BIT ERROR RATE ANALYSIS OF ADAPTIVE CODED MODULATION WITH MISMATCHED AND COMPLEXITY-LIMITED CHANNEL PREDICTION

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ABSTRACT

Adaptive coded modulation (ACM) has been shown to be a spectrally very efficient transmission scheme on flat-fading wireless channels, under certain idealized assumptions such as perfect channel knowledge at the transmitter. Recent analysis has focused on the reliability of ACM when this assumption is relaxed, and the channel state information available to the transmitter is instead an imperfect, but maximum a posteriori-optimal prediction of the channel state rather than the true channel state. In this paper we relax the assumptions on the available channel state knowledge even further, to account for a possible mismatch between the optimized predictor and the true channel model parameters. Furthermore we consider predictors with limited complexity, which is necessary for implementation and to achieve an acceptable processing delay. The results show that a high average spectral efficiency and an acceptable average bit error rate may still be simultaneously achieved for a wide operational range of terminal mobilities and average channel signal-to-noise ratios.

1. INTRODUCTION

Adaptive coded modulation (ACM) has in recent years emerged as a promising data transmission scheme for simultaneously achieving high spectral efficiency and an acceptably low bit error rate (BER) on wireless and mobile channels exhibiting flat fading [1]. The basic principle is to let the transmitted information rate vary (by varying the channel code and modulation size) according to the channel quality; transmitting a low number of bits per symbol when the channel is bad, and gradually increasing the rate as the channel quality becomes better. If the channel is in a long-lasting fade, such a scheme will also delay transmission until the channel becomes more reliable.

Assuming that each such channel is a subchannel in a multicarrier (e.g., orthogonal frequency division multiplexing (OFDM) [2]) scheme, the flat fading results can also be extended to frequency-dispersive channels. Under certain idealized assumptions it has been demonstrated that the spectral efficiency of such a scheme may surpass that of today's state-of-the-art wireless systems by several hundred percent [8].

Unfortunately, the idealized assumptions referred to above may not hold exactly in practical situations. One important assumption that may break down in many situations is that of perfect channel knowledge at the transmitter side. In practice, the time-varying channel quality (fading amplitude) must be estimated at the receiver—e.g., deduced from the channel influence on deterministically known and periodically transmitted pilot symbols—and sent back to the transmitter via a feedback return channel. Three mechanisms may contribute to the information thus received at the transmitter being incorrect:

- Nonzero feedback delay in the return channel, implying that the forward channel may have changed by the time the transmitter adapts its rate according to the received estimate.
- Additive channel noise, making for a noisy estimate regardless of the feedback delay.
- Bit errors occurring in the return channel.

In this paper, we focus on the first two mechanisms, still assuming that the return channel is noiseless. This is a reasonable assumption since, due to the low information rate needed in the feedback loop, the feedback channel can be heavily error protected.

2. SYSTEM MODEL

The system analyzed is depicted in Figure 1. Each fading channel corresponds to a wireless link between the transmitter and one out of H receive antenna elements. The adaptive coded modulator/demodulator

