Performance of Buffered Adaptive Transmission *

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Abstract

Adaptive transmission can maximize the average information rate on a time-varying channel for a given power constraint and target bit error rate (BER), by adapting certain parameters of the transmitted signal to the temporal variations of the channel. Since the outage probability of such schemes can be quite high, especially for channels with low average signal-to-noise ratio (SNR), adaptive systems will require buffering of the input data. In this paper, we present some analytical results for the performance analysis of buffered adaptive transmission in terms of the probability of over flow for two scenarios: (i) constant deterministic incoming data arrival process (which includes the discussions for a continuous fading channel model and a discrete Markov channel model, respectively) and (ii) Poisson data arrival process. Numerical examples are also provided and discussed to illustrate the mathematical formalism and the effects of various system parameters.

1 Introduction

Adaptive transmission can maximize the average information rate on a time-varying channel for a given power constraint and target bit error rate (BER), by adapting certain parameters of the transmitted signal to the temporal variations of the channel. This transmission technique relies essentially on real-time balancing of the link budget through adaptive variation of the transmitted power level, symbol transmission rate, constellation size, coding rate/scheme, or any combination of these parameters while meeting a target BER requirement. Thus, adaptive techniques, without loss of performance, provide a higher average link spectral efficiency by taking advantage of the time-varying nature of wireless channels: transmitting at high rates under favorable channel conditions and reducing the data throughput as the channel worsens [1, 2, 3]. Since the outage probability of such schemes can be quite high, especially for channels with low average signal-to-noise ratio (SNR), adaptive systems will require buffering of the input data. The focus of this paper is the overflow performance analysis of buffered adaptive transmission.

The remainder of this paper is organized as follows. The next section describes the system model under consideration and formulates the buffering problem in adaptive systems. In section 3, the performance of buffered adaptive transmission is analyzed in terms of the probability of overflow for two scenarios: (i) constant deterministic incoming data arrival process (which includes the discussions for a continuous fading channel model and a discrete Markov channel model, respectively) and (ii) Poisson data arrival process. Numerical examples are also provided and discussed in this section to illustrate the mathematical formalism of the derived analytical results and the effects of various system parameters.

2 System Model and Problem Setup

We consider an adaptive transmission scheme employing adaptive discrete rate (ADR) multi-level quadrature amplitude modulation (M-QAM) schemes with constellation sizes $M_n = 2^n$ for $n \in S_n = \{0, 1, 2, \cdots, N\}$

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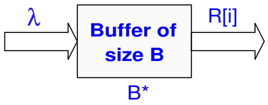

Figure 1: Model of buffered adaptive transmission.

and corresponding transmission rates $R = \{R_0, R_1, R_2, \cdots, R_N\}$. We assuming perfect channel estimation and negligible time delay between channel estimation and signal set adaptation. At the incoming data source side we maintain a buffer with fixed size B, see Fig. 1. Let λ be the data arrival rate, B^* denote the number of buffer units occupied at arbitrary time point, and $\tau[i]$ be the duration of time slots i for which the transmission system keeps its transmission rate $R[i] \in R$ unchanged. We are interested here in the probability of the overflow of the buffer when the transmission rate adapts to the time varying channel, which is defined as the probability of $B^* > B$. In this paper we will consider the following two scenarios: first we will study the case that the data arrival is a deterministic process with constant rate λ and then the case that the data arrive according to a Poisson process with arrival rate λ.

