Modeling and Analysis of a 40 GHz MIMO System for Fixed Wireless Access

Frode Bøhagen
UniK/Nera Research,
N-1375 Billingstad, Norway
Email: frode.bohagen@research.nera.no

Pål Orten
UniK/Nera Research,
N-1375 Billingstad, Norway
Email: pal.orten@research.nera.no

Geir E. Øien
NTNU Dept. of Electronics and Telecom.,
N-7491 Trondheim, Norway
Email: oien@iet.ntnu.no

***Abstract*— The throughput of a future fixed wireless access *multiple-input-multiple-output* (MIMO) system operating at high frequencies is investigated. We extend our previous theoretical work on MIMO for *line-of-sight* (LOS) channels to show that a considerable gain in throughput is achieved compared to *single-input-single-output* transmission for a practical system, which among other things is subject to antenna array size constraints. In our investigation we apply a propagation model applicable for high frequencies, where often LOS is required for sufficient coverage, and where weather phenomena like rain have considerable impact on the quality of the radio link. A state of the art transmission scheme is utilized with low density parity check coded modulation, as well as power control and bit loading.**

***Keywords*—MIMO, FWA, line-of-sight, LDPC, propagation model, Rice channel.**

I. Introduction

Fixed wireless access (FWA) is a flexible and cost efficient way to bring network applications such as Internet and telephony to organizations and homes. This technology requires no expensive and time consuming digging of cables, and is easily scalable with regard to the applications' demand. Some predictions on the role of FWA in the future communication environment are given in [1] and [2].

Most research efforts in the field of *multiple-input-multiple-output* (MIMO) communications utilize the fading given by the multipath environment, while FWA at high frequencies usually requires a strong *line-of-sight* (LOS) component for sufficient coverage. Some recent papers have shown the possibility of getting high MIMO gain for LOS channels (LOS-MIMO), i.e. [3], [4] and [5]. We combine these ideas with propagation models for high frequency FWA, where for instance the influence of rain has an important impact on the quality of the received signal. The transmission scheme utilized is based on eigenmode transmission with bit loading and power allocation. We present a novel analysis of constraints on maximum array size with regards to throughput for LOS-MIMO. The performance is evaluated both with respect to throughput utilizing LDPC coded modulation, and to Shannon capacity.

The rest of the paper is organized as follows. Section II presents the system model with the transmission scheme used. Then in section III the channel model and *signal-to-noise and-interference ratio* (SNIR) calculations are described, while results and conclusions are given in section IV and section V respectively.

II. System model

Fixed wireless LOS transmission at high-frequencies is generally not exposed to rapid channel variations. Still, it can be argued that it is beneficial to utilize *channel state information* (CSI) feedback to the transmitter in some scenarios, as e.g. a rain event can introduce a severe change in the channel characteristics (see section III). As described in the succeeding sections the CSI needed at the transmitter for our system is the MIMO channel matrix characteristics and the subchannels' SNIR. The channel estimation problem will not be studied in this paper. We will assume that perfect CSI is available in both the transmitter and receiver.

A. Transmission on eigenmodes

Transmission over a slowly varying and frequency flat fading MIMO channel in complex baseband can be expressed as

$$\mathbf{y} = \sqrt{\beta}\mathbf{H}\mathbf{x} + \mathbf{n}, \tag{1}$$

where $\mathbf{y}$ is the $M \times 1$ received vector, the factor β is the common power gain over the channel, $\mathbf{H}$ is the $M \times N$ normalized channel matrix, $\mathbf{x}$ is the $N \times 1$ transmitted signal vector, and $\mathbf{n}$ is the $M \times 1$ complex *additive white Gaussian noise* (AWGN) vector. The elements in $\mathbf{H}$ links the N transmit antennas with the M receive antennas. It is assumed that all the subchannels experience the same path loss. This common path loss is incorporated in the β factor. The channel matrix is normalized, which means that the elements have an average power of one.

