_{k=0}^{N-1}\), where the weighting factors \(w_k\) should be real-valued to maintain the orthogonality between subchannels. Each subchannel transmits one QAM symbol \(a_k[n] = a_k^R[n] + j a_k^I[n]\) per \(T/2\) seconds. QAM symbols are formed by shifting the imaginary part of the QAM symbol by \(T/2\). By summing up all the subchannels, the modulator generates a \(T/N\) spaced output sequence

\[
x[l] = \sum_{k=0}^{N-1} w_k \sum_{n=-\infty}^{\infty} (a_k^R[n] g[l-nN] + j a_k^I[n] g[l-nN-N/2]) e^{j(\frac{2\pi}{T}l+\phi)k}.
\]

(1)

Here we consider a time varying multipath channel. If the number of subchannels \(N\) is large enough, the equivalent channel response of subchannel \(k\) can be approximated as time varying flat-fading with a fading factor \(\mu_k[l]\). We assume that \(\mu_k[l]\) is a stationary random process with real-valued correlation function \(c_{\mu_k}[\tau]\) and variance \(\sigma_{\mu_k}^2\). The channel model also includes an additive circular white Gaussian noise source \(\nu[l]\) with variance \(\sigma_{\nu}^2\). We assume that input data symbols, channel and noise are mutually independent. The carrier frequency offset is normalized with respect to 1/T and to be denoted \(f_c\). Then we can write the received sequence
from the channel as
\[ r[l] = e^{j\frac{2\pi}{N} f_s l} \sum_{k=0}^{N-1} w_k \mu_k[l] \sum_{n=-\infty}^{\infty} (a_k^N[n] g[l - nN] + j a_k^N[n] g[l - nN - N/2]) e^{j\left(\frac{2\pi}{N}l + \frac{\pi}{2}\right) k} + \nu[l], \]
(2)

In subchannel \( k \) of the receiver, the received sequence is first down-converted by multiplying with \( e^{-j\frac{2\pi}{N} l + \frac{\pi}{2} k} \), then filtered by the receiver filter \( f[l] \) and \( N/2 \) times down-sampled to generate a \( T/2 \) spaced sequence
\[ b_k[s] = r[l] e^{-j\left(\frac{2\pi}{N} + \frac{\pi}{2}\right) k} * f[l] |_{l=\frac{s}{N}} \]
\[ = e^{j\pi f_s} \sum_{m=0}^{N-1} w_m \mu_m[s N/2] \sum_{n=-\infty}^{\infty} (a_m^N[n] p_{m,k}[s - 2n] + j a_m^N[n] p_{m,k}[s - 2n - 1]) + \nu_k[s], \]
(3)

where \( p_{m,k}[s] \) \( \equiv \) \( p_{m,k}^{(s)}[s N/2] \) and \( \nu_k[s] \) \( \equiv \) \( \nu_k^{(s)}[s N/2] \) are respectively the \( N/2 \) times down-sampled versions of \( p_{m,k}[l] \) and \( \nu_k^{(l)}[l] \) which are defined as
\[ p_{m,k}^{(s)}[l] = g[l] e^{j\left(\frac{2\pi}{N} + \frac{\pi}{2}\right) (m-k)} * f[l] \]
\[ \nu_k^{(s)}[l] = \nu[l] e^{-j\left(\frac{2\pi}{N} + \frac{\pi}{2}\right) k} * f[l] . \]
(4)

Note that although the sequence immediately before the decimator (or immediately after the receiver filter), \( r[l] e^{-j\left(\frac{2\pi}{N} + \frac{\pi}{2}\right) k} * f[l] \), contains more information than the \( N/2 \) down-sampled sequence \( b_k[s] \), this signal is not directly available in a receiver based on FFT and polyphase filters [11]. Therefore we will base our methods on \( b_k[s] \).

The convolution function of the subchannel signals is defined as \( c_k[s, \tau] = \mathbb{E}\{b_k[s + \tau] b_k^{*}[s]\} \). We further assume that the input QAM symbols have a unit power, and are i.i.d between different symbols and between the real and imaginary parts. Then, based on the expression of \( b_k[s] \) in (3), we find that \( b_k[s] \) is wide sense stationary in \( c_k[s, \tau] \) is not a function of \( s \). Thus we can use the notation \( c_k[\tau] \) expressed as
\[ c_k[\tau] = \frac{1}{2} \sum_{m=0}^{N-1} u_m^2 c_{\mu_m}[\tau N/2] A_{m,k}(\tau, f_e) + \sigma_p^2 p_{\tau}[\tau], \]
(5)

