Throughput Guarantees for Wireless Networks with Opportunistic Scheduling: A Comparative Study

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Abstract

In this letter we develop an expression for the approximate *throughput guarantee violation probability* (TGVP) for users in time-slotted networks for any scheduling algorithm with a given mean and variance of the bit-rate in a time-slot, and a given distribution for the number of time-slots allocated within a time-window. Based on this general result, we evaluate closed-form expressions for the TGVPs for four well-known scheduling algorithms. Through simulations we also show that our TGVP approximation is tight for a realistic network with moving users with correlated channels and realistic throughput guarantees.

Index Terms

Opportunistic scheduling, quality-of-service, wireless communications.

I. Introduction

In modern cellular network standards like HSPA, 1xEVDO, and Mobile WiMAX, the rate of a user is adapted to the channel quality [1]. By giving priority to users with high channel quality, the system capacity can be increased significantly [2], [3]. However, fulfilling the users' (quality-of-service) QoS requirements in such a system can be difficult since the users with the lowest channel quality will often be starved. Consequently, it is necessary to implement scheduling algorithms that take both the channel quality and the QoS demands of the users into account.

Many previous publications have concentrated on analyzing how *fair* the resource allocation is in the network [4], [5]. However, it can be difficult to quantify fairness and the concept of fairness can often be difficult to understand both for the operators and the mobile users. In commercial networks it is more useful to look at a more precise notion of QoS, namely *throughput guarantees*. The advantage of being able to quantify throughput guarantees will make it easier for the network operator to offer a service that is tailor-made to the applications that are going to be transmitted. In addition, the network operators do not have to over-dimension the wireless networks to satisfy the QoS demands of the customers.

There are two types of throughput guarantees that can be offered to customers, namely *hard* or *deterministic throughput guarantees*, and *soft* or *stochastic throughput guarantees*. The hard throughput guarantees promise, with unit probability, a certain throughput to the users within a given time-window, while soft throughput guarantees promise that each user will have a specified throughput within a given time-window, with a probability that is high, but less than unity. For telecommunications networks in general, and for wireless networks in particular, soft throughput guarantees are more suited for specifying QoS than hard throughput guarantees. This is because such networks often have a varying number of users and varying loads from the applications of these users. For wireless networks, the varying quality of the radio channel will further add uncertainty to the size of the throughput that can be guaranteed for short time-spans. In addition, will opportunistic scheduling give priorities to the users with the best channel conditions (subject to various constraints), and the waiting period between each time a user is scheduled can therefore vary significantly. This makes soft throughput guarantees suited as QoS metrics for modern wireless networks.

Obtaining analytical expressions for what soft throughput guarantees that can be offered in a wireless network makes it possible to calculate the QoS of the users in a very efficient way for a set of instantaneous system parameters. Such analytical expressions can therefore be used directly in adaptive radio resource algorithms for wireless networks where the users move around with high speed and where real-time applications constitute the dominating traffic load.

Contributions. Quantifying the soft throughput guarantees that can be given for a certain scheduling algorithm without conducting experimental investigations has, to the best of our knowledge, not been looked into before. We obtain a general expression for a tight approximation of the *throughput guarantee violation probability* (TGVP), for a given mean and variance of the number of bits transmitted in a time-slot, and a given distribution for the number of time-slots allocated to a user within a time-window. We also investigate the tightness of this approximation for a realistic scenario with users that have correlated channels[1].

Organization. The rest of this letter is organized as follows. In Section II we present the system model. We develop a general expression for the approximate TGVP in Section III. In Section IV we plot closed-form expressions for the approximate TGVP for four different scheduling algorithms and analyze the tightness of the approximation by comparing the analytical results with simulations for a realistic scenario. Our conclusions are presented in Section V.

II. System Model

We consider a single base station that serves N users using time-division multiplexing (TDM). The analytical results will be valid for the downlink, however also for the uplink if reciprocity can be assumed between the downlink and uplink. In any case we assume that the total bandwidth available for the users is W [Hz] and that the transmit power is constant for all transmitters. Each user measures his own CNR perfectly, and before performing scheduling, the base station is assumed to receive these measurements from all the users. For each time-slot the base station takes a scheduling decision and broadcasts this decision to the selected user before transmission starts. We assume that the channels of the users are flat Rayleigh block-fading channels with a constant average received CNR $\overline{\gamma}_i$ for user i. The variations in average CNR in real-life networks is often on the time-scale of several seconds, while realistic throughput guarantees are calculated for time-scales under 100 milliseconds. Consequently, it is realistic to assume that the average CNRs are constant over the time-window for which the throughput guarantees are calculated.

The block or time-slot duration, T_{TS} [seconds], is assumed to be less than one coherence time, i.e., the channels can be regarded more or less as constant during one time-slot. To obtain our analytical results we also assume that the CNR values from time-slot to time-slot are uncorrelated. This means that one user will very seldom experience two adjacent time-slots with the same

[1]Parts of this letter are based on work in [6] and [7].