The CSI assumed available at both sides of the communication link is utilized to transmit on the eigenmodes. By doing a *singular value decomposition* (SVD), $\mathbf{H}$ can be expressed as $\mathbf{H} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^H$, where $\mathbf{U}$ is a $M \times M$ unitary matrix, $\mathbf{\Sigma}$ is a $M \times N$ diagonal matrix with the singular values of $\mathbf{H}$ as its elements ($\sigma_{H,i}$), and $\mathbf{V}$ is a $N \times N$ unitary matrix. Eigenmode transmission is accomplished by premultiplying the transmitted signal vector with the unitary matrix $\mathbf{V}$. The resulting subchannels are orthogonal to each other, which means there is no cross-talk between them.

The CSI will also be employed to introduce power allocation on the eigenmode channels. This can be modeled by multiplying the symbol vector $\mathbf{s}$ with a diagonal matrix $\mathbf{W}$ of dimension $N \times N$. The diagonal elements of the power allocation matrix is denoted $\sigma_{W,i}$, and they are subject to the total transmit power constraint $\sum_{i=1}^{N} \sigma_{W,i}^2 = P_t^{tot}$. By doing this preprocessing of the symbol vector, and utilizing the SVD of $\mathbf{H}$, we get the following expression for the MIMO transmission:

$$\begin{aligned}\mathbf{y} &= \sqrt{\beta}\mathbf{H}\mathbf{x} + \mathbf{n} \\ &= \sqrt{\beta}\mathbf{U}\mathbf{\Sigma}\mathbf{V}^H(\mathbf{V}\mathbf{W}\mathbf{s}) + \mathbf{n} \\ &= \sqrt{\beta}\mathbf{U}\mathbf{\Sigma}\mathbf{W}\mathbf{s} + \mathbf{n}.\end{aligned}$$

Defining $\tilde{\mathbf{y}} = \mathbf{U}^H\mathbf{y}$ and $\tilde{\mathbf{n}} = \mathbf{U}^H\mathbf{n}$, we obtain

$$\tilde{\mathbf{y}} = \sqrt{\beta}\mathbf{\Sigma}\mathbf{W}\mathbf{s} + \tilde{\mathbf{n}}, \tag{2}$$

where $\tilde{\mathbf{y}}$ is the received vector. It is easy to show that $\tilde{\mathbf{n}}$ has the same statistical properties as $\mathbf{n}$ (same distribution). The first J elements of the received vector $\tilde{\mathbf{y}}$ can be looked at as parallel *single-input-single-output* (SISO) AWGN channels, each with gain $\sqrt{\beta} \cdot \sigma_{H,i} \cdot \sigma_{W,i}$, where J is the rank[1] of the channel matrix. The remaining $M - J$ elements contain no information, only noise. In Fig. 1, the transmission system described is illustrated, while the equivalent form from (2) is shown in Fig. 2.

B. Power allocation and bit loading

As mentioned earlier the system has total available transmission power P_t^{tot}. This power should be distributed between the subchannels in a smart way. Usually a transmission system is subject to some kind of QoS constraint given by the application. In this paper we introduce a *bit error rate* (BER) constraint, i.e. the system should have a BER lower than a given threshold, BER_0.

One simple but not optimal way to use the transmit power is to divide it equally between the subchannels. The bit loading is then done by maximizing the bit rate on each subchannel with the BER constraint based on the subchannels' SNIRs.

A more optimal but complex scheme was presented in [6]. It is referred to as QoS-based *waterfilling* (WF). The method starts with the WF distribution and is further constructed with the objective to use minimum power to achieve the BER threshold on each channel, and re-distribute the residual power in a smart way. The step by step procedure is described in [6].

[1]The rank of a matrix is equal to the number of independent rows and/or columns of the matrix, and is equivalent to the number of nonzero singular values.

