Performance Analysis of a Rate-Adaptive Dual-Branch Switched Diversity System

Bengt Holter* and Geir E. Øien†

Abstract—In this paper, a performance analysis of a dual-branch switched diversity system operating on identically distributed Nakagami-m fading channels is presented. An adaptive coded modulation (ACM) scheme is employed to increase the spectral efficiency of the system. The ACM scheme consists of a set of multidimensional trellis codes originally designed for additive white Gaussian noise channels, where the codes are based on quadrature amplitude modulation (QAM) signal constellations of varying size. The performance is evaluated by assuming perfect channel knowledge at the receiver and instantaneous feedback of channel state information, conveyed from the receiver to the transmitter on a zero-error feedback channel. Both spatially uncorrelated and correlated antenna branches are considered. The optimal switching threshold in terms of maximizing the average spectral efficiency is identified in the case of uncorrelated antenna branches.

I. INTRODUCTION

Adaptive modulation is an efficient transmission scheme to simultaneously achieve both high spectral efficiency and a low bit error rate (BER) in a wireless communication system. The basic principle is to utilize channel state information (CSI) conveyed from the receiver to the transmitter to adapt to changing channel conditions by transmitting with high information rates under favorable channel conditions, and reducing the information rate in response to channel degradation.

In [1], [2], a method for assessing performance merits of an adaptive coded modulation (ACM) system is employed to evaluate the average spectral efficiency (ASE) of a rate-adaptive coding scheme utilizing a set of multidimensional trellis codes originally designed for additive white Gaussian noise (AWGN) channels. The analysis is based on a single-input multiple-output (SIMO) channel model with statistically independent and identically distributed (i.i.d.) Rayleigh fading channels. Perfect coherent detection is assumed and maximum ratio combining (MRC) is employed to maximize the received signal-to-noise ratio (SNR).

The MRC receiver is frequently used in the literature for analysis purposes as a benchmark receiver, since it represents the optimal (in a maximum SNR sense) diversity scheme in the absence of interference. However, it also represents the highest possible spectral efficiency.

1When interference is present, the optimal combining scheme is denoted an optimum combiner, maximizing the instantaneous received signal-to-interference plus noise ratio (SINR) [3]

2In [16], an uncoded adaptive modulation scheme is utilized to jointly minimize the number of combined diversity branches while providing the highest possible spectral efficiency.
II. SYSTEM AND CHANNEL MODEL

A single user dual-branch SIMO system operating on two i.i.d. Nakagami-\(m\) fading channels is considered. The transmission rate on the wireless link is adaptively adjusted by utilizing a rate-adaptive coding scheme, using a set of \(N\) multi-dimensional trellis codes originally designed for AWGN channels. The codes are based on quadrature amplitude modulation (QAM) signal constellations, and rate adaptation is performed by splitting the received SNR range into \(N+1\) fading regions (bins) as depicted in Figure 1. The set \(\{\gamma_n\}_{n=1}^N\) contains the lower thresholds of the \(N\) fading regions, selected such that a target BER—denoted \(\text{BER}_0\)—is achieved for each available code in the rate-adaptive scheme. Let \(\gamma_{\text{sec}}\) denote the SNR per symbol at the output of the SSC combiner, and let \(\gamma_T\) denote the predetermined switching threshold. When \(\gamma_{\text{sec}}\) falls within fading region \(n\) (\(\gamma_n \leq \gamma_{\text{sec}} < \gamma_{n+1}\)), the associated CSI, i.e. the fading region index \(n\), is sent back to the transmitter through a dedicated feedback channel. Based on the reported CSI, the transmitter adapts its transmission rate according to the quality of the channel, by transmitting with a signal constellation realizing a spectral efficiency of \(R_n\). If \(0 \leq \gamma_{\text{sec}} < \gamma_1\), no information is transmitted (outage). In this paper, the following three idealized assumptions are employed: (i) the receiver has perfect channel knowledge (ii) instantaneous feedback (iii) the feedback channel is zero-error.