CNR values, and consequently, the opportunistic distribution of time-slots between the users appear to be more fair. This will influence our analytical results to some extent since it is easier to fulfill the throughput guarantees within a given time-window when such a channel model is assumed.

Another important assumption is that the users always have data to send or transmit. For real-time applications this is often a realistic assumption because the packet flow from the applications is relatively constant in this case.

III. How to Quantify the Throughput Guarantees

A soft throughput guarantee can be expressed as the probability of *not* fulfilling a given throughput guarantee, i.e., the *throughput guarantee violation probability*, TGVP. Defining the desired throughput guarantee as guaranteeing a throughput of B [bits] over a time-window T_W [seconds] for all N users with probability at least $1-\epsilon$, we can analytically define the problem as attempting to constrain the TGVP to be less than or equal to ϵ [8]:

$$\Pr(b_i < B) \leq \epsilon, \;\; i = 1, 2, \cdots, N, \tag{1}$$

i.e., the probability of the number of bits b_i being transmitted to or from user i within a time-window T_W being below B, should be less than or equal to ϵ.

A. Computing Throughput Guarantee Violation Probabilities

To be able to obtain an exact TGVP we would have to find a probability mass function (PMF) for the sum of bits that a user can transmit in the M time-slots he is allocated. From [9] and several other publications, we conclude that finding an exact closed-form expression for the value of the TGVP $\Pr(b_i < B)$ is a complex problem that has not yet been solved, and may very well not be solvable in closed form. We will therefore instead look at how we can *approximate* the TGVP.

We now formulate a proposition that can be used as a tool to specify an achievable soft throughput guarantee of B bits over a time-window T_W constituting K time-slots. For users transmitting over a time-slotted block fading channel, with $b_{i,j}$ bits being transmitted to or from user i in the jth time-slot he is scheduled, and the probability that user i gets $M = k$ out of K time-slots denoted as $p_M(k|i)$, the following holds:

Proposition: The probability that the throughput constraint B is violated over K time-slots for user i can be approximated as:

$$\Pr(b_i < B) \approx p_M(0|i) + \frac{1}{2}\sum_{k=1}^{K} p_M(k|i)\,\mathrm{erfc}\left(-\frac{B/k - \mu_{\bar{b}_{i,k}}}{\sqrt{2}\sigma_{\bar{b}_{i,k}}}\right), \tag{2}$$

where $\bar{b}_{i,k} = \frac{1}{k}\sum_{j=1}^{k} b_{i,j}$ is the average number of bits being transmitted to or from user i when he is allocated $M = k$ time-slots, and $\mu_{\bar{b}_{i,k}}$ and $\sigma^2_{\bar{b}_{i,k}}$ is the mean and variance of $\bar{b}_{i,k}$, respectively. Also, $\mathrm{erfc}(x) = \frac{2}{\sqrt{\pi}}\int_x^{\infty} e^{-t^2}dt$ is the *complementary error function*.

Proof: The allocation of different number of time-slots to a user constitute mutually exclusive events. The TGVP for user i over K time-slots can therefore be expressed as follows, using the law of total probability:

$$\begin{aligned}
\Pr(b_i < B) &= \Pr(b_i < B|0) \cdot p_M(0|i) \\
&+ \Pr(b_i < B|1) \cdot p_M(1|i) \\
&\dots \\
&+ \Pr(b_i < B|K) \cdot p_M(K|i),
\end{aligned} \tag{3}$$

where $\Pr(b_i < B|k)$ denotes the TGVP when user i is assigned $M = k$ time-slots and $p_M(k|i)$ denotes the probability that user i gets $M = k$ time-slots within the interval of K time-slots.

To be able to discuss a total throughput guarantee B within K time-slots, we first consider the number of bits transmitted to or from user i within the jth time-slot he is scheduled, and denote this number by $b_{i,j}$. For a system using constant transmit power and capacity-achieving codes which operate at the Shannon capacity limit we will have $b_{i,j} = T_{\mathrm{TS}} W \log_2(1 + \gamma_{i,j})$, where $\gamma_{i,j}$ is the CNR in the jth time-slot user i is scheduled.

We can now express the probability for violating the throughput guarantee B when k out of K time-slots are scheduled to user i as:

$$\begin{aligned}
\Pr(b_i < B|k) &= \Pr\left(\sum_{j=1}^{k} b_{i,j} < B\right) \\
&= \Pr\left(\bar{b}_{i,k} < \frac{B}{k}\right) \\
&\approx \frac{1}{2}\,\mathrm{erfc}\left(-\frac{B/k - \mu_{\bar{b}_{i,k}}}{\sqrt{2}\sigma_{\bar{b}_{i,k}}}\right),
\end{aligned} \tag{4}$$

where erfc $\left(-\frac{x-\mu}{\sqrt{2}\sigma}\right) = \Pr\left(X \leq x\right)$ is the cumulative distribution function (CDF) of a Gaussian distributed random variable X with mean μ and variance σ^2. In the expression in (4) we have $\mu_{\bar{b}_{i,k}} = \mu_{b_{i,j}}$ and $\sigma^2_{\bar{b}_{i,k}} = \sigma^2_{b_{i,j}}/k$ where $\mu_{b_{i,j}}$ and $\sigma^2_{b_{i,j}}$ are the mean and variance of the number of bits transmitted to or from user i in the jth time-slot he is scheduled. The approximation above has been obtained by using the *Central Limit Theorem* (CLT) [10, p. 1231].