where \( p_{\tau}[\tau] = g[l] * f[l] |_{l=\tau N/2} \) is the \( N/2 \) times downsampled version of the overall response of the cascade of \( g[l] \) and \( f[l] \), and
\[ A_{m,k}(\tau, f_e) = e^{j\pi f_{\tau} \tau} \sum_{n=-\infty}^{\infty} p_{m,k}[n + \tau] p_{m,k}^*[n] \]
\[ = \frac{1}{2} \int_{-1}^{1} |P_{m,k}(f)|^2 e^{j\pi(f+f_e)\tau} df, \]
(6)

where \( P_{m,k}(f) \) \( \equiv \) \( \sum_{n=-\infty}^{\infty} p_{m,k}[n] e^{-j2\pi fs} \).

Now we assume that the transmitter \( f[l] \) and receiver \( g[l] \) are identical real-valued symmetric pulses and band-limited to \([-1, 1]\) (normalized with respect to the subchannel spacing \( 1/T \)), for example, square root raised cosine pulse with a roll-off factor no larger than one. It is proved in the appendix that \( \sum_{m=0}^{N-1} A_{m,k}(\tau, f_e) \) is real-valued and independent of \( f_e \). Then for the case of unweighted systems and AWGN channel, i.e. \( w_k = 1 \) and \( \mu_k[l] = 1 \), the correlation function given by (5) is independent of \( f_e \) and thus contains no information of \( f_e \).

III. ESTIMATION ALGORITHMS

In the previous section, we have proved that for the case of \( w_k = 1 \) and \( \mu_k[l] = 1 \), the correlation function \( c_k[\tau] \) contains no information about CFO. One method to retain CFO information is subchannel weighting, i.e. distributing individual subchannel different power, which is similar to Bolcskei’s method [4]. Here we consider the method of null-subchannel inserting, i.e. setting \( w_k = 0 \) for some selected subchannels, which can be referred as null-subchannels, while the other factors are set to 1. If the null-subchannels are sparsely distributed and subchannel \( k \) is a null-subchannel, it can be verified that \( c_{k-1}[\tau] \), \( c_k[\tau] \), \( c_{k+1}[\tau] \) and \( c_{k+2}[\tau] \) contain information of \( f_e \) for \( 0 \leq f_e < 1 \).

In this paper, we will base our methods only on \( c_k[\tau] \) and consider only one null-subchannel. The extension to the multiple null-subchannels case is straightforward. We assume that subchannel \( k \) is a null-subchannel and \( |f_e| < 1 \). Then by substituting (20) into (6) then into (5), we obtain
\[ c_k[\tau] = \frac{1}{4} \int_{-1}^{1} \left( \sum_{m=k-2}^{k+2} c_{\mu_m}[\tau N/2] \right)^2 \left| P_{m,k}(f-f_e) \right|^2 e^{j\pi f_{\tau} \tau} df \]
\[ + \sigma_p^2 p_{\tau}[\tau] \]
\[ \simeq \frac{1}{4} e^{j\pi f_{\tau} / 2} c_{\mu_k}[\tau N/2] M_g(f_e, \tau) \]
\[ + (c_{\mu_k}[\tau N/2] + \sigma_p^2) p_{\tau}[\tau], \]
(7)

where the approximations \( c_{\mu_{k+2}}[\tau N/2] \simeq c_{\mu_{k+1}}[\tau N/2] \simeq c_{\mu_k}[\tau N/2] \) are used in the derivations, and
\[ M_g(f_e, \tau) = \int_{-1}^{1} G^2(\frac{f - f_e}{2}) G^2(\frac{f + f_e}{2}) \cos(\pi f \tau) df . \]

It is obvious that \( M_g(f_e, \tau) \) is real-valued. For the case of \( G(f) \) is the square root raised cosine pulse with a roll-off factor equal 1.0, i.e. \( G(f) = \sqrt{2} \cos(\pi f/2) \) (note that \( G(f) \) has a period \( N \) and is band-limited to \([-1, 1]\)), numerical results show that \( M_g(f_e, \tau) \) with \( \tau = 0, 1, 2 \) decreases quickly towards zero as increasing \( f_e \), while they all keep positive for \( |f_e| < 1 \).