3 Overflow Analysis of Buffered Adaptive Transmission

3.1 Case I: Deterministic Arrival Process

Consider the buffered adaptive transmission system in which the incoming data arrive deterministically with constant rate λ. Let $B[i]$ is the number of buffer units occupied at the end of time slots i which can be written as

$$B[i] = \max\{B[i-1] + \tau[i](\lambda - R[i]), 0\}. \tag{1}$$

Noting that the transmission rate is changed only at the beginning of each time slot, the overflow probability P_{of}^{D} for this case can be specified as $P_{\text{of}}^{\text{D}} = \Pr[B^* > B] = \Pr[B[i] > B]$. Since $B[i] > B$ is equivalent to $B[i-1] + \tau[i](\lambda - R[i]) > B$, this overflow probability can be written by applying the total probability theorem as

$$\begin{aligned} P_{\text{of}}^{\text{D}} &= \sum_{j=0}^{N} \Pr[B[i-1] + \tau[i](\lambda - R[i]) > B \mid R[i] = R_j] \Pr[R[i] = R_j] \\ &= \sum_{j=0}^{N} \Pr[B[i-1] + \tau_j(\lambda - R_j) > B] \Pr[R[i] = R_j], \end{aligned} \tag{2}$$

where we take $\tau[i] = \tau_j$ when $R[i] = R_j$. For the simplest memoryless buffer case (i.e., assume $B[i-1] = 0$), (2) reduces to

$$\begin{aligned} P_{\text{of}}^{\text{D}} &= \sum_{j=0}^{N} \Pr[\tau_j(\lambda - R_j) > B] \Pr[R[i] = R_j] \\ &= \sum_{j=0}^{J} \Pr\left[\tau_j > \frac{B}{(\lambda - R_j)}\right] \Pr[R[i] = R_j], \end{aligned} \tag{3}$$

where J is an integer such that $R_J < \lambda \leq R_{J+1}$ (we assume $R_{N+1} = +\infty$ for this comparison purpose so that the largest J could be N). In the following, we will use Eqn. (3) to evaluate the overflow performance of the buffered adaptive transmission with deterministic data arrival for two kinds of Rayleigh fading channel models: conventional continuous model and finite-state Markov model.

3.1.1 General Formula for the Conventional Continuous Model

For ADR M-QAM, $\Pr[R[i] = R_j]$ can be obtained by [2, Eqn. (34)] with $m = 1$ as

$$\Pr[R[i] = R_j] = \exp\left(-\frac{\gamma_j}{\bar{\gamma}}\right) - \exp\left(-\frac{\gamma_{j+1}}{\bar{\gamma}}\right), \tag{4}$$

where $\overline{\gamma}$ is the average received SNR. If the target BER is BER_0, the region boundaries $\{\gamma_j\}$ are given by [2, Eqn. (30)] as

$$\begin{aligned}
\gamma_1 &= [\text{erfc}^{-1}(2\ \text{BER}_0)]^2, \\
\gamma_j &= -\frac{2}{3}\ln(5\ \text{BER}_0)(2^j - 1); \qquad j = 0, 2, 3, \cdots, N, \\
\gamma_{N+1} &= +\infty,
\end{aligned} \tag{5}$$

where $\text{erfc}^{-1}(\cdot)$ denotes the inverse complementary error function. Let τ_{γ_j} be the duration for which the SNR process falls below level γ_j and $N(\gamma_j)$ be the level crossing rate of level γ_j for the SNR process, which is defined as the average number of times per unit interval that a fading signal crosses a given signal level in the positive direction only (or in the negative direction only). One can show that the duration τ_j of the transmission rate R_j can be approximated as

$$\tau_j = \frac{\tau_{\gamma_{j+1}} - \tau_{\gamma_j}\tau_{\gamma_{j+1}}N(\gamma_j)}{1 + \tau_{\gamma_{j+1}}N(\gamma_j)}, \tag{6}$$

where $N(\gamma_j)$ is provided by [4, Eqn. (3)] as

$$N(\gamma_j) = \sqrt{\frac{2\pi\gamma_j}{\overline{\gamma}}} f_m \exp\left(-\frac{\gamma_j}{\overline{\gamma}}\right), \tag{7}$$

where f_m is the maximum Doppler frequency. Recall that the asymptotic probability density function (PDF) of duration τ_{γ_j} is given by [5, Sec. IV] as