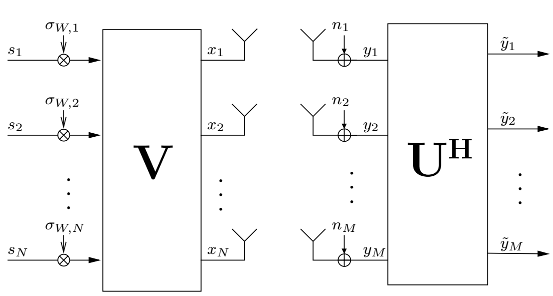

Fig. 1. Eigenmode transmission system

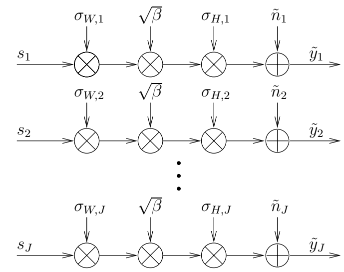

Fig. 2. Equivalent eigenmode transmission model

C. Coding and modulation

In this paper we utilize *Low density parity check* (LDPC) codes with QAM/PSK modulation. LDPC codes are block codes proposed by Gallager in 1962. Its main features is the high performance together with the iterative decoding scheme, where the complexity only grows linearly with the block length. As an example of LDPC codes we use the results from [7, Table 3], where the block length is 200 QAM/PSK channel symbols. We introduce a QoS requirement of BER less than 10^{-6}. The SNIR thresholds for where each code starts to fulfill the BER constraint are given in Table I.

D. LOS-MIMO transmission

Usually MIMO is employed to utilize the decorrelation of the subchannels that is introduced by a rich multipath environment. In [3] and [4] it was shown that a pure LOS channel can give high MIMO gain by placing the antennas in the transmitter and receiver in a smart way. In [5] we studied the *uniform linear array* (ULA) case, and a more general geometrical model was introduced and new parameters were incorporated. The optimal design parameters were given by

$$d_t d_r = \frac{\lambda r}{V \cos\theta_t \cos\theta_r}, \tag{3}$$

where d_t and d_r are the antenna separation distance at the transmitter and the receiver respectively, λ is the wavelength, r is the distance between transmitter and receiver, $V = \max(M, N)$, and θ_t and θ_r are the relative angles between the arrays [5]. From the equation we see that the product between d_t and d_r is the key design quantity. This product is referred to as the *antenna separation product* (ASP).

Number	Code rate	Bit rate (infobit/symbol)	SNIR threshold in dB
1	1/2	1	3.84
2	2/3	2	9.27
3	3/4	3	12.59
4	4/5	4	15.93
5	5/6	5	18.83
6	6/7	6	21.86
7	7/8	7	24.98
8	8/9	8	27.92

TABLE I
SNIR THRESHOLDS FOR LDPC CODED QAM/PSK MODULATION WITH $\text{BER}_0 = 10^{-6}$.

In [5] the sensitivity with regards to deviation from the optimal ASP value in (3) was studied, based on a deviation factor η. The deviation factor is defined as the ratio between the optimal ASP from (3) and the actual ASP (ASP_{opt}/ASP). For example, a deviation factor of 2 can be interpreted as an actual ASP of half the optimal, i.e. too small antenna arrays. It was shown that the LOS-MIMO scheme actually performs better than MIMO based on uncorrelated Rayleigh subchannels for small deviation factors.

E. Capacity

Optimal power allocation with respect to capacity when the channel is known at the transmitter is in accordance with the WF scheme [6]. The capacity is then given by

$$C_{WF} = \sum_{i=1}^{J} \log_2 \left(\xi \frac{\beta \sigma_{H,i}^2}{P_N} \right)^+ \qquad \text{bit/s/Hz}, \tag{4}$$

where P_N is the noise power, "+" denotes taking only the positive terms, and ξ is chosen to satisfy

$$\sigma_{W,i}^2 = \left(\xi - \frac{P_N}{\beta \sigma_{H,i}^2} \right)^+, \qquad i \in \{1, \ldots, J\}. \tag{5}$$

The constraint on the total transmit power is $\sum_{i=1}^{N} \sigma_{W,i}^2 = P_t^{tot}$, as described earlier ($\sigma_{W,i}^2 = 0$ for $i > J$).

III. Channel Modeling

In Fig. 2 there are three parameters which are dependent on the transmission environment, namely β, $\sigma_{H,i}$ and $\tilde{n}_i$. In this section we will discuss them further, and present models that is applicable for our system. We end this section by giving an expression for the SNIR.