In practice only one finite-length data record \{\( b_k[s]\)\}_{s=0}^{M-1} is available. We use the estimate \( \hat{c}_k[\tau] = \frac{1}{M} \sum_{s=0}^{M-1} b_k[s + \tau] b_k^{*}[s] \) (the undefined samples are assumed to be zero). Below we present two CFO estimators:

**Estimator 1:**

Since \( p_{\tau}[\tau] \) is a Nyquist pulse, we have \( p_{\tau}[\tau] = 0 \) for any non-zero even \( \tau \). We also assume that \( c_{\mu_k}[N] \) is positive. For time-invariant channel, this assumption is obviously satisfied. For slow enough fading, \( c_{\mu_k}[N] \) is approximately equal to \( \sigma_p^2 \) and thus positive. Then based on (7), we can estimate \( f_e \) based on the phase of \( c_k[2] \) as
\[ f_e = \angle \{ \hat{c}_k[2] \} / \pi, \]
(9)
where $\angle$ stands for the operation of taking phase in radians.

The acquisition range of this estimator is $|f_e| < 1$.

**Estimator 2:**
For $G(f) = \sqrt{2} \cos(\pi f/2)$, numerical results based on (8) and (7) show that $|c_k[0]|$ and $|c_k[1]|$ is higher than $|c_k[2]|$. Thus better performance is expected if we estimate $f_e$ based on $\hat{c}_k[0]$ and $\hat{c}_k[1]$ instead of $\hat{c}_k[2]$. Based on (7), we have

$$c_k[0] = -\frac{\sigma_{\mu}^2}{4} M_g(f_e, 0) + (\sigma_{\mu}^2 + \sigma_{v}^2)$$

$$c_k[1] = -\frac{c_{\mu k}[\mathbf{N}]}{4} e^{i\pi f_e/2} M_g(f_e, 1) + (\sigma_{\mu k}[\mathbf{N}/2] + \sigma_{v}^2) p_t[1].$$

We see that both $c_k[0]$ and $c_k[1]$ contain noise terms. For slow enough fading, $c_{\mu k}[\mathbf{N}/2] \approx \sigma_{\mu k}^2$ (for time-invariant channel they are actually equal), we have

$$p_t[1] c_k[0] - c_k[1] \approx \frac{1}{4} \sigma_{\mu k}^2 M_g(f_e, 1) \left( e^{i\pi f_e/2} - \frac{M_g(f_e, 0)}{M_g(f_e, 1)} p_t[1] \right).$$

(10)

Now the effect of noise has been successfully eliminated, and the CFO can be estimated as

$$\hat{f}_e = \arg \left( \phi(f_e) = \hat{\phi} \right),$$

(11)

where $\phi(f_e) \overset{\text{def}}{=} \angle \left( e^{i\pi f_e/2} - \frac{M_g(f_e, 0)}{M_g(f_e, 1)} p_t[1] \right)$ and $\hat{\phi} = \angle \{p_t[1] c_k[0] - \hat{c}_k[1]\}$.

Now the estimation problem is turned into solving the nonlinear equation $\phi(f_e) = \hat{\phi}$, and we should look closer at the properties of $\phi(f_e)$. For the case of $G(f) = \sqrt{2} \cos(\pi f/2)$, it can be proved strictly that $\phi(f_e)$ increases monotonously with increasing $f_e$ in the region of $-\pi < \phi(f_e) < \pi$ based on (8) and (7). Thus $\hat{f}_e$ is uniquely determined for $-\pi < \hat{\phi} < \pi$. The acquisition range of this estimator is $|f_e| < 2$ theoretically. While numerical results show that both $M_g(f_e, 0)$ and $M_g(f_e, 1)$ are close to zero for $|f_e| > 1.5$, thus the estimation is not reliable for large values of $f_e$.

**IV. ASYMPTOTIC ANALYSIS**

In this section, we will derive the asymptotic mean square error (MSE) for both estimator 1 and 2 by using reasoning similar to that in [12]. The estimation error of CFO is defined as $\Delta f_e = \hat{f}_e - f_e$ and the MSE is then defined as $\mathbb{E}[(\Delta f_e)^2]$.

It is obvious that $\lim_{M \to \infty} \mathbb{E}[\hat{c}_k[\tau] - c_k[\tau]]^2 \to 0$ with probability 1. Thus the amplitude of the estimate error $\Delta c_k[\tau] \overset{\text{def}}{=} \hat{c}_k[\tau] - c_k[\tau]$ should be small for large values of data record length $M$.

**Estimator 1:**
First we analysis estimator 1 given by (9). The estimated CFO can be rewritten as

$$\hat{f}_e = \text{Im} \{\ln(-c_k[2])\} / \pi = \text{Im} \{\ln(-c_k[2] - \Delta c_k[2])\} / \pi$$


(12)

where the last step follows from the first order Taylor approximation.