For i.i.d. channels, the probability density function (PDF) of the output SNR of a dual-branch SSC combiner may be written as [4]

\[
p_{\gamma_{\text{sec}}} (\gamma) = \begin{cases} P_b(\gamma_T) \cdot p_{\gamma} (\gamma) & \gamma < \gamma_T \\ (1 + P_b(\gamma_T)) \cdot p_{\gamma} (\gamma) & \gamma \geq \gamma_T \end{cases}
\]

where \(p_\gamma(\gamma)\) and \(P_\gamma\) are the PDF and the cumulative distribution function (CDF) of the output SNR per symbol \(\gamma\) for a single branch receiver, respectively. In Table I, expressions for the PDF and CDF are presented for the Nakagami-\(m\) fading model, where \(\overline{\gamma}\) represents the average SNR on a single channel, \(\beta = \overline{\gamma}/m\), \(\Gamma(\cdot)\) is the gamma function [17, Sec. 8.31], and \(\Gamma(\cdot, \cdot)\) is the complementary incomplete gamma function [17, Sec. 8.35].

III. ASE AND BER ANALYSIS

The ASE [bits/s/Hz] of the system is obtained as a weighted sum of the spectral efficiencies of each individual code, i.e. \(\text{ASE} = \sum_{n=1}^N R_n P_n\), where \(P_n = \int_{\gamma_n}^{\gamma_{n+1}} p_{\gamma_{\text{sec}}} (\gamma) d\gamma\) is the probability that code \(n\) is used. The average BER (averaged over all codes and all SNRs) is given as the average number of bits in error, divided by the average number of bits transmitted [2]

\[
\text{BER} = \frac{\sum_{n=1}^N R_n \cdot \text{BER}_n}{\sum_{n=1}^N R_n \cdot P_n},
\]

where \(\text{BER}_n\) is the average BER experienced when code \(n\) is applied. It is obtained as \(\text{BER}_n = \frac{1}{\text{BER}_0} \int_{\gamma_n}^{\gamma_{n+1}} p_{\gamma_{\text{sec}}} (\gamma) d\gamma\), where \(\text{BER}_n = a_n \exp \left(-b_n \gamma / M_n\right)\) is the instantaneous BER experienced for code \(n\) and \(a_n\) and \(b_n\) are code-dependent constants found by least-square fitting to simulated data on AWGN channels. Since the expression for \(\text{BER}_n\) is invertible, the thresholds in the set \(\{\gamma_n\}_{n=1}^N\) can be identified as \(\gamma_n = (M_{n+1}/b_n) \ln (a_n / \text{BER}_0)\). In Table II, values for \(a_n, b_n, M_n,\) and \(\gamma_n\) are summarized for the target \(\text{BER}_0 = 10^{-4}\) and \(N = 8\) (number of codes used in the numerical examples). Restricting \(\gamma_T\) to be larger than \(\gamma_1\), \(P_n\) may in general be divided into three parts:

\[
P_n = \begin{cases} \rho_{\text{left}} (n) & \text{for } \gamma_{n+1} < \gamma_T \\ \rho_{\text{middle}} (n) & \text{for } \gamma_1 < \gamma_T < \gamma_{n+1} \\ \rho_{\text{right}} (n) & \text{for } \gamma_T \geq \gamma_n \end{cases}
\]

where \(I(n) = \int_{\gamma_n}^{\gamma_{n+1}} p_{\gamma_{\text{sec}}} (\gamma) d\gamma\) and \(I_{\gamma_T} (n) = \int_{\gamma_{\text{sec}}}^{\gamma_T} p_{\gamma} (\gamma) d\gamma\).

The superscripts in (3) are introduced to distinguish between the different expressions for \(P_n\), depending on the placement.

\(\gamma_n + 1 = \infty.\)

![Fig. 1. The SNR range is split into \(N+1\) bins. When the instantaneous SNR falls in the lowest interval, an outage occurs; whereas in the upper \(N\) intervals, a code with spectral efficiency \(R_n\) is employed.](image)

<table>
<thead>
<tr>
<th>Model</th>
<th>Parameter</th>
<th>PDF ((p_\gamma(\gamma)))</th>
<th>CDF ((P_\gamma(\gamma)))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nakagami-(m)</td>
<td>(m \geq \frac{1}{2})</td>
<td>(\overline{\gamma}^{m-1} e^{-\overline{\gamma}/m})</td>
<td>(1 - \frac{\Gamma(m, \gamma/\beta)}{\Gamma(m)})</td>
</tr>
</tbody>
</table>

\(\overline{\gamma}\)

\(\gamma_n\) [dB]

\(n\)

\(M_n\)

\(a_n\)

\(b_n\)