By inserting (4) into (3), we see that the expression for the total throughput guarantee can be expressed as in (2). ■

IV. Numerical Results

In this section we plot and compare the expressions for the approximate TGVPs for four different scheduling algorithms. We also evaluate the accurateness of these expressions. However, before evaluating the plots, we choose to comment on the system parameters used in this section.

A. Realistic System Parameters for Cellular Networks

For the wireless standards 1xEVDO, HSDPA, and Mobile WiMAX, the time-slot length for the downlink is respectively 1.67, 2, and 5 ms [1]. The European IST research project WINNER I has suggested a time-slot duration of 0.34 ms for a future wireless system [11]. According to [12], the maximum one-way delay over a wireless HSDPA link should lie between 80 and 150 ms for voice over IP (VoIP) conversations to achieve good speech quality. If we assume that $T_W = 80$ ms, K equals 235, 48, 40, and 16 time-slots for WINNER I, 1xEVDO, HSDPA, and Mobile WiMAX, respectively.

The raw throughput needed for one-way, telephone-quality speech varies from about 5 kbit/s up to 64 kbit/s [13]. The corresponding raw throughput needed for one-way videoconferencing varies from 64 kbit/s up to 500 kbit/s. In addition a minimum of 4 percent protocol overhead has to be added. From these throughput demands and the value of T_W, realistic values for B can be calculated for each application session for a given set of system parameters.

B. Comparison of the TGVP of Different Scheduling Algorithms

Figs. 1 and 2 show the TGVP-performance of different scheduling algorithms for 10 users requesting B bits within a time-window $T_W = 80$ ms for a system with the time-slot length of Mobile WiMAX and WINNER I, respectively. We have plotted the TGVP performance for four algorithms, namely Round Robin Scheduling (RR), Maximum CNR Scheduling (MCS), Normalized CNR Scheduling (NCS) and Normalized Opportunistic Round Robin Scheduling (ORR). By using the expressions in Table I and inserting $p_M(k|i)$, $\mu_{\bar{b}_{i,k}} = \mathrm{E}[b_{i,j}]$ and $\sigma^2_{\bar{b}_{i,k}} = (\mathrm{E}[b^2_{i,j}] - (\mathrm{E}[b_{i,j}])^2)/k$ into (2), we obtain the TGVP approximations for these four scheduling algorithms. For the RR policy, the time-slots are allocated to the users in a sequential manner, i.e. totally non-opportunistically. The most opportunistic algorithm is the MCS policy because it always schedules the user with the highest CNR, and hence the highest rate. The NCS policy is a more fair policy because it schedules the users with the highest CNR relative to their own average CNR [14]. The ORR policy was introduced in [15] and is a combination of the RR and MCS policies. For this algorithm, the time-slots are allocated in rounds of N competitions where the users are guaranteed to be assigned one time-slot in each round. For the first competition the best user is chosen. This user is than taken out from the rest of the competitions in the round, and for the second time-slot the best of the remaining users are chosen. For each competition a new user is taken out and for the last time-slot in a round the channel is assigned to the remaining user. If the users average CNRs are spread far apart, the ORR algorithm will have the same spectral efficiency as conventional RR Scheduling. To have a more efficient ORR algorithm for this scenario, we have modified this algorithm such that the user with the highest normalized CNR is chosen in each competition. We refer to this algorithm as the Normalized-ORR (N-ORR) algorithm.

Figs. 1 and 2 are plotted for a user with $\overline{\gamma}_i = 5$, where the all the users' channels are Rayleigh distributed with constant average CNRs that have a total average of 15 dB. The user with the worst channel has an average CNR of $\overline{\gamma}_i = 5$ dB and the user with the best channel has $\overline{\gamma}_i = 17.79$ dB. We have chosen to plot the TGVP for the user with the worst channel because this user will have the lowest TGVP values of all the users in the system. The most interesting parts the figures are where the TGVP is close to zero, since for these low TGVP values it is a high probability that the throughput guarantee is fulfilled. We can observe that for both the Mobile WiMAX and the WINNER I systems, the N-ORR algorithm shows the best TGVP-performance. This algorithm can support close to hard throughput guarantees up to about 0.5 bits/sec/Hz for Mobile WiMAX, while the corresponding throughput guarantee limit for the WINNER I system is over 2 bits/sec/Hz. The reason why this value more than quadruples from $K = 16$ time-slots to $K = 235$ time-slots is that the more time-slots we have within the time-window, the higher is the likelihood that all the users will be assigned some time-slots with good channel conditions. Hence, it will be easier to obtain a low TGVP for large values of K.