$$f_{\tau_{\gamma_j}}(t) = \frac{3\gamma_j^{\frac{3}{2}}}{4\pi^{\frac{5}{2}} f_m^3 \overline{\gamma}^{\frac{3}{2}} t^4}\, {}_1F_1\left(\frac{5}{2}, 3, -\frac{\gamma_j}{\pi^2 f_m^2 \overline{\gamma} t^2}\right), \tag{8}$$

where ${}_1F_1(\cdot,\cdot,\cdot)$ is the degenerate hypergeometric function [6]. Assuming τ_{γ_j} and $\tau_{\gamma_{j+1}}$ are independent, the PDF of τ_j in (6) can then be obtained by applying the Jacobian transformation method as

$$\begin{aligned}
f_{\tau_j,\tau_{\gamma_{j+1}}}(t, t_{j+1}) &= f_{\tau_{\gamma_{j+1}}}(t_{j+1}) f_{\tau_{\gamma_j}}\left(\frac{t_{j+1} - t(1 + t_{j+1}N(\gamma_j))}{t_{j+1}N(\gamma_j)}\right) \frac{1 + t_{j+1}N(\gamma_j)}{t_{j+1}N(\gamma_j)} \\
&= \frac{9\gamma_j^{\frac{3}{2}}\gamma_{j+1}^{\frac{3}{2}N^3(\gamma_j)}}{16\pi^5 f_m^6 \overline{\gamma}^3 (t_{j+1} - t(1 + t_{j+1}N(\gamma_j)))^4} \frac{1 + t_{j+1}N(\gamma_j)}{t_{j+1}} \\
&\times\ {}_1F_1\left(\frac{5}{2}, 3, -\frac{\gamma_{j+1}}{\pi^2 f_m^2 \overline{\gamma} t^2}\right) {}_1F_1\left(\frac{5}{2}, 3, -\frac{\gamma_j t_{j+1}^2 N^2(\gamma_j)}{\pi^2 f_m^2 \overline{\gamma} t^2 (t_{j+1} - t(1 + t_{j+1}N(\gamma_j)))^2}\right),
\end{aligned} \tag{9}$$

and further averaging over $\tau_{\gamma_{j+1}}$ which yields $f_{\tau_j}(t) = \int_0^\infty f_{\tau_j,\tau_{\gamma_{j+1}}}(t, t_{j+1}) dt_{j+1}$. With $f_{\tau_j}(t)$ in hand, $\Pr\left[\tau_j > \frac{B}{(\lambda - R_j)}\right]$ is just the complementary of the cumulative distribution function (CDF) of τ_j which can be obtained by

$$\begin{aligned}
&\Pr\left[\tau_j > \frac{B}{(\lambda - R_j)}\right] \\
&= \int_{\frac{B}{\lambda - R_j}}^\infty \int_0^\infty \frac{9\gamma_j^{\frac{3}{2}}\gamma_{j+1}^{\frac{3}{2}N^3(\gamma_j)}}{16\pi^5 f_m^6 \overline{\gamma}^3 (t_{j+1} - t(1 + t_{j+1}N(\gamma_j)))^4} \frac{1 + t_{j+1}N(\gamma_j)}{t_{j+1}} \\
&\times\ {}_1F_1\left(\frac{5}{2}, 3, -\frac{\gamma_{j+1}}{\pi^2 f_m^2 \overline{\gamma} t^2}\right) {}_1F_1\left(\frac{5}{2}, 3, -\frac{\gamma_j t_{j+1}^2 N^2(\gamma_j)}{\pi^2 f_m^2 \overline{\gamma} t^2 (t_{j+1} - t(1 + t_{j+1}N(\gamma_j)))^2}\right) dt_{j+1} dt.
\end{aligned} \tag{10}$$

Hence, substituting (10) and (4) into (3), we get the overflow probability for the Rayleigh fading channel in the conventional continuous model.