\(\gamma_n\) [dB]| 1 | 4 | 188.7471 | 9.8118 | 7.77 |
<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>8</td>
<td>288.8051</td>
<td>6.8792</td>
<td>12.4</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>16</td>
<td>161.6898</td>
<td>7.8862</td>
<td>14.6</td>
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</tr>
<tr>
<td>4</td>
<td>32</td>
<td>142.6920</td>
<td>7.8264</td>
<td>17.6</td>
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<tr>
<td>5</td>
<td>64</td>
<td>126.2118</td>
<td>7.4931</td>
<td>20.8</td>
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</tr>
<tr>
<td>6</td>
<td>128</td>
<td>121.5189</td>
<td>7.7013</td>
<td>23.7</td>
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<tr>
<td>7</td>
<td>256</td>
<td>79.8360</td>
<td>7.1450</td>
<td>26.9</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>512</td>
<td>34.6128</td>
<td>6.9190</td>
<td>29.7</td>
<td></td>
</tr>
</tbody>
</table>

For simplicity, \(\gamma_T\) is restricted not to reside within the interval range \(0 \leq \gamma < \gamma_1\) (outage region).

| \(n\) | \(M_n\) | \(a_n\) | \(b_n\) | \(\gamma_n\) [dB] |
|---|---|---|---|---|---|
| 1 | 4 | 188.7471 | 9.8118 | 7.77 |
| 2 | 8 | 288.8051 | 6.8792 | 12.4 |
| 3 | 16 | 161.6898 | 7.8862 | 14.6 |
| 4 | 32 | 142.6920 | 7.8264 | 17.6 |
| 5 | 64 | 126.2118 | 7.4931 | 20.8 |
| 6 | 128 | 121.5189 | 7.7013 | 23.7 |
| 7 | 256 | 79.8360 | 7.1450 | 26.9 |
| 8 | 512 | 34.6128 | 6.9190 | 29.7 |
of $\gamma_T$ on the SNR axis in Figure 1. For the fading bins located to the left of $\gamma_T$, $P_n = P_{\text{left}}^n$. For the fading bin containing $\gamma_T$, $P_n = P_{\text{middle}}^n$. For the fading bins located to the right of $\gamma_T$, $P_n = P_{\text{right}}^n$. Similarly, $\gamma_T$ can be expressed as

$$
\begin{align*}
\gamma_T^{\text{left}} &= J(n) \cdot P_{\gamma_T}^{\gamma_T} \gamma_{n+1} \leq \gamma_T \\
\gamma_T^{\text{middle}} &= J(n) + \gamma_T^{\text{right}} \quad \gamma_n < \gamma_T \leq \gamma_{n+1}, \quad (4)
\end{align*}
$$

where $J(n) = \int_{\gamma_n}^{\gamma_{n+1}} a_n e^{-\frac{\gamma}{\beta} m_q} p_{\gamma}(\gamma) d\gamma$ and $J_{\gamma_T(n)} = \int_{\gamma_n}^{\gamma_{n+1}} a_n e^{-\frac{\gamma}{\beta} m_q} p_{\gamma}(\gamma) d\gamma$. The expressions in (3) and (4) are applicable to any fading distribution.

In the following, the index $q \in [1, 2, \ldots, N]$ is used to indicate the fading bin in which $\gamma_T$ is placed. In particular, $q = n$ if $\gamma_n \leq \gamma_T \leq \gamma_{n+1}$. For a system operating on i.i.d. Nakagami-$m$ fading channels, the ASE and the average BER may then be written compactly as

$$
\text{ASE} = (R_q - A_1) \bar{\Gamma}(m, \gamma_T/\beta) + T_q,
$$

and

$$
\text{BER} = \frac{R_q C_T \Gamma(m, \mu_q \gamma_T) - B_T \Gamma(m, \gamma_T/\beta) + S_q}{(R_q - A_1) \bar{\Gamma}(m, \gamma_T/\beta) + T_q},
$$

respectively. The expressions involved are defined as $A_1 = \sum_{n=1}^{N} R_n \bar{\Gamma}(n)$, $B_1 = \sum_{n=1}^{N} R_n J(n)$, $T_q = A_1 + \gamma_q^{n+1} - R_q \bar{\Gamma}(m, \gamma_q^{n+1}/\beta)$, $S_q = B_1 + \gamma_q^{n+1} - R_q C_T \Gamma(m, \mu_q \gamma_q^{n+1})$, $\gamma_q^{n+1} = \sum_{n=q+1}^{N} R_n \bar{\Gamma}(n)$, $C_q = \frac{\mu_q}{\gamma_q^{n+1}}$, and $\mu_q = (\beta q + M_q)/(M_q \beta)^2$. Note that both (5) and (6) are functions of $q$, $\gamma_T$, $\beta$, and $m$.