Also the RR algorithm shows a relatively good TGVP-performance for $K = 16$ time-slots. This is because this algorithm can promise that all the user get at least one time-slot within a time-window of N time-slots. The MCS algorithm is not very useful to guarantee any throughput for the user with the worst channel. This is because the user with the highest CNR is chosen at all times and there is therefore a low probability that the user with $\overline{\gamma}_i = 5$ dB is chosen.

In this paper we have assumed that only one user is scheduled in each time-slot. Since both Mobile WiMAX and WINNER I are based on orthogonal frequency-division multiplexing (OFDM) with respectively 720 and 1664 sub-carriers for user data, it is possible to schedule more user within the same time-slot for these systems, if we assume that channel estimates of each sub-carrier are available at the base station [1], [11]. Consequently, the corresponding TGVP performance for OFDM-based systems will be higher than the results shown in this paper. How much the TGVP performance will increase for a OFDM-based system model depends on the CNR correlation between the sub-carriers. Our closed-form expressions can also be used to obtain TGVP approximations for this system model by replacing K with $K \cdot N_{SC}$ and W with W_{SC}, where N_{SC} is the number of sub-carriers and W_{SC} is the bandwidth of each sub-carrier.

C. On the Accuracy of the Approximate TGVP

Figs. 3 and 4 show the TGVP approximations for N-ORR together with the corresponding Monte Carlo simulated TGVPs for respectively Mobile WiMAX and WINNER I. The approximate results are based on the assumption that the time-slots are uncorrelated, while the Monte Carlo simulations are for users that have a correlated CNR from time-slot to time-slot. We have used Jakes' correlation model with carrier frequency of $f_c = 1$ GHz and a user speed of $v = 30$ m/s. The channel gain is modeled as a sum of sinusoids with correlation coefficient $f_D T_{\text{TS}} = \frac{v f_c}{c}$, where f_D is the Doppler frequency shift and c is the speed of light [16].

The tightness of the approximation is both influenced by K and T_{TS}. Since the CLT is used to obtain the formula for the approximative TGVPs, we therefore need to calculate the TGVP for a relatively large number of time-slots K to obtain a tight approximation. However, if we have shorter time-slots, we will also experience a higher correlation between the time-slots. Since we have assumed uncorrelated time-slots to obtain our TGVP approximation, we will therefore have a less tight approximation for short time-slots. For both Mobile WiMAX ($K = 16$ time-slots) and WINNER I ($K = 235$ time-slots) we see that our approximate results are too optimistic for TGVPs close to zero. However, we see that the TGVP approximation close to TGVP$= 0$ is slightly better for WINNER I and we can therefore conclude that the number of time-slots K within the time-window T_W will affect the tightness of TGVP-approximation more than the fact that the shorter time-slots are more correlated.

For long values of T_W, the value of K is higher and the correlation over the time-window is smaller. We can therefore conclude that long time-windows will lead to more tight TGVP approximations.

V. CONCLUSION

In this letter we have developed a general approximation for the TGVP which can be obtained in a time-slotted wireless network with any scheduling policy with (i) a given set of system parameters, (ii) known first two moments of the bits transmitted to or from the scheduled user in a time-slot, and (iii) a given distribution of the number of time-slots allocated to a user within a time-window. We have evaluated closed-form expressions for the corresponding TGVP approximations for four well-known scheduling algorithms, namely Round Robin, Maximum CNR Scheduling, Normalized CNR Scheduling and Normalized Opportunistic Round Robin. Our TGVP approximations were also compared to Monte Carlo simulations for users with correlated channels. From our numerical investigations, it can be concluded that correlated time-slots have a small effect on the tightness of the approximations. It can also be concluded that the TGVP approximations are tighter for relatively long time-windows T_W.

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TABLE I

CLOSED-FORM EXPRESSIONS FOR $p_M(k\mid i)$, $\mathrm{E}[b_{i,j}]$ AND $\mathrm{E}[b_{i,j}^2]$

