General Outline

- Part 0: Background, Motivation, and Goals.
- Part I: Some Basics.
- Part II: Diversity Systems.
- **Part III: Co-Channel Interference.**
Outline - Part III: Co-Channel Interference

1. Co-Channel Interference (CCI) Analysis
   • Effect of Shadowing
   • Effect of Multipath Fading
     – Single Interferer
     – Multiple Interferers
     – Minimum Desired Signal Requirement
     – Random Number of Interferers

2. CCI Mitigation
   • Diversity Combining
   • Optimum Combining/Smart Antennas
   • Optimized MIMO Systems in Presence of CCI
Effect of Shadowing

• Finding the statistics of the sum of log-normal random variables.

• No known exact closed-form available.

• Several analytical techniques have been developed over the years.

• As an example, we cover the Farley bounds on the sum of log-normal random variables.
Effect of Multipath Fading (1)

- Single Interferer
  - Let the carrier-to-interference ratio (CIR) \( \lambda = \frac{s_d}{s_i} \),
  
  where \( s_d = \alpha_d^2 \) (with average \( \Omega_d \)) and \( s_i = \alpha_i^2 \) (with average \( \Omega_i \)) are the instantaneous fading powers of desired and interfering users.

- Outage probability
  \[
  P_{out} = \text{Prob}[\lambda \leq \lambda_{th}] = \int_0^\infty p_{s_d}(s_d) \text{Prob} \left[ s_i \geq \frac{s_d}{\lambda_{th}} \middle| s_d \right] ds_d
  \]
  \[
  = \int_0^\infty p_{s_d}(s_d) \int_{s_d/\lambda_{th}}^\infty p_{s_i}(s_i) ds_i ds_d.
  \]

- Exp: Rician (Rician factor \( K_d \))/Rayleigh case
  \[
  P_{out} = \frac{\lambda_{th}}{\lambda_{th} + b} \exp \left( -\frac{Kb}{\lambda_{th} + b} \right),
  \]
  where
  \[
  b = \frac{\Omega_d}{\Omega_i(K_d + 1)}.
  \]
Effect of Multipath Fading (2)

- $N_I$ independent identically distributed interferers
  - Let the carrier-to-interference ratio
    \[ \lambda = \frac{s_d}{s_I}, \]
    where $s_I = \sum_{i=1}^{N_I} s_i$ and all the $s_i$ have the same average fading power $\Omega_i$.
  - Outage probability
    \[ P_{\text{out}} = \text{Prob} [\lambda \leq \lambda_{\text{th}}] = \int_0^{\infty} p_{s_d}(s_d) \int_{s_d/\lambda_{\text{th}}}^{\infty} p_{s_I}(s_I) \, ds_I \, ds_d, \]
  - Exp: Nakagami/Nakagami scenario
    \[ P_{\text{out}} = I_x(m, mN_I) \]
    where
    \[ x = \left( 1 + \frac{\Omega_d}{\Omega_i \lambda_{\text{th}}} \right) \]
    and
    \[ I_x(a, b) = \frac{\Gamma(a + b)}{\Gamma(a)\Gamma(b)} \int_0^x t^{a-1} (1 - t)^{b-1} \, dt \]
    is the incomplete Beta function ratio.
Effect of Multipath Fading (3)

- $N_I$ independent non-identically distributed interferers
  - Let the carrier-to-interference ratio
    \[ \lambda = \frac{s_d}{s_I}, \]
    where $s_I = \sum_{i=1}^{N_I} s_i$ and all the $s_i$ can have different average fading power $\Omega_i$.
  - Outage probability
    \[ P_{out} = \text{Prob}[\lambda \leq \lambda_{th}] = \text{Prob} \left[ s_d - \lambda_{th} \sum_{i=1}^{N_I} s_i \leq 0 \right]. \]
  - Define $\alpha = \lambda_{th} \sum_{i=1}^{N_I} s_i - s_d$.
    * $\alpha \geq 0$ corresponds to an outage.
    * $\alpha \leq 0$ corresponds to satisfactory transmission.
  - Find characteristic function of $\alpha$ and then use Gil-Palaez lemma.
Gil-Palaez Lemma and Application