**IV. MAXIMIZING THE ASE**

In the following, $\gamma_T$ which maximizes the ASE within a single fading bin $q$ is determined by assuming that $\beta$ and $m$ are fixed. Then, based on this result, the optimal fading bin $q^*$ and the associated optimal threshold $\gamma_T^{*}$, maximizing the ASE over all fading bins, is determined. Since the ASE is a key performance measure only when the system is operating at acceptable BER levels (below BER$_0$), the optimization of (5) can formally be defined as an optimization problem with a side constraint: maximize ASE subject to BER $\leq$ BER$_0$.

However, since BER $\leq$ BER$_0$ is used as a design criteria to obtain the thresholds $\{\gamma_n\}_{n=1}^{N}$, both the instantaneous and the average BER are guaranteed to be lower than or equal to BER$_0$ as long as perfect channel knowledge is assumed [18]. Hence, the side constraint is already introduced, and the ASE and BER in $\gamma_T$ may be optimized as if it represented an unconstrained optimization problem. The partial derivative of (5) with respect to $\gamma_T$ is equal to

$$
\frac{\partial \text{ASE}}{\partial \gamma_T} = -(R_q - A_1) \cdot G_{\gamma_T}(m, \beta),
$$

where $G_{\gamma_T}(m, \beta) = \frac{\gamma_{n+1} - \gamma_T/\beta}{\gamma_{n+1} - \gamma_T/\beta}$. Since $G_{\gamma_T}(m, \beta) > 0$ for $1/2 \leq m \leq 4$, $\bar{\gamma} \geq 0$ dB, and $0 < \gamma_T < \infty$, the derivative has the following properties when $\gamma_q \leq \gamma_T < \gamma_{q+1}$: (1) monotonically decreasing if $R_q - A_1 > 0$, which implies $\gamma_T = \gamma_q$ (2) monotonically increasing if $R_q - A_1 < 0$, which implies $\gamma_T = \gamma_{q+1}$. Hence, the threshold $\gamma_T$ which maximizes the ASE within bin $q$ is one of the two fading bin endpoints. In Figure 2, $R_q - A_1$ is depicted as function of $\bar{\gamma}$, $m = 1$, and $q = [1, 2, \ldots, 7]$. It is observed that $R_q - A_1$

![Fig. 2. The size of $R_q - A_1$ for $m = 1$, governing the sign of the gradient in (7).](image)

is positive at low average SNR and negative at high average SNR. Hence, $\gamma_T = \gamma_q$ for low average SNR, and $\gamma_T = \gamma_{q+1}$ for high average SNR. The switching point between these two solutions, denoted $\bar{\gamma}^*$, can be determined from the equality condition $R_q = A_1$. Using Newton’s method, $\bar{\gamma}^*$ can be determined by means of the recursion in (13) (see next page), where $\rho_i = 0, 1, \ldots$ and $\beta_i = \bar{\gamma}^*(i)/m$. The initial value $\bar{\gamma}^*(0)$ can be selected as $\gamma_i$. Knowing that the set of thresholds maximizing the ASE within bin $q$ is limited to either $\gamma_q$ or $\gamma_{q+1}$, the optimal threshold $\gamma_T^{*}$, maximizing the ASE among all fading bins, is derived as follows:

$$
\gamma_T^{*} = \gamma_q^* \quad \text{where} \quad q^* = \arg \max_{i=1}^{N} \text{ASE}_{\gamma_T = \gamma_i}.
$$

In Figure 4, $\gamma_T^{*}$ is depicted as a function of $\bar{\gamma}$ for $m = \{1, 2, 4\}$.