RR[2,3]	$p_M(k \mid i) = \begin{cases} \frac{(k+1)N-K}{N}, & k = \lfloor \frac{K}{N} \rfloor \\ \frac{K-(k-1)N}{N}, & k = \lceil \frac{K}{N} \rceil \\ 0, & \text{otherwise} \end{cases}$
	$\mathrm{E}[b_{i,j}] = \frac{WT_{\text{TS}}}{\ln 2} e^{1/\overline{\gamma}_i} E_1\left(\frac{1}{\overline{\gamma}_i}\right)$
	$\mathrm{E}[b_{i,j}^2] = \frac{(WT_{\text{TS}})^2}{\overline{\gamma}_i (\ln 2)^2} \Psi\left(\frac{1}{\overline{\gamma}_i}\right)$
MCS[2,3,4]	$p_M(k \mid i) = \binom{K}{k} p_i^k (1-p_i)^{K-k}$, $p_i = \frac{1}{\overline{\gamma}_i} \sum_{\tau \in T_i^N} \text{sign}(\tau) \frac{1}{\frac{1}{\overline{\gamma}_i} + \lvert\tau\rvert}$
	$\mathrm{E}[b_{i,j}] = \frac{WT_{\text{TS}}}{p_i \overline{\gamma}_i \ln 2} \sum_{\tau \in T_i^N} \text{sign}(\tau) \frac{e^{\left(\frac{1}{\overline{\gamma}_i} + \lvert\tau\rvert\right)}}{\frac{1}{\overline{\gamma}_i} + \lvert\tau\rvert} E_1\left(\frac{1}{\overline{\gamma}_i} + \lvert\tau\rvert\right)$
	$\mathrm{E}[b_{i,j}^2] = \frac{(WT_{\text{TS}})^2}{p_i \overline{\gamma}_i (\ln 2)^2} \sum_{\tau \in T_i^N} \text{sign}(\tau) \Psi\left(\frac{1}{\overline{\gamma}_i} + \lvert\tau\rvert\right)$
NCS[2,3]	$p_M(k \mid i) = \binom{K}{k} \frac{1}{N}^k (1 - \frac{1}{N})^{K-k}$
	$\mathrm{E}[b_{i,j}] = \frac{NWT_{\text{TS}}}{\ln 2} \sum_{j=0}^{N-1} \binom{N-1}{j} \frac{(-1)^j}{1+j} e^{\frac{1+j}{\overline{\gamma}_i}} E_1\left(\frac{1+j}{\overline{\gamma}_i}\right)$
	$\mathrm{E}[b_{i,j}^2] = \frac{N(WT_{\text{TS}})^2}{\overline{\gamma}_i (\ln 2)^2} \sum_{j=0}^{N-1} \binom{N-1}{j} (-1)^j \Psi\left(\frac{1+j}{\overline{\gamma}_i}\right)$
N-ORR[5,6]	$p_M(k) = \begin{cases} \frac{(k+1)N-K}{N}, & k = \lfloor \frac{K}{N} \rfloor \\ \frac{K-(k-1)N}{N}, & k = \lceil \frac{K}{N} \rceil \\ 0, & \text{otherwise} \end{cases}$
	$\mathrm{E}[b_{i,j}] = \begin{cases} \frac{WT_{\text{TS}}}{N} \sum_{n=1}^{N} A_i(n), & k = \lfloor \frac{K}{N} \rfloor \\ WT_{\text{TS}} \left(\frac{(k-1)\sum_{n=1}^{N} A_i(n)}{kN} + \frac{\sum_{n=kN-K+1}^{N} A_i(n)}{k(K-(k-1)N)} \right), & k = \lceil \frac{K}{N} \rceil \end{cases}$
	$\mathrm{E}[b_{i,j}^2] = \begin{cases} \frac{WT_{\text{TS}}}{N} \sum_{n=1}^{N} B_i(n), & k = \lfloor \frac{K}{N} \rfloor \\ WT_{\text{TS}} \left(\frac{(k-1)\sum_{n=1}^{N} B_i(n)}{kN} + \frac{\sum_{n=kN-K+1}^{N} B_i(n)}{k(K-(k-1)N)} \right), & k = \lceil \frac{K}{N} \rceil \end{cases}$

[2] $E_1(x) = \int_1^\infty e^{-xt}/t \, dt$ is the *exponential integral function*

[3] $\Psi(\mu) = e^\mu \left\{ \frac{1}{\mu} \left[\frac{\pi^2}{6} + (C + \ln(\mu))^2 \right] - 2 {}_3F_3(1,1,1;2,2,2;-\mu) \right\}$, where C is Euler's constant [17, (9.73)] and ${}_pF_q(a_1, \cdots, a_p; b_1, \cdots, b_q; \cdot)$ is the *generalized hypergeometric function* [18].

[4] T_i^N denotes a set containing the terms that arise from an expansion of the product $\prod_{\substack{j=1 \\ j \neq i}}^{N} P_{\gamma_j}(\gamma)$ as described in [19, Sec. III-D-2], where $P_{\gamma_j}(\gamma) = 1 - e^{-\gamma/\overline{\gamma}_j}$ is the CDF of the CNR γ of user j.