A. Propagation model

To predict the signal attenuation for our high frequency case we utilize a propagation model called *Free space + RMD* [8, p.98], where RMD is short for *Reflection/Multiple Diffraction*. This is a physical propagation model, which indicates that it is based on the basic principles of physics rather than statistical outcomes from experiments. Only the LOS attenuation part of the model will be used, and it is given by [8, p.102]

$$L_{LOS} = 32.45 + 20 \log f + 20 \log r + A_r + A_{fr}. \tag{6}$$

Here f is the frequency in GHz, r is the distance, A_r is a reflection attenuation term, and A_{fr} is the attenuation because of Fresnel zone obstruction. For simplicity the two attenuation terms, A_r and A_{fr}, are set to zero dB. This is common for the A_r term when the reflection point is not precisely known. To justify the removal of A_{fr}, we assume that the signal path has sufficient clearance from potential obstacles. If the transmitted signal is exposed to rain, additional attenuation is introduced. The power gain from the transmitter to the receiver at a distance r is given by

$$\beta(dB) = G_t(dB) + G_r(dB) - L_{LOS} - L_{rain}, \tag{7}$$

where G_t and G_r are the transmitter and receiver antenna gain respectively, and L_{rain} is the additional rain attenuation.

To model the rain attenuation we use Crane's rain fade model [8, p.144]. In this model the attenuation is a function of the rain rate, and the distance between the transmitter and receiver. Two regression coefficients, k and α, which are functions of frequency and polarization, are also incorporated in the model. Values for these two parameters can be found in [9].

B. Short term variation

In addition to the extra attenuation, rain introduces a short term stochastic behavior for the received signal. In [10] it was suggested that the received amplitude follows a Ricean distribution. Ricean MIMO channels can be modeled as a sum of two channel matrices, one LOS and one *non-LOS* (NLOS), where the Ricean K factor is defined as the ratio between the power of the two [4]. As mentioned earlier the channel matrix $\mathbf{H}$ is assumed to be normalized, i.e. common path loss is moved out of the matrix. Under a rain event the channel matrix can thus be modeled as

$$\mathbf{H} = \sqrt{\frac{K}{K+1}} \mathbf{H_{LOS}} + \sqrt{\frac{1}{K+1}} \mathbf{H_{NLOS}}. \tag{8}$$

Here K is the Ricean K-factor which is given by [10]

$$K(dB) = \begin{cases} 16.88 \text{ - } 0.04\ R_{rain} & \text{dB} \quad R_{rain} \neq 0 \\ \infty & \text{dB} \quad R_{rain} = 0, \end{cases} \tag{9}$$

where R_{rain} is the rain rate in mm/h. As we see from (8) and (9) the channel matrix becomes a pure LOS matrix when there is no rain.

To be able to investigate the MIMO system the elements of $\mathbf{H_{LOS}}$ and $\mathbf{H_{NLOS}}$ have to be determined. For the LOS matrix the geometrical model presented in [5] is used. This model is based on ULAs at both sides, and the arrays can take any orientation. The amplitudes of the matrix elements are equal to one as described earlier, while a ray tracing technique is used to find the different phases over the receive array. The NLOS matrix elements are determined by the definition of the Rice model as independent complex gaussian (Rayleigh matrix), with variance equal to one because of the normalization.

The equations above show that the performance of a radio link operating at high frequencies is very much dependent on the rain rate. Rain rates are usually given by the probability that the rain exceeds a given value in millimeters per hour. Areas that have high total rainfall could have fewer deep rain fade periods than areas with lower total rainfall. This is because it is the intensity of the rain that decides the fading depth. Worldwide rain rate contour maps can be found in the literature, e.g. [11].