- Let $X$ be a random variable with cumulative distribution function (CDF) $P_X(x) = \text{Prob}[X \leq x]$ and characteristic function (CF) $\phi_X(t) = E\left[e^{jXt}\right]$ then

$$P_X(x) = \frac{1}{2} - \frac{1}{\pi} \int_0^\infty \frac{\text{Im} \left[ \phi_X(t)e^{-jXt}\right]}{t} dt.$$

- Application to outage probability

$$P_{\text{out}} = \text{Prob}[\alpha \geq 0] = 1 - \text{Prob}[\alpha \leq 0]
= 1 - P_\alpha(0) = \frac{1}{2} + \frac{1}{\pi} \int_0^\infty \frac{\text{Im} \left[ \phi_\alpha(t)\right]}{t} dt,$$

where for the Nakagami/Nakagami fading case

$$\phi_\alpha(t) = \phi_{s_d}(-t) \prod_{i=1}^{N_I} \phi_{s_i}(\lambda_{\text{th}}t)$$

$$= \left(1 + \frac{jt\Omega_d}{m_d}\right)^{-m_d} \prod_{i=1}^{N_I} \left(1 - \frac{jt\Omega_i\lambda_{\text{th}}}{m_i}\right)^{-m_i}.$$
Random Number of Interferers

• Depending on traffic conditions, the number of active interferers \( n_I \) is a random variable from 0 to \( N_I \) which is the maximum number of active interferers.

• Outage probability is given by

\[
P_{\text{out}} = \sum_{n_I=0}^{N_I} \text{Prob}[n_I] \cdot P_{\text{out}}[n_I].
\]

• Assume \( N_c \) available channels per cell each with activity probability \( p_a \).

• The blocking probability

\[
B = p_a^{N_c}.
\]

• PDF of \( n_I \) is Binomial

\[
\text{Prob}[n_I] = \binom{N_I}{n_I} p_a^{n_I} (1 - p_a)^{N_I-n_I}, \quad n_I = 0, \ldots, N_I.
\]

\[
= \binom{N_I}{n_I} B^{n_I/N_c} \left( 1 - B^{1/N_c} \right)^{N_I-n_I}.
\]
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Diversity Combining

• Reduce the effect via selective (or switched) antenna diversity combining techniques.

• Consider a dual-antenna diversity system with one co-channel interferer. Let $\alpha_{11}$ denote the fading amplitude from desired user to antenna 1, $\alpha_{12}$ denote the fading amplitude from desired user to antenna 2, $\alpha_{21}$ denote the fading amplitude from interfering user to antenna 1, and $\alpha_{22}$ denote the fading amplitude from interfering user to antenna 2.

• Three main decision algorithms for selective diversity.
Decision Algorithms

• CIR algorithm: picks and process the information from the antenna with the highest CIR. For the scenario describe previously:

\[
\text{Max} \left[ \left( \frac{\alpha_{11}}{\alpha_{21}} \right)^2, \left( \frac{\alpha_{12}}{\alpha_{22}} \right)^2 \right].
\]

• Desired signal algorithm: picks and process the information from the antenna with the highest desired signal, i.e.,

\[
\text{Max} \left[ \alpha_{11}^2, \alpha_{12}^2 \right].
\]

• Signal plus interference algorithm: picks and process the information from the antenna with the highest desired plus interfering signal, i.e.,

\[
\text{Max} \left[ \alpha_{11}^2 + \alpha_{21}^2, \alpha_{12}^2 + \alpha_{22}^2 \right].
\]
Interference Mitigation

• More advanced interference mitigation techniques
  – Optimum combining
  – Optimized MIMO systems
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• Part IV: Multi-Hop Communication Systems.
Multi-Hop Communication Systems

- Advantages of transmission with relays:
  - Broader coverage
  - Lower transmitted power (higher battery life and lower interference)
  - “Cooperative/Collaborative/Multi-user” diversity.