[5] $A_i(n) = \frac{n}{\ln 2} \sum_{j=0}^{n-1} \binom{n-1}{j} \frac{(-1)^j}{1+j} e^{\frac{1+j}{\overline{\gamma}_i}} E_1\left(\frac{1+j}{\overline{\gamma}_i}\right)$

[6] $B_i(n) = \frac{n}{\overline{\gamma}_i (\ln 2)^2} \sum_{j=0}^{n-1} \binom{n-1}{j} (-1)^j \Psi\left(\frac{1+j}{\overline{\gamma}_i}\right)$

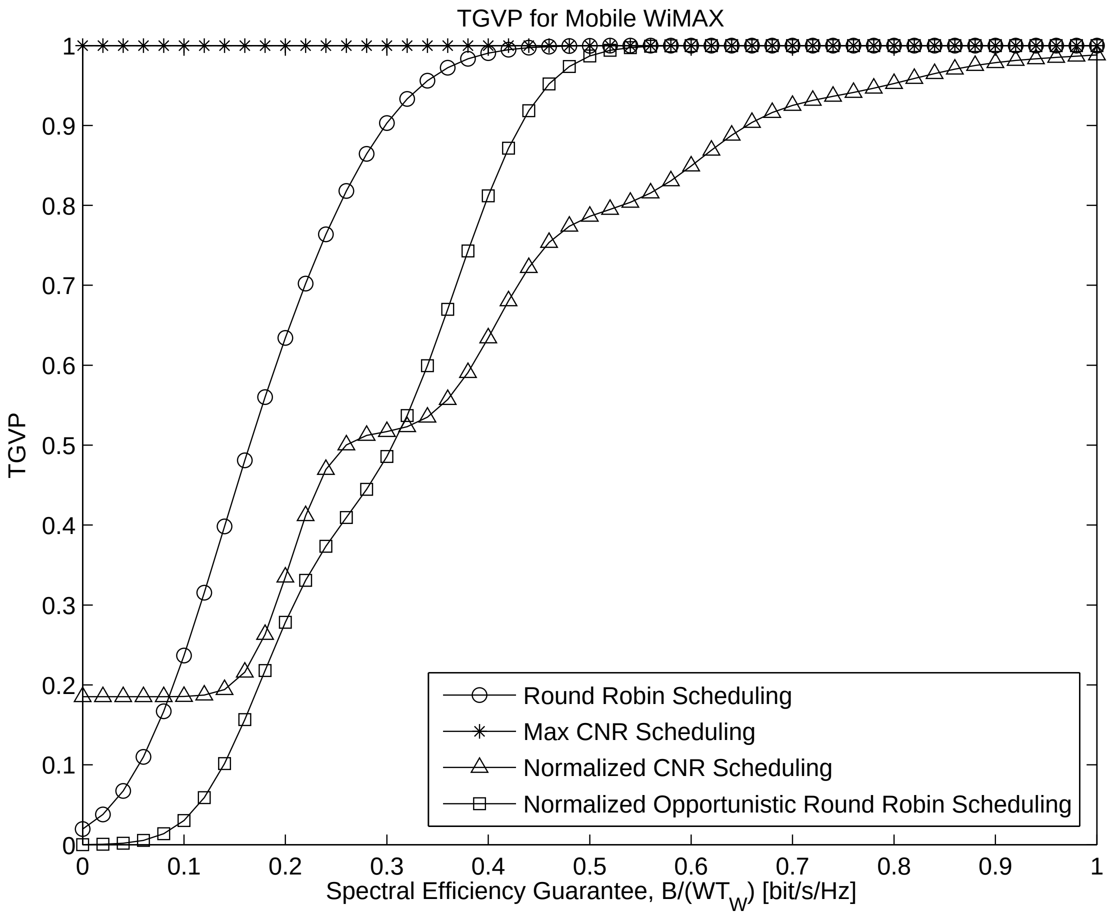

Fig. 1. Approximated Throughput Guarantee Violation Probability for a specific user i experiencing Rayleigh fading with $\overline{\gamma}_i = 5$ dB. There are 9 other users in the cell. Plotted for the Mobile WiMAX time-slot length of 5 ms and a time-window of $T_W = 80$ ms, corresponding to $K = 16$ time-slots.