Since $\lim_{M \to \infty} \mathbb{E}[\Delta c_k[\tau]] = \mathbb{E}[\hat{c}_k[\tau] - c_k[\tau]] = 0$, we have that $\lim_{M \to \infty} \mathbb{E}[\hat{f}_e - f_e] = 0$. This implies that the estimation of $f_e$ is asymptotically unbiased. Based on (12), we can write the MSE of estimator 1 as

$$\text{MSE}_1 \approx \frac{\mathbb{E}[(\Delta c_k[2])^2]}{2 \pi^2 |c_k[2]|^2}$$

(13)

**Estimator 2:**
Now we analysis estimator 2 given by (11). First we define $\beta = p_t[1] c_k[0] - c_k[1]$ and $\Delta \beta = p_t[1] \Delta c_k[0] - \Delta c_k[1]$. Based on (11), we have

$$\phi(f_e + \Delta f_e) = \text{Im} \{\ln(\beta + \Delta \beta)\}.$$

Then by using the first order Taylor approximation in both sides and the fact that $\phi(f_e) \approx \mathbb{E}[\ln(\beta)]$, we have

$$\phi'(f_e) \Delta f_e = \text{Im} \{\Delta \beta \beta^{-1} \Delta f_e \} \approx \frac{\Delta \beta \beta^{-1} \Delta f_e}{2 j |\beta|^2},$$

where $\phi'(f_e)$ stands for the derivative of $\phi(f_e)$.

Then after some straightforward derivations, the MSE of estimator 2 can be written as

$$\text{MSE}_2 \approx \frac{\mathbb{E}[(\Delta c_k[1])^2]}{2 |\beta|^2 |\phi'(f_e)|^2}.$$
V. SIMULATION RESULTS

The following conditions apply to all simulations:

- The number of subchannels $N$ is set to 16, and subchannel 4 is the only null-subchannel;
- 16OQAM modulation is used in all subchannels, and the input symbols are uniformly distributed;
- $g[l]$ and $p[l]$ are square root raised cosine pulses with a roll-off factor $\alpha = 1.0$;
- Each result is obtained by averaging over 1000 Monte Carlo trials.

**Simulation 1: performance of estimator 1 and 2 versus SNR over an AWGN channel**

In this simulation, we set the data record length $M = 256$ (corresponding to 128 OFDM symbols since the OQAM symbols are two times over-sampled), and $f_e = 0.2$ and 0.8. The simulation results are shown in Fig. 1. We see that the MSE of both estimators converges to a certain floor value (determined by $A_1(f_e)$ and $A_2(f_e)$ in (16)) as increasing SNR. We also note that the simulated results match well with theoretical predictions, except estimator 1 with $f_e = 0.8$. The large gap between simulated results and theoretical predictions for estimator 1 with $f_e = 0.8$ is caused by the threshold effect: as $f_e \to 1$, $\angle \{-c_k[2]\} \to \pi$, even a small estimation error of $c_k[2]$ may cause the estimated CFO to jump to negative due to the discontinuity of function $\angle$, then cause a large estimation error of CFO. In addition, since the amplitude of $c_k[2]$ is relatively smaller, estimator 1 is more sensitive to estimation error than estimator 2. We will show in Simulation 3 that such threshold disappears for large enough $M$.

**Simulation 2: performance of estimator 1 and 2 versus $f_e$ over an AWGN channel**

Now we simulate the performance of estimators versus CFO. We set the data record length $M = 256$, and SNR to be 0 and 40 dB. The simulation results are shown in Fig. 2. We see that for SNR = 40 dB, estimator 1 is sensitive to $f_e$ and better than estimator 2 for $|f_e| < 0.15$, and the MSE of estimator 1 is very small for $f_e$ close to zero. This can be explained by noting that $A_1(0) = 0$ in (16), thus no MSE floor is present. Estimator 2 is largely insensitive to $f_e$, especially in the region of $|f_e| < 0.2$. Then if closed-loop estimation is used, estimator 1 can be better than estimator 2. We also note that both estimators suffer threshold effect for large values of $f_e$, especially for estimator 1 in the region of $|f_e| > 0.6$. Such threshold effect will disappear for large enough $M$.