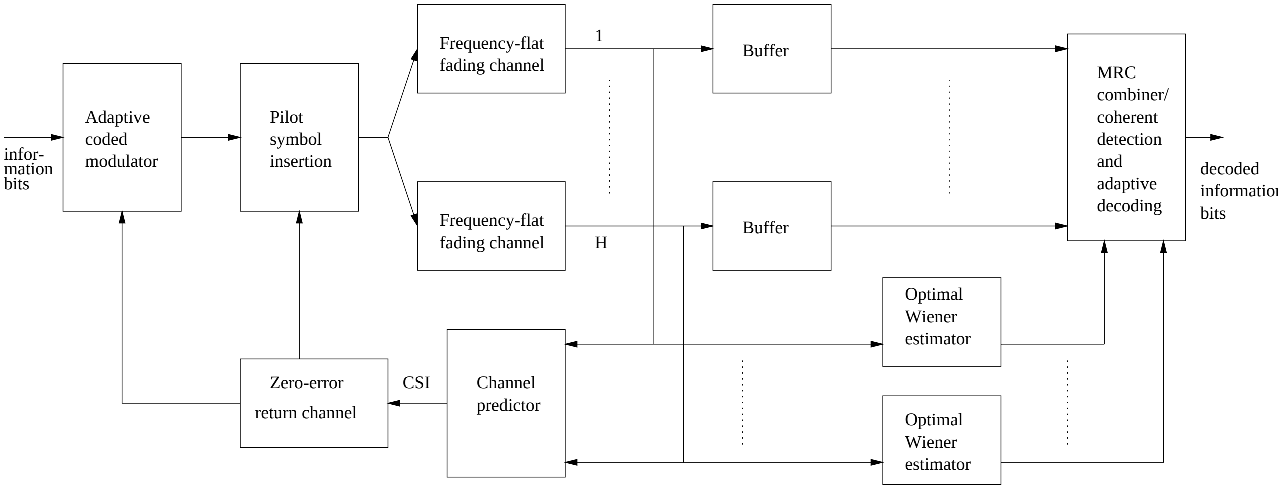

Figure 1: ACM system with pilot-aided channel estimation (for detection) and prediction (for adaptation).

contains N transmitter-receiver pairs, indexed by $n = 1, \ldots, N$. Transmitter n has a spectral efficiency of R_n information bits/s/Hz (equal to the number of information bits per symbol if Nyquist signaling is assumed), such that $R_1 < R_2 < \ldots < R_N$. Transmitter-receiver pair n is used when the *channel signal-to-noise ratio* (CSNR), $\gamma = P_r/P_n$ where P_r is received signal power and P_n is noise power, lies in $[\gamma_n, \gamma_{n+1}\rangle$. For each n, γ_n is computed as the lowest CSNR necessary for transmitter-receiver pair n to operate at a BER below some specified target BER_0, at transmit power P [1]. Also, we let $\gamma_0 = 0$ and $\gamma_{N+1} = \infty$, so for all $n \in \{0, \ldots, N\}$, $\gamma_{n+1} > \gamma_n$. We assume that no available transmitter-receiver pair satisfies the BER requirement in $[0, \gamma_1\rangle$, so no information is transmitted when γ falls here.

When performing rate adaptation, the transmitter must rely on the accuracy of the CSNR *as predicted by the receiver* at discrete time k. The *true* channel quality at the transmitter update time $k + \tau$, where τ is the discrete return channel delay (corresponding to τT_s seconds where T_s [s] is the channel symbol duration), may deviate from the prediction; hence the transmitter may adapt to the wrong channel quality.

Denoting the transmitted complex baseband signal after pilot symbol insertion at time index k by $x(k)$, the received signal on the hth subchannel can be written $y_h(k) = z_h(k) \cdot x(k) + n_h(k)$. Here $z_h(k)$ is the *complex fading amplitude*, and $n_h(k)$ is complex-valued additive white gaussian noise (AWGN) with statistically independent real and imaginary components. $x(k)$ is the information signal, except for time instants $k = mL$ (m any integer, L a constant integer), when deterministic *pilot symbols* are periodically transmitted. We assume that the pilot symbols all have the same (absolute) value, $x(mL) = a$.

Assuming flat Rayleigh fading on each subchannel, $z_h(k)$ is complex-valued gaussian with zero mean, and will be assumed approximately constant between two successive pilot symbols (block fading). Its variance is $\Omega = E[|z_h|^2]$, independent of h, with Ω assumed time-invariant; i.e., we assume a wide-sense stationary (WSS) channel.

If a constant average transmit power P [W] is used, and the one-sided power spectral density of the complex AWGN is N_0 [W/Hz] in every subchannel, the received CSNR on subchannel h at a given time k is $\gamma_h(k) = |z_h(k)|^2 \cdot P/(N_0 B)$, where B [Hz] is the one-sided information bandwidth. The expected CSNR is $E[\gamma_h] = \overline{\gamma}_h = \Omega P/(N_0 B)$. With H statistically independent antenna branches combined by MRC at the receiver, the *overall* received CSNR at time k is $\gamma(k) = \sum_{h=1}^{H} \gamma_h(k)$, which is *Gamma* distributed [4] with expectation $\overline{\gamma} = H\overline{\gamma}_h$. This CSNR is predicted and sent back to the transmitter for each received pilot symbol. The delay between CSNR prediction and the subsequent allowed transmitter update is an integer number of pilot symbol periods, $\tau = jL$.