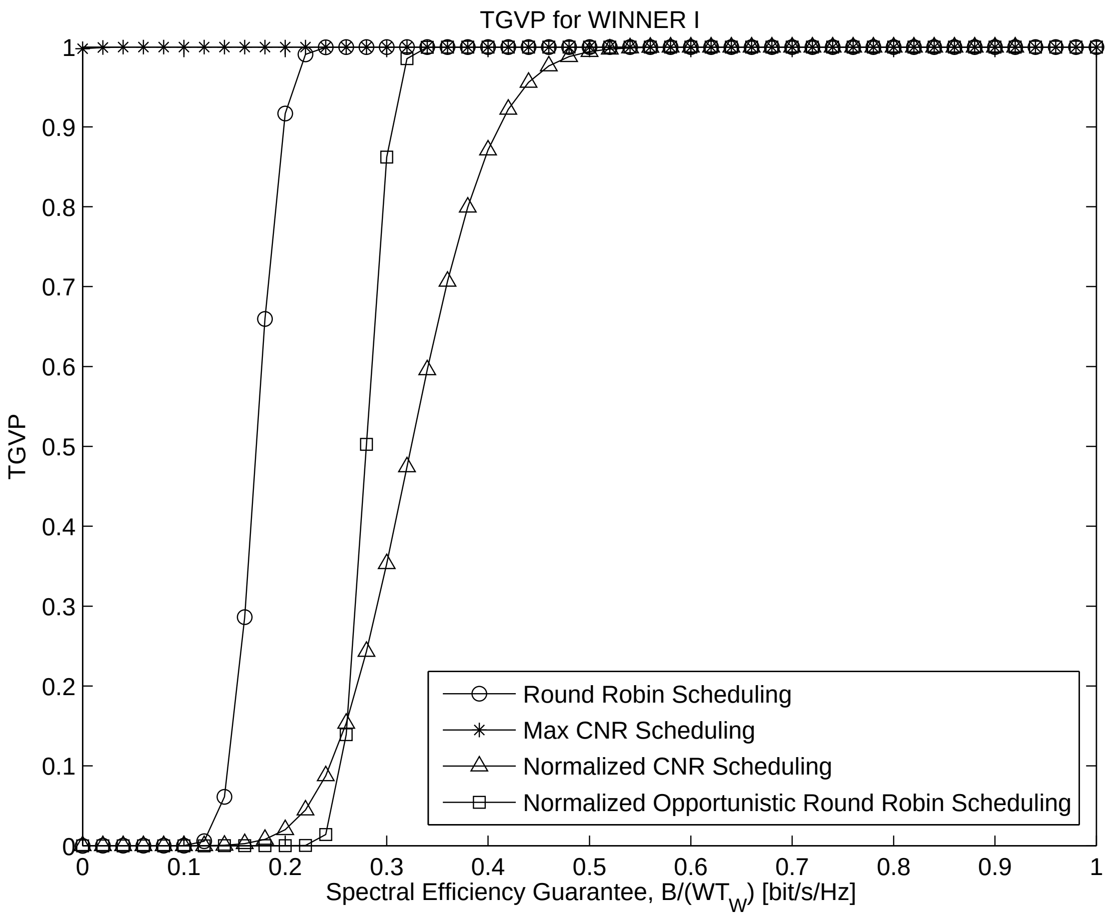

Fig. 2. Approximated Throughput Guarantee Violation Probability for a specific user i experiencing Rayleigh fading with $\overline{\gamma}_i = 5$ dB. There are 9 other users in the cell. Plotted for the WINNER I time-slot length of 0.34 ms and a time-window of $T_W = 80$ ms, corresponding to $K = 235$ time-slots.