C. Signal-to-noise and-interference ratio

The radio link is exposed to additive white noise. The noise source can for instance be thermal noise at the receiver, or due to interference from co-channel users. The noise power at the detector is given by

$$P_N(dB) = 10 \cdot \log(kT_0B \cdot F + P_I) \qquad \text{dBW}, \tag{10}$$

where k is Boltzmann's constant ($1.38 \cdot 10^{-23}$ J/K), B is the channel bandwidth, T_0 is room temperature (293 K), F is the receiver noise figure, and P_I is the co-channel interference power. The SNIR for subchannel number i then becomes

$$\text{SNIR}_i(dB) = 20\log(\sigma_{W,i}) + 20\log(\sigma_{H,i}) + \beta(dB) - P_N(dB). \quad (11)$$

For a SISO system, $\sigma^2_{W,i}$ is the total transmit power, while $\sigma^2_{H,i}$ has an average value of one. In this case we will denote the average SNIR as $\overline{\gamma}_{SISO}$. This parameter will be utilized when comparing different systems in the results section.

IV. Results

We start by investigating the system performance as a function of the antenna array design for a fixed average SNIR. The quality of the antenna array design is quantized through the deviation parameter η, as described in section II-D. The case investigated could for instance be a *subscriber unit* (SU) in a fixed position with a given average SNIR, and we want to explore how the array design affect the performance.

In Fig. 3 the throughput variation as a function of η is illustrated for 3×3 LDPC coded transmission system with different power allocations schemes. The waterfilling Shannon capacity is also plotted as a reference. From the figure we can clearly see the performance gain by doing a correct antenna array design as the throughput decreases with increasing deviation factor. The figure also illustrates the gain by using QoS-based WF in contrast to the equal power scheme.

Fig. 4 shows the impact of rain rate on the throughput for the same system as in Fig. 3. We observe that when there is no rain the channel is deterministic and so is the throughput (discontinuous curve), while for rain rates grater than zero the channel becomes stochastic and the throughput is averaged (smooth curves). It is interesting to see how the throughput is increased for large values of the deviation factor as the rain rate increases. This is because $\mathbf{H_{LOS}}$ becomes low rank for large deviation factors [5], but increasing rain rates increases

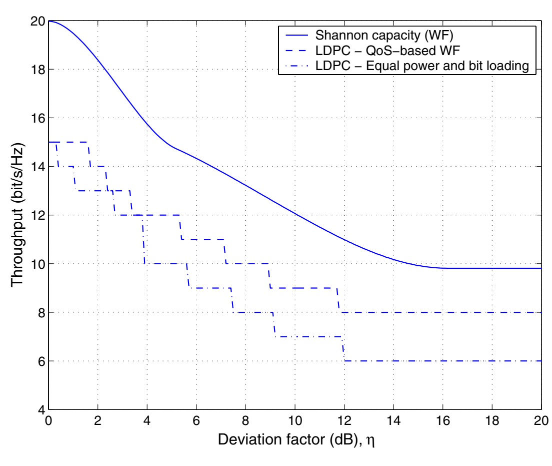

Fig. 3. Throughput as a function of deviation factor (η) for a 3×3 MIMO system with LDPC coded modulation and different power and bit allocation schemes. No rain situation (pure LOS) and $\overline{\gamma}_{SISO} = 20$ dB.

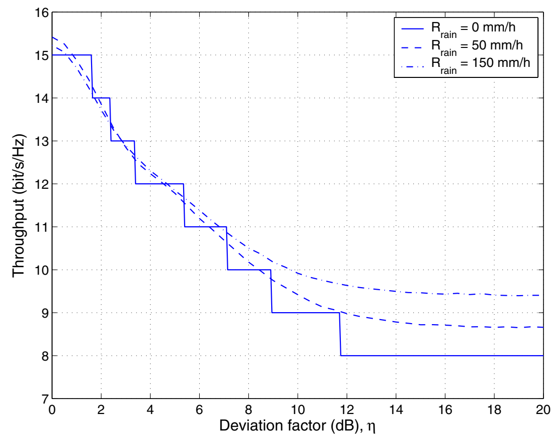

Fig. 4. Throughput as a function of deviation factor (η) for a 3×3 MIMO system with LDPC coded modulation and QoS-based WF power and bit allocation scheme. Different rain rates and $\overline{\gamma}_{SISO} = 20$ dB.

the influence of $\mathbf{H_{NLOS}}$, which is high rank. It is important to note that we have isolated this effect by using a constant average SNIR. From Crane's rain fade model we know that an increasing rain rate also increases the attenuation, and this effect is not taken into account her.