For the receiver detection, the signal may be buffered before channel estimation, and an optimal *noncausal Wiener interpolator filter* may be used [5]. This will smooth the noise and allow for true coherent detection. Hence, we assume that perfect channel state information (CSI) is used during *detection*.

3. OPTIMAL CHANNEL PREDICTION

In a recent doctoral thesis [6], the effect of imperfect CSI on ACM was analyzed on Rayleigh fading channels with receiver antenna diversity (*maximum ratio combining*—MRC [2]).

The performance measures analyzed in [6] are average BER, average spectral efficiency (ASE), and the probability of no transmission (the percentage of time when nothing is transmitted). The assumption is that pilot symbol assisted *maximum a posteriori- (MAP)-optimal* prediction can be used to periodically estimate the CSNR which is fed back to the transmitter.

3.1. Mathematical foundation of MAP-optimal prediction

For any pilot symbol time instant $k - lL$ (l positive integer), define

$$\tilde{z}_h(k - lL) = z_h(k - lL) + \frac{n_h(k - lL)}{a} \qquad (1)$$

which is the maximum-likelihood (ML) estimate of $z_h(k - lL)$ based on one received observation [5]. The two terms are statistically independent complex gaussians, so their sum is a complex gaussian with variance equal to the sum of their variances. At time $k + jL$ of transmitter update we have available $\tilde{z}_h(k), \tilde{z}_h(k - L), \tilde{z}_h(k-2L), \ldots, \tilde{z}_h(k-(K-1)L)$ from which to predict $z_h(k + jL)$. Here, $K \geq 1$ is the chosen *predictor order*. For gaussian processes, the *optimal* predictor in the *maximum a posteriori* (MAP) sense is a *linear* function of the observations [5]. *Any* linear predictor of order K can be written

$$\widehat{z}_h(k + jL) = \mathbf{f}_j^T \tilde{\mathbf{z}}_h \qquad (2)$$

where $\mathbf{f}_j = [f_j(0), \ldots, f_j(K-1)]^T$ is the predictor filter coefficient vector associated with delay jL (we do not need a subchannel index h, since the optimal filter will depend only on the feedback delay when all subchannels have the same fading properties), and

$$\tilde{\mathbf{z}}_h = [\tilde{z}_h(k), \tilde{z}_h(k - L), \ldots, \tilde{z}_h(k - (K-1)L)]^T.$$

The predicted fading envelope is a linear combination of gaussians;, thus itself a complex gaussian. Now, define $\widehat{\alpha}_h = |\widehat{z}_h|$, with $E[\widehat{\alpha}_h^2] = \widehat{\Omega}$. Then, there exists an r such that $\widehat{\Omega} = r\Omega$. Thus $\widehat{\alpha}_h$ is Rayleigh distributed, and the corresponding predicted overall CSNR $\widehat{\gamma} = \sum_{h=1}^{H} \widehat{\alpha}_h^2 P/(N_0 B)$ is Gamma distributed with $E[\widehat{\gamma}] = r\overline{\gamma}$. In [7] expressions are derived for r which apply directly also to our case. Let

$$[\mathbf{R}]_{kl} = \frac{\mathrm{Cov}(z_h(kL), z_h(lL))}{\Omega}, \qquad (3)$$

element (k, l) in the *normalized covariance matrix* $\mathbf{R}$ (dimension $K \times K$) of the fading—at *pilot* time instants—on the subchannel in question. Due to the WSS assumption this will only be a function of the lag between the two pilot symbol time instants kL and lL, $[\mathbf{R}]_{kl} = R(\tau_{kl})$, with $\tau_{kl} = |k - l|LT_s$. For $a = \sqrt{P}$, we obtain [7]

$$r = \mathbf{f}_j^T \mathbf{R} \mathbf{f}_j + \frac{|\mathbf{f}_j|^2}{\overline{\gamma}_h}. \qquad (4)$$