3.1.2 General Formula for the Finite-State Markov Model

In this section we assume that the channel is described by a discrete-time $(N+1)$-state Markov model. Each of the states corresponds to one of the $N+1$ possible transmission rates $R = \{R_0, R_1, R_2, \cdots, R_N\}$

and the region boundaries (or switching thresholds) is given in (5). The channel is in state k if the SNR is between γ_k and γ_{k+1} (or equivalently the transmission rate is R_k). If we assume that the transitions only happen between adjacent states which is applicable for the slow fading case, we have transition probabilities

$$P_{k,i} = 0, \quad \text{if } |k-i| > 1 \tag{11}$$

and approximately [4, Eqns. (10) and (11)]

$$\begin{aligned} P_{k,k+1} &\approx \frac{N(\gamma_{k+1})T_p}{\pi_k}, \quad \text{if } k = 0, 1, \cdots, N-1 \\ P_{k,k-1} &\approx \frac{N(\gamma_k)T_p}{\pi_k}, \quad \quad \text{if } k = 1, 2, \cdots, N, \end{aligned} \tag{12}$$

where $N(\gamma_k)$ is given in (7), T_p is one packet time period, and π_k is the steady-state probability which is provided by [4, Eqn. (5)]

$$\pi_k = \exp\left(-\frac{\gamma_k}{\bar{\gamma}}\right) - \exp\left(-\frac{\gamma_{k+1}}{\bar{\gamma}}\right). \tag{13}$$

For this finite-state Markov model, $\Pr[R[i] = R_j]$ is just the steady-state probability π_j. By applying the method provided in [7], the probability $\Pr[\tau_j > \tau T_p]$ can be obtained as

$$\Pr[\tau_j > \tau T_p] = \frac{\boldsymbol{\pi}_{\bar{j}} \boldsymbol{P}_j^{\tau-1} \boldsymbol{P} \boldsymbol{e}_j}{\boldsymbol{\pi}_{\bar{j}} \boldsymbol{P} \boldsymbol{e}_j}, \tag{14}$$

where $\boldsymbol{\pi}_{\bar{j}}$ is obtained from $\boldsymbol{\pi} = [\pi_0, \pi_1, \cdots, \pi_N]$ by setting to zero the entry corresponding to state j, $\boldsymbol{P}$ is the transition matrix with entries $P_{k,i}$, $\boldsymbol{e}_j$ is a column vector whose $(j+1)$-th entry is 1 and others are 0s, and $\boldsymbol{P}_j$ is obtained from $\boldsymbol{P}$ by setting to zero all $P_{k,i}$ with $i \neq j$. Therefore, one can get $\Pr\left[\tau_j > \frac{B}{(\lambda - R_j)}\right]$ by evaluating (14) at $\tau = [\frac{B}{T_p(\lambda - R_j)}]$, where $[\cdot]$ denotes the operation rounding up to the closest integer. Finally, the overflow probability for the Rayleigh fading channel in the finite-state Markov model case can be given as

$$P_{\text{of}}^{\text{D}} = \sum_{j=0}^{J} \pi_j \frac{\boldsymbol{\pi}_{\bar{j}} \boldsymbol{P}_j^{[\frac{B}{T_p(\lambda - R_j)}]-1} \boldsymbol{P} \boldsymbol{e}_j}{\boldsymbol{\pi}_{\bar{j}} \boldsymbol{P} \boldsymbol{e}_j}. \tag{15}$$

3.1.3 Numerical Examples

In our numerical examples for the case of deterministic data arrival process, we focus on the discrete Markov model. Fig. 2 and 3 plot the effects of the buffer size, the target BER, and the average received SNR on the performance of the overflow probability when the data arrival rate $\lambda = 350$ kb/s, the number of states $N + 1 = 8$, one packet time period $T_p = 0.384$ ms, bandwidth $w = 100$ kHz, and maximum Doppler frequency $fm = 10$ Hz. It can be observed from Fig. 2 that the overflow probability decreases as the buffer size increases, as expected. Both of these two figures demonstrate that a lower target BER will increase the overflow probability. This is because achieving a lower target BER requires a slower transmission which in turn increases the possibility of overflow. Fig. 3 shows that a higher average received SNR increases the transmission rate which will result in a smaller overflow probability.