We now go one step further and include the propagation model; thus the average SNIR is not constant any more but a function of the distance r. In this paper we look at the link from the SU to the *base station* (BS), i.e. uplink. The only difference when investigating the downlink is that we need to change the parameter values used; the models remain the same. A set of realistic parameter values are chosen to produce the results. Most of the values are found in [12, app. E], and reproduced for convenience in Table II together with the other parameters employed. The noise power used to produce the results corresponds to a receiver noise figure of 8 dB, and three identical interfering SUs transmitting with the same power and propagation model (no short term variation) as the desired SU at a distance of 5 km (because of frequency reuse).

In Fig. 5 we investigate the effect of constraining the maximum array size. Waterfilling Shannon capacity is plotted for different array size constraints. The BS array size is fixed while the SU array size is adapted perfectly with distance

Parameter	Value
Carrier, f_c	40 GHz
BS antenna gain, G_{BS}	15 dBi
SU transmit power, P_{SU}	-6 dBW
SU antenna gain, G_{SU}	32 dBi
Receiver Noise figure, F	8 dB
Bandwidth, B	28 MHz
Number of interfering SUs, N_I	3
Distance to interfering SUs, r_I	5 km
Relative array angle, θ_t and θ_r from (3)	0°
Regression coefficient, α_v	0.929
Regression coefficient, k_v	0.31

TABLE II

Values used to produce the results

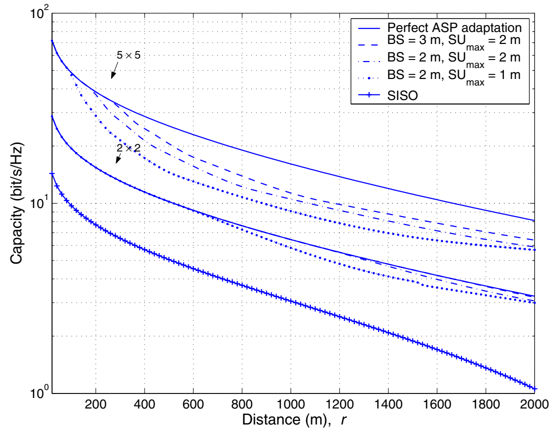

Fig. 5. Capacity as a function of distance for a no rain situation (pure LOS) with array size constraint.

(Eq. (3)) until it reaches its limit, SU_{max}, where the ASP is frozen for longer distances (i.e. not optimal design for these distances). Two different MIMO dimensions are plotted together with a SISO system as reference. A first observation is that the figure supports the promising results of LOS-MIMO found in [5], and a substantial gain over SISO is predicted. Further, the figure shows that to follow the perfect ASP one needs a larger array for larger MIMO dimensions. For the 5×5 case the performance in the different array size constraint situations deviate from the unconstrained performance as the distance increases. The first scenario (BS = 2 m, SU_{max} = 1 m) starts to deviate at approximately 100 m, while the deviation begins at 300 m for the last scenario (BS = 3 m, SU_{max} = 2 m). For the 2×2 case the constrained situations follow the unconstrained much better. The last scenario seems to follow the unconstrained capacity all the way to 2000 m.

In Fig. 6 the throughput for LDPC coded modulation with QoS-based WF is plotted for different rain rates. A 3×3 system is considered, and perfect ASP adaptation is assumed. The figure shows that as the rain rate increases the throughput

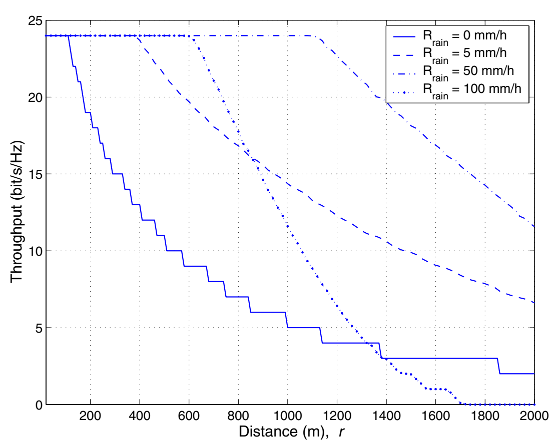

Fig. 6. Throughput as a function of distance for a 3×3 MIMO system with LDPC coded modulation and QoS-based WF power and bit allocation for different rain rates.