**Simulation 3: performance of estimator 1 and 2 versus $M$ over an AWGN channel**

In this simulation, we simulate the performance of estimator 1 and 2 versus the data record length $M$. To observe the disappearing of threshold effect with increasing $M$, we set $f_e = 0.8$. The simulation results are shown in Fig. 3. We see that the MSE decreases with increasing $M$ for both estimators. We also note that for SNR = 40 dB, the threshold effect of estimator 1 disappears for $M > 2600$. For estimator 2, the small gap between simulated results and theoretical predictions, which is due to the asymptotic approach of theoretical analysis, disappears asymptotically with increasing $M$.

**Simulation 4: performance comparison of estimator 1, estimator 2 and the modified Bolcskei estimator over a time-invariant multipath fading channel**

In this simulation, Bolcskei estimator [4] is included to make a comparison to our estimators. We will show that Bolcskei estimator doesn’t work under the assumptions that the real and imaginary parts of input QAM symbols have the same power, and then present a modified Bolcskei estimator. First by replacing $M$ by $N$, we write (15) in [4] as

$$C_r[k, \tau] = \frac{1}{N} e^{jk f_e \tau} e^{-j k \delta_h} \sum_{\tau=0}^{N-1} \frac{\Gamma_N[\theta] A(g, g)}{N} \left[ \Gamma_r \frac{k}{N} \right] - \mathbb{K}_r + (-1)^k \sigma^2_c[I] + c_p[\tau] \delta[k],$$
where $\sigma_{c,R}^2$ and $\sigma_{c,I}^2$ are respectively the average power of the real and imaginary parts of the input QAM symbols, $\Gamma_N[\tau] = \sum_{k=0}^{N-1} |w_k|^2 e^{j\tau k}$ and

$$A^{(g,g)} \left[ \tau, \frac{k}{N} \right] = \sum_{l=-\infty}^{\infty} g[l] g[l-\tau] e^{-j\frac{2\pi}{N} kl}. \quad (17)$$

Note that the frequency offset $\theta_c$ in [4] is normalized with respect to $N/T$. By using Parseval’s relation, we can rewrite $A^{(g,g)} \left[ \tau, \frac{k}{N} \right]$ in frequency domain as

$$A^{(g,g)} \left[ \tau, \frac{k}{N} \right] = \int_{-\infty}^{0.5} G_0(f) G_0(f + \frac{k}{N}) e^{-j2\pi f \tau} df,$$

where $G_0(f) = \sum_{l=-\infty}^{\infty} g[l] e^{-j2\pi f l}$. Since $G_0(f)$ is band-limited to $[-1/N, 1/N]$ (normalized with respect to $N/T$), $A^{(g,g)} \left[ \tau, \frac{k}{N} \right]$ is nonzero only if $k \in \{-1,0,1\}$. In this paper, we have assumed that $\sigma_{c,R}^2 = \sigma_{c,I}^2$. Then $\sigma_{c,R}^2 + (-1)^k \sigma_{c,I}^2 = 0$ for $k = \pm 1$, and only $C_0[0,\tau]$ can be used for the CFO estimation. Recall that the channel noise is assumed to be white, i.e. $c_d[\tau] = \sigma_d^2 \delta[\tau]$. Then the effect of noise can still be excluded (in theory) since only $C_0[0,\tau]$ with nonzero lag $\tau$ can be used for CFO estimation. Finally, the modified Bolcskei estimator can be expressed as

$$\hat{f}_c = N^2 \theta_c = \frac{N}{2\pi} \sum_{\tau=1}^{L-1} \frac{1}{\tau} \left\{ \hat{C}_\tau[0,\tau]/\Gamma_N[\tau] \right\}. \quad (18)$$

Subchannel weighting is still needed to keep $\Gamma_N[\tau]$ nonzero for some $\tau \in [0, N-1]$. For the case of subchannel 4 is the only null-subchannel, one can verify that $\Gamma_N[\tau] \neq 0, \forall \tau \in [1, N-1]$, thus $L_\tau$ is set to 15.

In the simulations, we assume a time-invariant three-path channel with impulse response

$$h[l] = \sum_{d=0}^{2} \lambda_d \delta[l-d], \quad (19)$$

where $\lambda_d$ are the attenuation factors of paths.