- “Pionnering” work on this topic:
  - Sendonaris, Erkip, and Aazhang, [ISIT’98].
  - Laneman, Wornell, and Tse [WCNC’00, Allerton’00, ISIT’01].
  - Emamian and Kaveh, [ISC’01].

- Goal:
  Develop an analytical framework for the exact end-to-end performance analysis of dual-hop then multi-hop relayed transmission over fading channels.
Dual-Hop Systems

- Consider the following dual-hop communication system with a relay

![Diagram of dual-hop communication system]

- Two relaying options:
  - Non-regenerative relaying (known also as analog or amplify-and-forward relaying)
  - Regenerative relaying (known also as digital or decode-and-forward relaying)
Non-Regenerative Systems

- Received signal at the relay input (B) is
  \[ r_b(t) = \alpha_1 s(t) + n_1(t). \]

- Received signal at the destination (C) is
  \[ r_c(t) = \alpha_2 G r_b(t) + n_2(t) \]
  \[ = \alpha_2 G (\alpha_1 s(t) + n_1(t)) + n_2(t). \]

- Equivalent end-to-end SNR
  \[ \gamma_{eq} = \frac{\alpha_1^2 \alpha_2^2 G^2}{\alpha_2^2 G^2 N_0 + N_0} = \frac{\alpha_1^2}{N_0} \frac{\alpha_2^2}{N_0} + \frac{1}{G^2 N_0}. \]
Choice of the Relay Gain

- One possible choice of the relay gain is just channel inversion, i.e.,

\[ G^2 = \frac{1}{\alpha_1^2}, \]

- Resulting equivalent end-to-end SNR

\[ \gamma_{eq2} = \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2}. \]

- Lower bound on the performance of practical relays
- Related to the Harmonic Mean of \( \gamma_1 \) and \( \gamma_2 \)
A Second Choice of the Relay Gain

• Another possible choice of the relay gain [Lane-man et al. ’00]

\[ G^2 = \frac{1}{\alpha_1^2 + N_0}. \]

– Limits the gain of the relay when first hop is deeply faded
– Resulting equivalent end-to-end SNR

\[ \gamma_{eq1} = \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2 + 1}, \]
Monte Carlo Simulation

- Comparison of the outage probability for the two choices of the relay gain
Harmonic Mean

- Given two numbers $X_1, X_2$:
  
  - Arithmetic Mean
    \[
    \mu_A(X_1, X_2) = \frac{X_1 + X_2}{2}
    \]
  
  - Geometric Mean
    \[
    \mu_G(X_1, X_2) = \sqrt{X_1 X_2}
    \]
  
  - Harmonic Mean
    \[
    \mu_H(X_1, X_2) = \frac{2X_1 X_2}{X_1 + X_2} = \frac{2}{\frac{1}{X_1} + \frac{1}{X_2}}
    \]

- Relation with end-to-end SNR

  \[
  \gamma_{eq2} = \frac{1}{2} \mu_H(\gamma_1, \gamma_2) \geq \gamma_{eq1},
  \]

  where $\gamma_1$ and $\gamma_2$ are the instantaneous SNRs of hops 1 and 2, respectively.
Harmonic Mean of Exponential Variates

- Theorem:
  Let $X_1$ and $X_2$ be two independent exponential variates with parameters $\beta_1$ and $\beta_2$ respectively. Then, the PDF of $X = \mu_H(X_1, X_2)$, $p_X(x)$, is given by

$$p_X(x) = \frac{1}{2} \beta_1 \beta_2 x e^{-\frac{x}{2}(\beta_1 + \beta_2)} \left[ \left( \frac{\beta_1 + \beta_2}{\sqrt{\beta_1 \beta_2}} \right) K_1 \left( x \sqrt{\beta_1 \beta_2} \right) + 2K_0 \left( x \sqrt{\beta_1 \beta_2} \right) \right] U(x),$$

where $K_i(\cdot)$ is the $i$th order modified Bessel function of the second kind and $U(\cdot)$ is the unit step function.