Finally, assuming the much-used *Jakes spectrum* [2] for the fading process, the MAP-optimal filter coefficient vector on a Rayleigh fading channel can be deduced from [5, p. 742, Eq. (14.36)] as

$$\mathbf{f}_{j,\mathrm{MAP}}^T = \mathbf{r}_j^T \left(\mathbf{R} + \frac{1}{\overline{\gamma}_h} \mathbf{I} \right)^{-1}. \qquad (5)$$

where $\mathbf{r}_j = \frac{2}{\Omega}[R_I(jL), \ldots, R_I((j + K - 1)L)]^T$, with $R_I(\tau) = \frac{\Omega}{2} J_0(2\pi f_D \tau)$. Here, $J_0(x)$ is the 0th order Bessel function of the first kind, and f_D [Hz] is the Doppler frequency shift due to terminal movement.

3.2. Visualization of MAP-optimal prediction

In Fig. 2 is shown, for certain parameter value choices, how such an optimal predictor works on a Rayleigh fading channel. The Doppler frequency $f_d = 250$ Hz corresponds to a terminal velocity of 54 km/h, or $v = 15$ m/s, in this case. It is apparent upon study of the curves how the tracking of the fading amplitude degrades somewhat when we predict 5 pilot symbols (200 symbols) instead of 1 pilot symbol (40 symbols) ahead in time. This can be attributed to decreasing fading correlation as the lag increases.

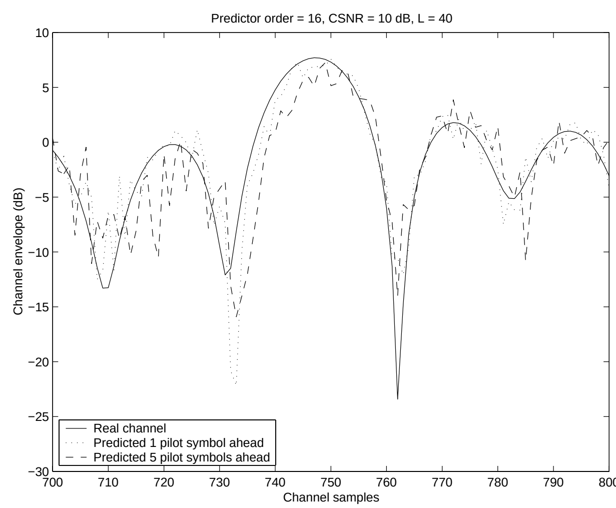

Figure 2: Predicted and true fading amplitude for prediction $j = 1$ and $j = 5$ pilot symbols ahead. Carrier frequency f_c=5 GHz; Doppler frequency $f_d = 250$ Hz; $H = 1$; $K = 16$; $L = 40$; $\overline{\gamma} = 10$ dB.

3.3. BER analysis

Closed-form approximative expressions for the BER, as averaged over all available codes and all CSNRs, can now be derived based on the above results [6]. The most important parameters when evaluating these expressions, which are too involved to relate here, are the correlation between the predicted and the true fading amplitudes (which is predictor and channel dependent), the expected CSNR (which is channel dependent), and the CSNR bin levels associated with each available code (which are code and target BER dependent). The results, also published in [8], show that the successful implementation of ACM might be feasible over a wide range of expected CSNRs and terminal velocities (Doppler shifts), if MAP-optimal channel information is available. However, these results do not take into account the following facts:

- In practice, the channel model statistics (fading correlation, expected CSNR) which are needed to optimize the predictor must also be estimated from noisy observations. This can lead to a mismatch between the actual channel parameters and the predictor derived from the parameter estimates. Such a mismatch might have a deteriorating effect on the BER performance and on the possible region of operation in the "expected CSNR–terminal velocity" domain.

- In practice, the system complexity must be limited by limiting the predictor filter order (this might also be an issue if, as is often the case, the channel is not truly WSS). The lower the predictor order, the lower the possible prediction gain and, theoretically, the worse the achievable quality of the channel estimate.