also increases. This can be attributed to the reduced co-channel interference power because of the extra attenuation introduced by the rain, giving an increased SNIR. However, as the rain rate continues to increase we see that the throughput decreases, and for large distances becomes less than that of a rain rate of zero. The reason for this is the large extra attenuation that the rain forces on the desired signal, and even if the interference power is reduced, we still have a constant thermal noise. The transmission scheme used seems to fit best to the propagation scenario in the no rain case, as the throughput adapts best with distance. For the other cases, the throughput is constant equal to 24 bit/s/Hz in large regions near the BS, which is maximum throughput for the LDPC codes described ($3 \cdot 8 = 24$). Codes offering higher bit rates could be used here to increase the performance.

V. Conclusion

The combination of transmission on eigenmodes with power and bit allocation using LDPC codes in a LOS-MIMO system is shown to be an interesting way of implementing FWA links at high frequencies. It is shown that much higher throughput is obtained compared to traditional SISO transmission. This is true even if we have some throughput loss when we restrict the array sizes used.

The influence of rain on the radio link is dependent on the combination of many factors. It is beneficial with regards to reduced interference from co-channel users and increased rank for the channel matrix for large deviation factors, but it is negative with respect to extra attenuation of the desired signal.

References

[1] W. Webb, "Broadband fixed wireless access as a key component of the future integrated communications environment," *IEEE Commun. Mag.*, vol. 39, pp. 115–121, Sep. 2001.

[2] T. Tjelta et al., "Future broadband radio access systems for integrated service with flexible resource management," *IEEE Commun. Mag.*, vol. 39, pp. 56–63, Aug. 2001.

[3] D. Gesbert, H. Bölcskei, D. A. Gore, and A. J. Paulraj, "Outdoor MIMO wireless channels: Models and performance prediction," *IEEE Trans. Commun.*, vol. 50, pp. 1926–1934, Dec. 2002.

[4] P. F. Driessen and G. Foschini, "On the capacity formula for multiple input-multiple output wireless channels: A geometric interpretation," *IEEE Trans. Commun.*, vol. 47, pp. 173–176, Feb. 1999.

[5] F. Bøhagen, P. Orten, and G. E. Øien, "Construction and capacity analysis of high-rank line-of-sight MIMO channels," *to appear in Proc. of IEEE WCNC*, 2005.

[6] X. Zhang and B. Ottersten, "Power allocation and bit loading for spatial multiplexing in MIMO systems," *Proc. ICASSP*, vol. 5, pp. 53–56, April 2003.

[7] O. Jetlund, G. E. Øien, K. J. Hole, V. Markhus, and B. Myhre, "Rate-adaptive coding and modulation with LDPC component codes," *COST 273 TD(02) 108*, Sep. 2002.

[8] H. R. Anderson, *Fixed broadband wireless system design*. Wiley, 1 ed., April 2003.

[9] ITU-R Rec. P.838-2, *Specific attenuation model for rain for use in prediction methods*. April 2004.

[10] H. Xu, T. S. Rappaport, R. J. Boyle, and J. H. Schaffner, "Measurements and models for 38-GHz point-to-multipoint radiowave propagation," *IEEE J. Select. Areas Commun.*, vol. 18, pp. 310–321, April 2000.

[11] ITU-R Rec. P.837-4, *Characteristics of precipitation for propagation modeling*. April 2003.

[12] IEEE computer society, *802.16.2, IEEE recommended practice for local and metropolitan area networks, coexistence of fixed broadband wireless access systems*. Sep. 2001,
Available from: http://standards.ieee.org/getieee802/802.16.html.