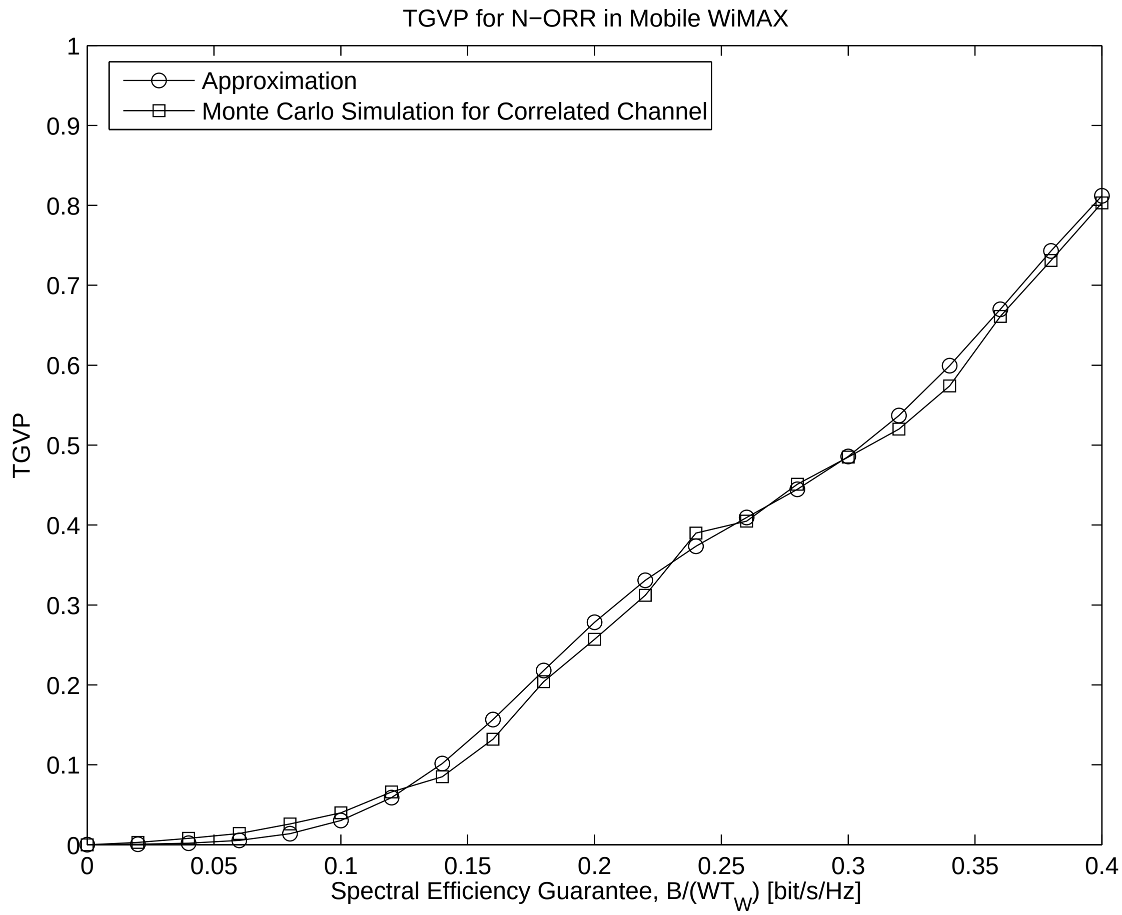

Fig. 3. Approximated TGVP vs. Monte Carlo simulated TGVP for a user with $\overline{\gamma}_i = 5$. There are 9 other users in the cell and N-ORR scheduling is used. Plotted for the Mobile WiMAX time-slot length of 5 ms and a time-window of $T_W = 80$ ms, corresponding to $K = 16$ time-slots. Each value in the simulated graph is an average over 1000 Monte Carlo simulations.

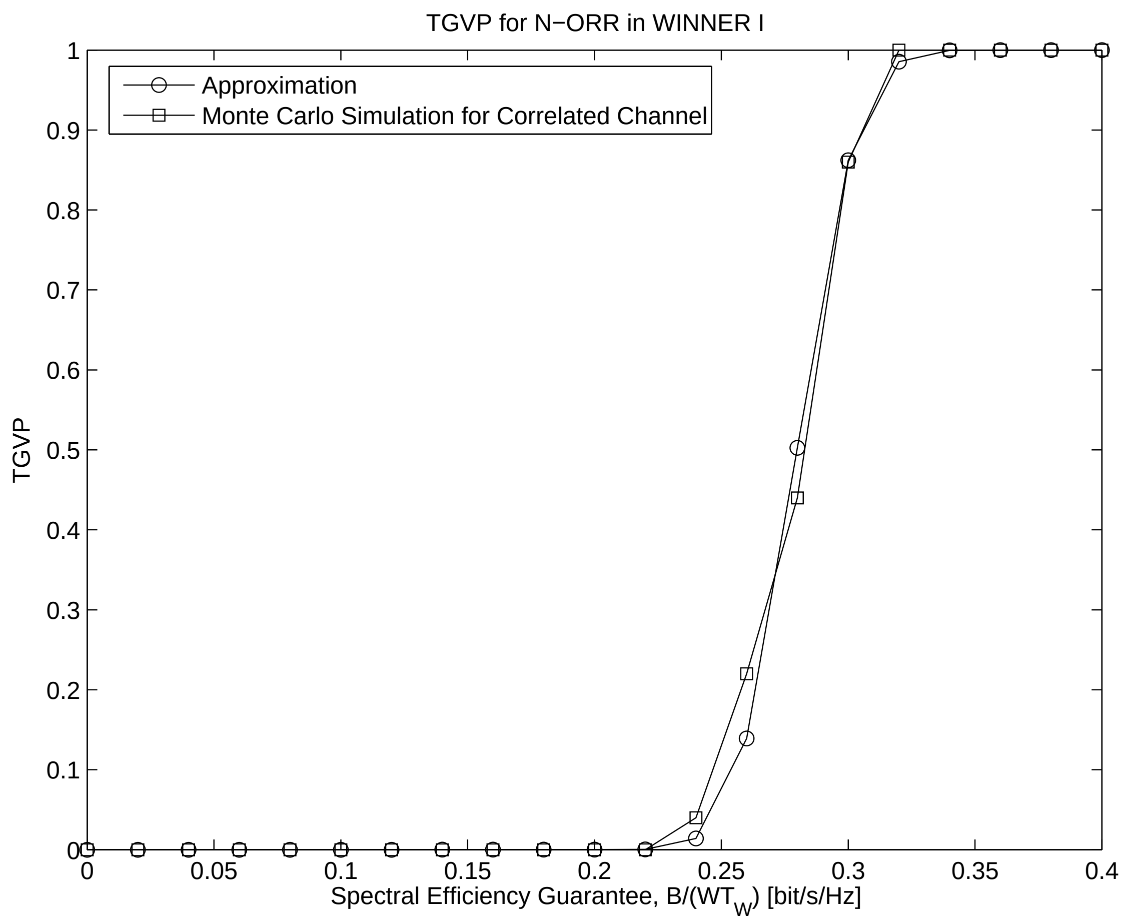

Fig. 4. Approximated TGVP vs. Monte Carlo simulated TGVP for a user with $\overline{\gamma}_i = 5$. There are 9 other users in the cell and N-ORR scheduling is used. Plotted for the WINNER I time-slot length of 0.34 ms and a time window of $T_W = 80$ ms, corresponding to $K = 235$ time-slots. Each value in the simulated graph is an average over 50 Monte Carlo simulations.