- The CDF and MGF of the harmonic mean of two independent exponential variates are also available in closed-form.
Validation by Monte-Carlo Simulations

Monte Carlo Simulation for $p_\Gamma(\gamma)$

Probability Density Function, $p_\Gamma(\gamma)$

Analysis
Simulation

$\gamma$

0 0.5 1 1.5 2 2.5 3 3.5 4

0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6

Analysis
Simulation
Derivation of the CDF of $X = \mu_H(X_1, X_2)$

- Let
  \[ Z = \frac{1}{X} = \frac{1}{2} \left( \frac{1}{X_1} + \frac{1}{X_2} \right). \]

- The CDF of $X$, $P_X(x)$, is given by
  \[ P_X(x) = \Pr(X < x) \]
  \[ = \Pr \left( \frac{1}{X} > \frac{1}{x} \right) = \Pr \left( Z > \frac{1}{x} \right) \]
  \[ = 1 - \Pr \left( Z < \frac{1}{x} \right) = 1 - P_Z \left( \frac{1}{x} \right), \]
  where $P_Z(\cdot)$ is the CDF of $Z$. 

Derivation of the CDF of $X = \mu_H(X_1, X_2)$ (Continued)

• If $X$ is an exponential random variable with parameter $\beta$ then the MGF of $Y = 1/X$ can be shown to be given by

$$M_Y(s) = E\left[e^{-sY}\right] = 2\sqrt{\beta s} \ K_1\left(2\sqrt{\beta s}\right).$$

• Using the differentiation property of the Laplace transform, $P_Z(z)$ can be written as

$$P_Z(z) = \mathcal{L}^{-1}\left(\frac{M_Z(s)}{s}\right) = 1 - \mathcal{L}^{-1}\left(2\sqrt{\beta_1\beta_2} K_1\left(2\sqrt{\beta_1 s}\right) K_1\left(2\sqrt{\beta_2 s}\right)\right) \bigg|_{z=\frac{1}{x}},$$

which is a tabulated inverse Laplace transform leading to

$$P_X(x) = 1 - P_Z\left(\frac{1}{x}\right) = 1 - x\sqrt{\beta_1\beta_2} e^{-\frac{x}{2}(\beta_1+\beta_2)} K_1\left(x\sqrt{\beta_1\beta_2}\right).$$
Derivation of the PDF of $X = \mu_H(X_1, X_2)$

• Taking the derivative of the CDF of $X$ with respect to $x$ results in

$$\frac{d}{dx} \left( P_X(x) \right) = - \left[ \sqrt{\beta_1 \beta_2} e^{-\frac{x}{2}(\beta_1 + \beta_2)} K_1 \left( x \sqrt{\beta_1 \beta_2} \right) + x \sqrt{\beta_1 \beta_2} \left( -\frac{1}{2} (\beta_1 + \beta_2) e^{-\frac{x}{2}(\beta_1 + \beta_2)} \right) K_1 \left( x \sqrt{\beta_1 \beta_2} \right) + e^{-\frac{x}{2}(\beta_1 + \beta_2)} \frac{d}{dx} \left[ K_1 \left( x \sqrt{\beta_1 \beta_2} \right) \right] \right] \right) \right] .$$

• Using

$$z \frac{d}{dz} K_v(z) + v K_v(z) = -z K_{v-1}(z)$$

leads to the final desired result

$$p_X(x) = \frac{1}{2} \beta_1 \beta_2 x e^{-\frac{x}{2}(\beta_1 + \beta_2)} \left[ \left( \frac{\beta_1 + \beta_2}{\sqrt{\beta_1 \beta_2}} \right) K_1 \left( x \sqrt{\beta_1 \beta_2} \right) + 2 K_0 \left( x \sqrt{\beta_1 \beta_2} \right) \right] .$$
Formulas for the Outage Probability