We set $M = 256$, $f_c = 0.3$ and $[\lambda_0, \lambda_1, \lambda_2] = [\frac{1}{2}, 1, -0.5]$. To calculate the theoretical MSE of estimator 1 and 2, the attenuation factor $\mu_k$ is approximated as $\sum_{d=0}^{2} \lambda_d e^{-j \frac{2\pi}{N} kd}$. The simulation results are shown in Fig. 4. We see that both estimator 1 and 2 outperform the modified Bolcskei estimator over an AWGN channel, except that Bolcskei estimator is slightly better than estimator 1 for low SNR. This is expected since the information about CFO is intensified by the receiver filter(s) of null-subchannel(s). We also note that as expected, both estimator 1 and 2 are robust to the multipath effects, while the modified Bolcskei estimator is not robust to that. For multipath channel, the MSE of both estimator 1 and 2 is even slightly lower than that for AWGN channel. This can be explained by noting that the equivalent SNR of subchannel 4 is higher than that of AWGN channel.

**Simulation 5: performance versus SNR for estimator 1 and 2 over a Rayleigh multipath fading channel**

At last, we simulate the performance of estimator 1 and 2 over a Rayleigh multipath fading channel. The same channel model as (19) is used except the factors $\lambda_d$ are time-variant here. In this simulation, the factors $\lambda_d[l]$ are i.i.d. and to be modelled as autoregressive processes, and generated by filtering a circular white Gaussian noise source by lowpass filters with a frequency response $H(f) = 1/(1 - \rho e^{-j2\pi f})^5$ and a 3 dB bandwidth denoted $B_{\mu}T$. The power of the driving noise source is set to let $\sum_{d=0}^{2} \lambda_d[l] e^{-j \frac{2\pi}{N} kd}$. It can be easily verified that $\sigma_{\mu_k}^2 = 1$.

We set $M = 256$ and $f_c = 0.2$. The 3 dB bandwidth $B_{\mu}T$ is set to two cases: 0.001 and 0.01. The simulation results are shown in Fig. 5. We see that the Rayleigh multipath fading will cause only slight degradation compared to the AWGN case for both estimator 1 and 2, and the simulated results matches well.
Fig. 5. MSE versus SNR for estimator 1 and 2 over a Rayleigh multipath fading channel ($f_e = 0.2$).

with theoretical predictions. It is shown in [15] that for both estimator 1 and 2, the MSE floor over a slow Rayleigh fading channel with $\sigma^2_{pk} = 1$ will be approximately 2 times (3 dB) higher than that over an AWGN channel.

VI. CONCLUSION

We have shown how to estimate CFO based on the correlation function of subchannel signals, and that this method requires non-uniform power distribution (weighting) over sub-channels. Two estimators are presented and studied based on null-subchannels. Asymptotic analysis is performed and shows that the MSE of proposed estimators decreases as $O(M^{-1})$. Simulation results, which match well with the theoretical predictions, show that both estimators are robust to both time-invariant and Rayleigh multipath fading, and outperform the method based on the correlation function of the signal before demodulation. For slow Rayleigh multipath fading with the same signal power, only 3 dB MSE degradation is observed compared to AWGN case for our methods.

ACKNOWLEDGMENT

This work was funded by the Research Council of Norway through the BEATS project.

APPENDIX

Proof: From the definition of $p_{m,k}[l]$ shown in (4) and the relationship of $p_{m,k}[s] = p_{m,k}(s \Sigma N^2)$, we can write the frequency response of $p_{m,k}[s]$ in one period $[-1,1]$ as

$$P_{m,k}(f) = G(f - (m - k)) G(f + f_e) e^{j \pi (m-k)},$$

where $G(f) = \sqrt{2/N} \sum_{l=\infty}^{\infty} g[l] e^{-j \frac{2\pi}{\tau} f}.$

Then by substituting (20) into (6), and using the fact that $P_{m,k}(f)$ is periodic in $f$ with a period 2, we have

$$\sum_{m=0}^{N-1} A_{m,k}(\tau, f_e) = \frac{1}{1} \int_{-1}^{1} \left( \sum_{m=0}^{N-1} |P_{m,k}(f - f_e)|^2 \right) e^{j \pi \beta \tau} df$$

$$= \frac{1}{1} \int_{-1}^{1} \left( \sum_{m=0}^{N-1} G^2(f - (m - k) - f_e) \right) G^2(f) e^{j \pi \beta \tau} df$$

$$\equiv \int_{-1}^{1} G^2(f) e^{j \pi \beta \tau} df = \int_{-1}^{1} G^2(f) \cos(\pi f \tau) df,$$

(21)

where (a) follows from the fact that $\sum_{m=0}^{N-1} G^2(f - (m - k)) \equiv 2$ since $G^2(f)$ is a Nyquist pulse.

Then we see that $\sum_{m=0}^{N-1} A_{m,k}(\tau, f_e)$ is real-valued and independent of $f_e$.}

REFERENCES