3.2 Case II: Poisson Arrival Process

3.2.1 General Formula

When the incoming data arrive according to a Poisson process, the buffered adaptive transmission system can be modelled as an $M/G/1$ queue. Specifically, in addition to the Poisson input with arrival rate λ to this queue, the general service time x is a discrete random variable with the set of possible values $x = \{\frac{1}{R_1}, \frac{1}{R_2}, \cdots, \frac{1}{R_N}\}$[1] and discrete PDF

$$p\left(x = \frac{1}{R_j}\right) = \exp\left(-\frac{\gamma_j - \gamma_1}{\bar{\gamma}}\right) - \exp\left(-\frac{\gamma_{j+1} - \gamma_1}{\bar{\gamma}}\right), \qquad j = 1, \cdots, N, \tag{16}$$

[1]When the transmission rate is $R_0 = 0$, the server can be viewed as on vacation. In practice, the probability of this occurs is very low, so for simplicity, we omit it in our analysis.

where γ_j is given in (5). Therefore the mean service time T is obtained by

$$T = \bar{x} = \sum_{j=1}^{N} \frac{1}{R_j} \left[\exp\left(-\frac{\gamma_j - \gamma_1}{\overline{\gamma}} \right) - \exp\left(-\frac{\gamma_{j+1} - \gamma_1}{\overline{\gamma}} \right) \right]. \quad (17)$$

For $M/G/1$ queues, the distribution of the number of customers in the system can be found e.g. in [8, Chap. 5]. Applying their results to the buffered adaptive transmission system, when the traffic intensity $\rho = \lambda T < 1$ (for which the system can obtain statistical equilibrium after a sufficiently long time), the overflow probability P_{of}^{Q} of the buffered adaptive transmission with Poisson data arrival can be shown to be given by

$$P_{\text{of}}^{\text{Q}} = 1 - \frac{1}{\pi_0^* + \rho}, \quad (18)$$

where π_0^* is determined from the linear equations

$$\pi_{j+1}^* = p_0^{-1} \left(\pi_j^* - p_j \pi_0^* - \sum_{i=1}^{j} p_{j-i+1} \pi_i^* \right), \qquad j = 0, 1, \cdots, B-1,$$

$$\text{and} \qquad \sum_{i=0}^{B} \pi_i^* = 1, \quad (19)$$

where π_j^*'s are the limiting distribution of the number of buffer units occupied at the transmission completion points (i.e., the time point at which the transmission of a unit of data (e.g., a bit, a packet, etc) is just finished) and also referred to as the departing customer's distribution of an imbedded Markov chain. In (19), the coefficient p_j is given by the Riemann-Stieltjes integral

$$p_j = \int_0^\infty \frac{(\lambda x)^j}{j!} \exp(-\lambda x) dF(x), \quad (20)$$

where $F(x)$ is the CDF of the service time. Following the results for the Riemann-Stieltjes integral provided in [8, Chap. 5], (20) can be shown to be written as

$$p_j = \sum_{k=1}^{N} \frac{\left(\frac{\lambda}{R_k} \right)^j}{j!} \exp\left(-\frac{\lambda}{R_k} \right) \left[\exp\left(-\frac{\gamma_k - \gamma_1}{\overline{\gamma}} \right) - \exp\left(-\frac{\gamma_{k+1} - \gamma_1}{\overline{\gamma}} \right) \right]. \quad (21)$$

3.2.2 Numerical Examples

Fig. 4 plots the effect of the data arrival rate on the performance of the overflow probability when the incoming data arrive according to a Poisson process. The simulation setup is the following: the number of states $N+1 = 8$, buffer size $B = 5$ packets, average received SNR $\overline{\gamma} = 20$ dB, and transmission rate is $R = [10, 20, 30, 40, 50, 60, 70]$ packets/s[2]. One can see from Fig. 4 that the overflow probability increases as the Poisson arrival rate increases, as expected. Similar to the deterministic data arrival process case, Fig. 4 also shows that a lower target BER increases the overflow probability of buffered adaptive system with Poisson arrival data.