4. MISMATCH BETWEEN OPTIMIZED PREDICTOR AND CHANNEL MODEL PARAMETERS

One way to analyze the effects of channel parameter mismatch is to assume that a MAP-optimized predictor is still used, but that it has been optimized using channel parameter estimates that have a certain percentage of error in them. This approach has been taken in [9], on which the results of this section are based. We study the following cases:

1. Introducing a specified percentage of error in the expected CSNR value used in the MAP-optimal predictor expression.

2. Introducing a certain percentage of error in the *normalized feedback delay* value.

The normalized feedback delay for an absolute feedback delay τT_s [s] is defined as

$$\tau_d = \frac{\tau T_s}{T_d} \qquad (6)$$

where $T_d = 1/f_d$ [s] is the *Doppler period*; i.e., the inverse of the Doppler frequency $f_d = v f_c / c$ where v [m/s] is velocity of relative transmitter-receiver movement, f_c [Hz] is carrier frequency, and c [m/s] is the speed of light.

Thus, τ_d measures the feedback delay in the number of Doppler periods. This value is important when computing the correlation between the fading amplitude to be predicted, and the noisy pilot symbol observations used for predicting it. Thus the predictor expression (Eq. 5) may contain the wrong correlation vector and matrix if there is an error in τ_d.

In both the above cases, the analytical tools derived in [6] can still be applied for performance analysis. Applied to the example ACM system of [1], the results give increased insight into the conditions under which an ACM system with mismatched channel prediction can still operate successfully (i.e., fulfill given average BER requirements while simultaneously providing a large ASE). Examples of the results are shown in the Figs. 3 and 4. In these figures, the BER is smaller than the target value to the right of each curve, while to the left it is bigger. In other words, the more the curves are shifted to the left, the more robust the system gets.

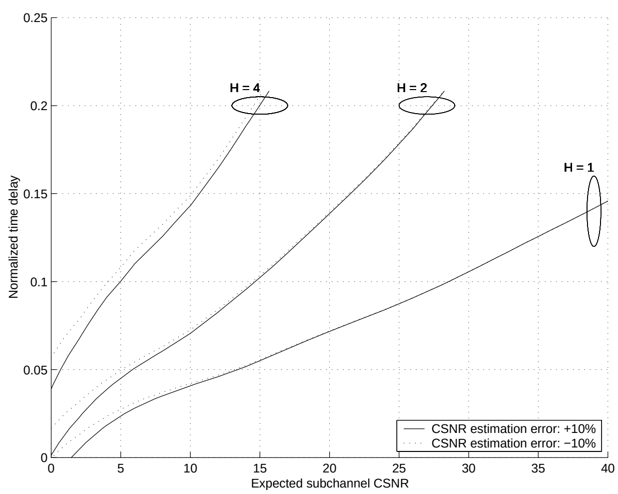

Figure 3: $\overline{\mathrm{BER}}$ contours at $\mathrm{BER}_0 = 10^{-4}$ as functions of normalized delay and $\overline{\gamma}_h$ [dB] for example system. $H = 1, 2, 4$; $\pm 10\%$ error in $\overline{\gamma}_h$.

It is apparent that the example system is more robust towards estimation errors in the expected CSNR than it is towards estimation errors in the Doppler period or feedback delay. However, even with errors in the feedback delay it is still possible to obtain large ASE gains and a reliable BER performance from an ACM system compared to a fixed rate system, for a relatively wide (gracefully decreased relative to the results of [6]) range of expected CSNRs and Doppler shifts. Also, it is seen that the use of MRC in the receiver considerably decreases the sensitivity towards channel parameter estimation errors.

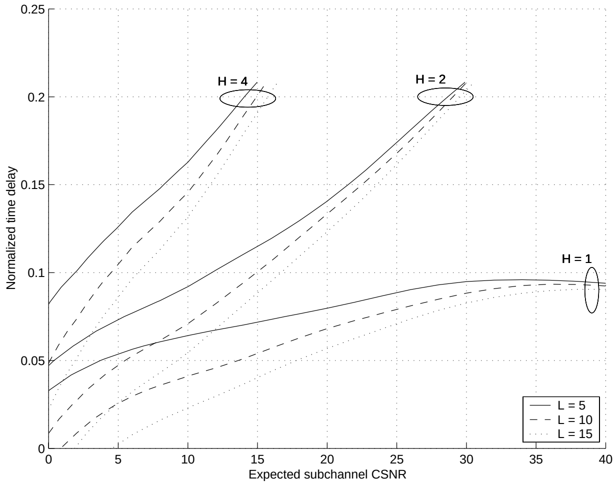

Figure 4: $\overline{\text{BER}}$ contours at $\text{BER}_0 = 10^{-4}$ as functions of normalized delay and $\overline{\gamma}_h$ [dB] for example system. $H = 1, 2, 4$; $+5\%$ error in the normalized delay.