**V. IMPACT OF BRANCH CORRELATION**

Spatial correlation between the antenna branches may arise e.g. from insufficient antenna spacing in small-size terminals. For identically distributed but spatially correlated Nakagami-$m$ fading channels, the PDF of the output SNR may be written as [7]

$$
\begin{align*}
p_{\gamma_{\text{sc}}}(\gamma) = \left\{ \begin{array}{ll}
g_{\gamma}(m, \beta) \cdot (1 - Q_m(\alpha_s \sqrt{\gamma}, \beta_s)) & \gamma \leq \gamma_T \\
g_{\gamma}(m, \beta) \cdot (2 - Q_m(\alpha_s \sqrt{\gamma}, \beta_s)) & \gamma > \gamma_T
\end{array} \right.,
\end{align*}
$$

where $g_{\gamma}(m, \beta) = p_{\gamma}(\gamma)$ in Table I, $Q_m(\cdot, \cdot)$ is the generalized Marcum-Q function, $\alpha_s = \sqrt{\frac{2 \rho}{2 - \rho}}$, $\beta_s = \frac{1 - p_{\gamma}}{\sqrt{1 - p_{\gamma}}}$, and $\rho$ denotes the normalized spatial power correlation coefficient. Using the same type of analysis as presented in Section III,
closed-form expressions for the ASE the average BER can be derived. However, due to space limitations, the explicit expressions are omitted in this paper.

VI. NUMERICAL RESULTS

Numerical results are presented for BER$_0 = 10^{-4}$, $N = 8$, $\{M_n\}_{n=1}^{N} = \{4, 8, 16, 32, 64, 128, 256, 512\}$, and $\{R_n\}_{n=1}^{N} = \{1.5, 2.5, \ldots, 8.5\}$. In Figure 3, the maximum ASE of the SSC combiner is compared to dual-branch MRC and selection combining (SC) when operating on i.i.d. Rayleigh fading channels. For the SSC combiner, the ASE is maximized by employing the optimal switching threshold $\gamma_T^*$ for each value of $\overline{\gamma}$. As a result, the performance of SSC is close to the performance of both MRC and SC. In Figure 4, $\gamma_T^*$ used to generate the SSC curve in Figure 3 is depicted as a function of $\overline{\gamma}$ and the Nakagami-$m$ fading parameter. It is a stepwise function where the discrete values of $\gamma_T^*$ are identical to the predefined thresholds $\{\gamma_n\}_{n=1}^{N}$ of the rate-adaptive scheme. As the Nakagami-$m$ parameter is increased, the stepwise function is shifted to the left, reflecting that an increase in $\gamma_T^*$ occurs at an earlier stage when the channels become more stable.

In Figure 5, the average BER is depicted when operating on i.i.d. Nakagami-$m$ fading channels with $m \in \{1, 2\}$. For SSC, results are depicted when $\gamma_T^*$ is used as switching threshold. The target BER$_0 = 10^{-4}$ is achieved, and the performance is almost identical to MRC and SC at low and medium $\overline{\gamma}$. Note that the diversity advantage of SSC is lost at high $\overline{\gamma}$. The reason is that $\gamma_T^*$ is not increased beyond $\gamma_N$, the highest threshold of the rate-adaptive scheme. Since the instantaneous SNR easily exceeds $\gamma_N$ when $\overline{\gamma}$ is large, the switching rate between the branches will tend to decrease. When the switching rate is reduced, the SSC receiver approaches a single branch receiver. This effect is reflected in Figure 5 for SSC at high average SNR, since the average BER curves have slopes equal to one and two for $m = 1$ and $m = 2$, respectively. The slopes are then identical to the average BER slopes of a single branch receiver for $m = 1$ and $m = 2$, respectively.

In Figures 6 and 7, the impact of fading correlation is depicted when operating on identically distributed Rayleigh fading channels. The switching threshold $\gamma_T$ is now chosen to be fixed at $\gamma_T = 17.6$ dB, to illustrate how the performance is affected by not adapting $\gamma_T$ to $\overline{\gamma}$ on the channels. According to Figure 4, $\gamma_T = 17.6$ dB maximizes the ASE for spatially uncorrelated channels in the vicinity of $\overline{\gamma} = 20$ dB. Indeed, in Figure 6, it is observed that the ASE for spatially uncorrelated channels and $\gamma_T = 17.6$ dB is identical to the maximum ASE

\[
\gamma^*(i + 1) = \overline{\gamma}^*(i) + \frac{R_0 - A_1}{\sum_{n=1}^{N} R_n \left( \frac{\gamma_n}{(G_{m,\beta})} \gamma_n(m, \beta_n) - \frac{\gamma_{n+1}}{(G_{m,\beta})} \gamma_{n+1}(m, \beta_n) \right)}.
\]
the optimal switching threshold, the performance is close to average BER, but at a lower complexity. It has also been shown that in order to maximize the ASE for a given average SNR on the channels, the switching threshold $\gamma_T$ of the switched diversity receiver must be identical to one of the predefined thresholds of the rate-adaptive scheme.

**REFERENCES**