References

[1] A. J. Goldsmith and S. -G. Chua, "Variable-rate variable-power M-QAM for fading channels," *IEEE Trans. Commun.*, vol. COM-45, pp. 1218–1230, October 1997.

[2] M. -S. Alouini and A. Goldsmith, "Adaptive modulation over Nakagami fading channels," *Kluwer Journal on Wireless Communications*, vol. 13, pp. 119–143, May 2000.

[3] K. J. Hole, H. Holm, and G. E. Øien, "Adaptive multidimensional coded modulation over flat fading channels," *IEEE Jour. Sel. Areas Commun.*, vol. 18, pp. 1153–1158, July 2000.

[2]Note that the units for λ and R can be bits/s, packets/s, etc, as long as they are consistent with each other and the corresponding unit for the buffer size B should be bits or packets accordingly.

[4] Q. Zhang and S. A. Kassam, "Finite-state markov model for Rayleigh fading channels," *IEEE Trans. Commun.*, vol. COM-47, pp. 1688–1692, November 1999.

[5] S. Mukherjee and H. Viswanathan, "Minimum duration outages for diversity systems," in *Proc. IEEE Global Telecommun. Conf. (GLOBECOM'98)*, Sydney, Australia, pp. 3663–3668, November 1998.

[6] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products.* Orlando, FL: Academic Press, corr. and enlarg. ed., 1980.

[7] M. Zorzi, "Minimum duration outages in Markov channels," in *Proc. IEEE 6th Int. Conf. Univ. Pers. Commun. (ICUPC'97)*, San Diego, CA, pp. 12–16, October 1997.

[8] R. B. Cooper, *Introduction to Queueing Theory.* Elsevier North Holland Inc., New York, Oxford, 1981.

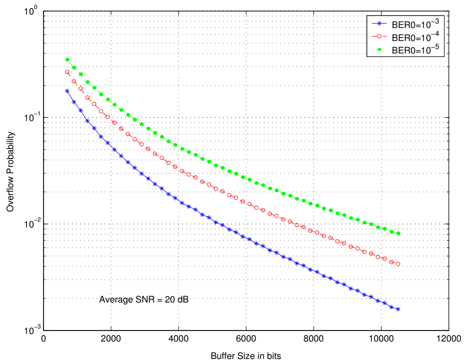

Figure 2: Overflow probability versus buffer size for various values of the target BER.

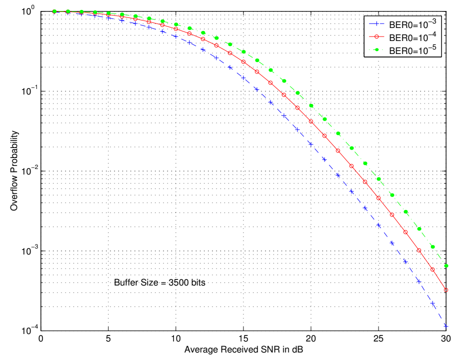

Figure 3: Overflow probability versus average received SNR for various values of the target BER.

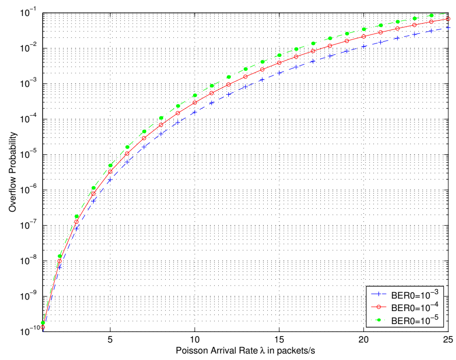

Figure 4: Overflow probability versus Poisson arrival rate λ for various values of the target BER.