- For non-regenerative systems, $P_{\text{out}}$ is given by
  \[
P_{\text{out}} = 1 - \frac{2\gamma_{\text{th}}}{\sqrt{\bar{\gamma}_1 \bar{\gamma}_2}} K_1 \left( \frac{2\gamma_{\text{th}}}{\sqrt{\bar{\gamma}_1 \bar{\gamma}_2}} \right) e^{-\gamma_{\text{th}} \left( \frac{1}{\bar{\gamma}_1} + \frac{1}{\bar{\gamma}_2} \right)},
  \]
  where $\bar{\gamma}_1$ and $\bar{\gamma}_2$ are the average SNRs of hops 1 and 2, respectively.

- For regenerative systems, $P_{\text{out}}$ is given by
  \[
P_{\text{out}} = 1 - e^{-\gamma_{\text{th}} \left( \frac{1}{\bar{\gamma}_1} + \frac{1}{\bar{\gamma}_2} \right)}.
  \]

- Both formulas are equivalent at high average SNR since for small $x$
  \[
  K_1(x) \approx \frac{1}{x}.
  \]
Outage Probability: Numerical Example

Comparison of Outage Probability for Regenerative and Non-Regenerative Systems

- Non-Regenerative System, $\gamma_1 = \gamma_2$
- Regenerative System, $\gamma_1 = \gamma_2$
- Non-Regenerative System, $\gamma_1 = 2\gamma_2$
- Regenerative System, $\gamma_1 = 2\gamma_2$
Outage Probability of Collaborative Systems

- Consider a wireless communication system with one direct link and $L$ collaborating paths.
- Assume direct link with average SNR $\bar{\gamma}_0$ and that the two hops in collaborating path $l$ have the same average SNR $\bar{\gamma}_l$.
- Assume that the strongest path is selected at any given time.
- Resulting outage probability

$$P_{\text{out}} = \left(1 - e^{-\frac{\gamma_{\text{th}}}{\bar{\gamma}_0}}\right) \times \prod_{l=1}^{L} \left(1 - \frac{2\gamma_{\text{th}}}{\bar{\gamma}_l} e^{-\frac{2\gamma_{\text{th}}}{\bar{\gamma}_l} K_1 \left(\frac{2\gamma_{\text{th}}}{\bar{\gamma}_l}\right)}\right).$$
Formulas for the Average BER

- The MGF of $\gamma_{eq}$, $\mathbb{E}(e^{-\gamma s})$, for identical and independent faded hops, i.e. $\bar{\gamma}_1 = \bar{\gamma}_2 = \bar{\gamma}$, is given by

$$M_\Gamma(s) = \sqrt{\frac{\bar{\gamma}}{4} s} \left(\frac{\bar{\gamma}}{4} s + 1\right) + \text{arcsinh}\left(\sqrt{\frac{\bar{\gamma}}{4} s}\right)$$

$$\frac{2\sqrt{\frac{\bar{\gamma}}{4} s} \left(\frac{\bar{\gamma}}{4} s + 1\right)^{\frac{3}{2}}}{2\sqrt{\frac{\bar{\gamma}}{4} s} \left(\frac{\bar{\gamma}}{4} s + 1\right)^{\frac{3}{2}}}$$

- For non-regenerative systems with DPSK the average BER is

$$P_b(E) = \frac{1}{2} M_\gamma(1).$$

- For regenerative systems with DPSK over identical and independent faded hops

$$P_b(E) = \frac{1 + 2\bar{\gamma}}{2 (1 + \bar{\gamma})^2}.$$
Comparison of Bit Error Rates for Regenerative and Non-Regenerative Systems