5. LIMITING THE PREDICTOR COMPLEXITY

The final problem analyzed in this paper is that of limiting the predictor filter order K, and the relation between this order and the pilot symbol period L. In [6] and [9] a predictor order of $K = 1000$ was used, something that is clearly too complex for many practical situations. In [10], the effects of lowering this order has been analyzed, again assuming a truly MAP-optimal predictor. Systems with limited mobility requirements—such as wireless local area networks (WLANs)—are studied in particular.

As an example, a HiperLAN/2-like system (carrier frequency $f_c = 5$ GHz; channel bandwidth $B = 400$ kHz), with a terminal speed of $v = 2.0$ m/s (or 7.2 km/h), corresponding to $f_d = 33$ Hz, has been considered. No MRC combining were used in these experiments; i.e., $H = 1$ for all results discussed in this section. With the above parameter choices it is shown that a predictor order of $K = 128$ taps and a pilot symbol period of $L = 20$ (i.e., one pilot symbol transmitted every 20th channel symbol) is sufficient to obtain satisfactory BER performance over a wide range of expected CSNRs and typical feedback delays. This is illustrated by Figs. 5 and 6.

In Fig. 5 is shown, for various predictor lengths, how the average BER depends on the expected CSNR and on the number of pilot symbols predicted ahead in time. The BER is smaller than the target $\text{BER}_0 = 10^{-4}$ to the right of each contour, and higher than the target to the left. Note that for the chosen system parameters, predicting 20 pilot symbols ahead is in this case equivalent to predicting 400 channel symbols ahead, or $\tau T_s = 1.0$ millisecond ahead in absolute

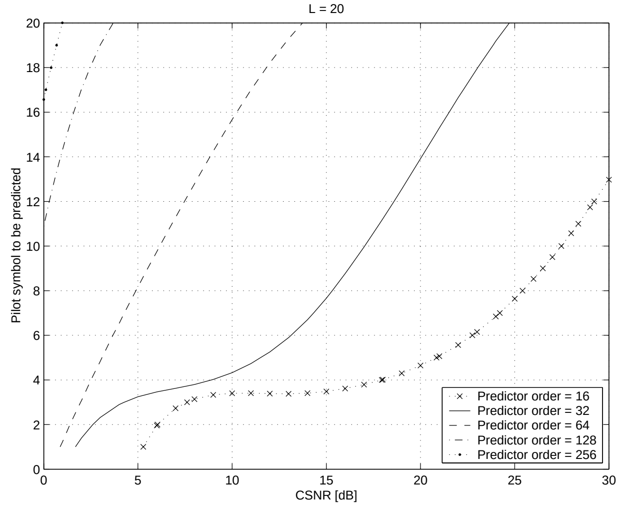

Figure 5: $\overline{\text{BER}}$ contours at $\text{BER}_0 = 10^{-4}$ as functions of the number of pilot symbols predicted ahead, and $\overline{\gamma}$ [dB], for example system. $L = 20$; $H = 1$; $f_c = 5$ GHz; $f_d = 33$ Hz.

time (assuming Nyquist signaling). This delay should be more than enough to account for a typical WLAN feedback link delay. It is seen that for $K \geq 128$, the average BER lies below 10^{-4} for almost the whole expected CSNR range.