Average BER: Numerical Example
Average BER with Collaboration

- Consider one direct link and $L$ i.i.d. faded collaborating paths.
- Using maximal-ratio combining at the receiver, the overall SNR can be written as

$$\gamma_t = \gamma_0 + \sum_{l=1}^{L} \gamma_l.$$ 

- Under these conditions the MGF of the overall combined SNR $\gamma_t$ is given by

$$\mathcal{M}_{\gamma_t}(s) = \mathcal{M}_{\gamma_0}(s) \prod_{l=1}^{L} \mathcal{M}_{\gamma_l}(s).$$
Diversity Gain due to Collaboration

Effect of Collaborative Diversity on Average BER Performance

Average Bit Error Rate vs. Average SNR per bit for different loadings (L)
Formulas for the Shannon Capacity

- For non-regenerative systems

\[ C/W = \log_2(1 + \gamma_{eq}) \text{bps/Hz} \]

- Capacity PDF is given by

\[ p_C(c) = \frac{2^{c+1} \ln 2 (2^c - 1) e^{-(2^c-1)\left(\frac{1}{\gamma_1} + \frac{1}{\gamma_2}\right)}}{(\gamma_1 + \gamma_2) \sqrt{\gamma_1 \gamma_2} \left[K_1\left(\frac{2(2^c - 1)}{\sqrt{\gamma_1 \gamma_2}}\right) + 2K_0\left(\frac{2(2^c - 1)}{\sqrt{\gamma_1 \gamma_2}}\right)\right]} \times \left[\left(\frac{\sqrt{\gamma_1 \gamma_2}}{\gamma_1 + \gamma_2}\right) K_1\left(\frac{2(2^c - 1)}{\sqrt{\gamma_1 \gamma_2}}\right) + 2K_0\left(\frac{2(2^c - 1)}{\sqrt{\gamma_1 \gamma_2}}\right)\right] \].

- For regenerative systems

\[ C_{eq} = \min(C_1, C_2) \]

- Capacity PDF is given by

\[ p_C(c) = \ln 2 \left(\frac{1}{\gamma_1} + \frac{1}{\gamma_2}\right) 2^c e^{-(2^c-1)\left(\frac{1}{\gamma_1} + \frac{1}{\gamma_2}\right)}. \]
Comparison of Capacity Outage for Regenerative and Non–Regenerative Systems

- **Non–Regenerative System**
- **Regenerative Systems**

![Graph showing comparison of capacity outage for regenerative and non-regenerative systems.](image-url)
Extension to Systems with $N$ Hops

• Analog relaying with channel inversion of the previous link

$$\gamma_{eq2} = \left[ \sum_{n=1}^{N} \frac{1}{\gamma_n} \right]^{-1}$$

– Related to the harmonic mean of the hop’s SNRs.

• Analog relaying with bounded relay gains

$$\gamma_{eq1} = \left[ \prod_{n=1}^{N} \left( 1 + \frac{1}{\gamma_n} \right) - 1 \right]^{-1}$$

– Example of a triple-hop system:

$$\frac{1}{\gamma_{eq1}} = \frac{1}{\gamma_1} + \frac{1}{\gamma_2} + \frac{1}{\gamma_3} + \frac{1}{\gamma_1 \gamma_2} + \frac{1}{\gamma_1 \gamma_3} + \frac{1}{\gamma_2 \gamma_3}.$$
Where to Regenerate?

Effect of the Regeneration Position

Normalized Average SNR per Hop [dB]

Outage Probability

Non-regenerative System
Regeneration after hop 1 or 3
Regeneration after hop 2
Increasing the Number of Hops

Effect of Number of Hops on Outage Probability

Normalized Average SNR per Hop [dB]

Outage Probability

N=1  N=10
Analog versus Digital Relaying

Comparison Between Regenerative and Non-regenerative Systems

- Non-regenerative System
- Regenerative System

Number of Hops
Outage Probability
Other Topics of Interest

• Analog relaying with “fixed” relay gain.
• Variable-power and/or variable rate relays.
• Latency and delay associated with multi-hop systems.
• Global optimization versus local optimization.
• Power consumption and fairness issues.