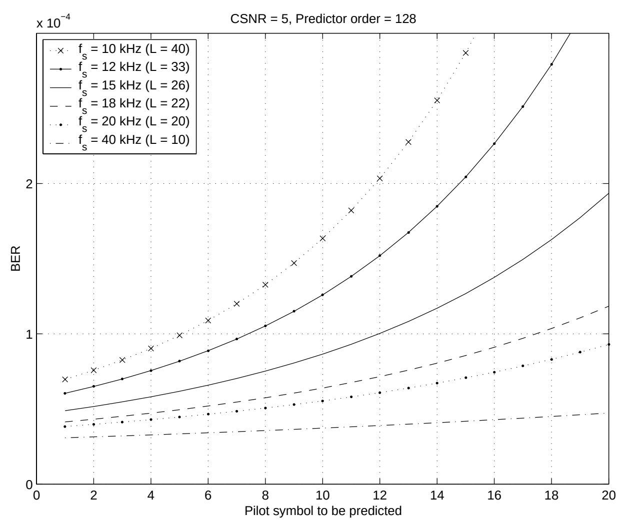

Figure 6: $\overline{\text{BER}}$ for various L as function of the number of pilot symbols predicted ahead, for example system. $\overline{\gamma} = 5$ dB; $H = 1$; $K = 128$.

In Fig. 6, an alternative view of the BER performance is given. Here, the predictor order is kept constant at $K = 128$, and the expected CSNR is kept constant at the low value of 5 dB, while the pilot symbol period L is varied between 10 and 40. It is seen that $L = 20$ is the lowest value that guarantees that the average BER always falls below the target value, if we are to predict up to 20 pilot symbols ahead.

The above BER results are confirmed by an analysis of the mean squared error (MSE) of the predictor, which is shown to be near-stabilized for predictor length $K = 128$, as long as the expected CSNR is not very low or the normalized delay is not very high. This is shown by Fig. 7, which gives an example of the predictor MSE as a function of the expected CSNR and the number of pilot symbols ahead to be predicted.

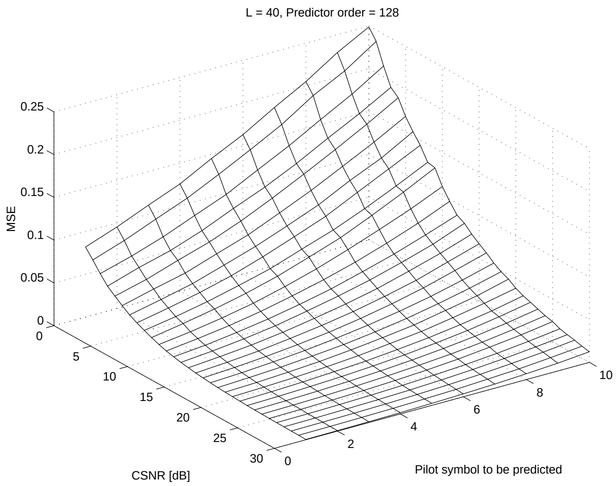

Figure 7: Predictor MSE as function of number of pilot symbols ahead to be predicted, and $\overline{\gamma}$ [dB]. $H = 1$; $K = 128$; $L = 40$;$f_c = 5$ GHz; $f_d = 250$ Hz.

6. CONCLUSIONS AND FUTURE WORK

For the example ACM system under study, predictor mismatches and limited predictor complexity lead to a "graceful" shrinking of the acceptable operation region in the expected CSNR-normalized feedback delay domain. Of the two investigated types of channel parameter errors, the system is most sensitive towards errors in the Doppler frequency. However, it is still possible to obtain large ASE gains and a reliable BER performance from an ACM system, for a relatively wide range of expected CSNRs and normalized feedback delays. Also, the use of MRC receive diversity increases system robustness.

It is also demonstrated, for systems with limited mobility requirements, that the predictor order may be reduced to a feasible complexity without a too dramatic effect on the system performance. In general, the predictor order and the pilot symbol period must be co-optimized to obtain the best possible performance. Future work will include application of actual estimates of the channel parameters, using the work done on optimal estimation of Nakagami-m fading parameters by Ko and Alouini [11]. This will enable a more well-founded evaluation of the percentage of parameter estimation error to expect. Also, systems employing other component codes than those used in [1] should be studied, in order to verify the generality of the conclusions. Finally, a *combined* analysis of predictor order reduction and channel parameter estimation should be performed.

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