Multiuser Switched Diversity Transmission

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In a spatial diversity system, the diversity gain arises from independent signal paths received by multiple antennas.

In a multiuser communication system, a multiuser diversity gain arises from independent fading channels across different users.

Multiuser diversity can be exploited to maximize the average system throughput by always serving the user with the strongest channel.
A key observation is that a multiuser diversity system may be looked upon as a traditional spatial diversity system, in which the antennas of the spatial diversity combiner (acting as a base station (BS)) have been replaced by users, each having a single antenna.

A time-division multiplexed (TDM) system is considered, and the challenge for the BS is to identify the user having the best channel quality of all the connected users on a time-slot basis.
• A traditional way of performing this task is to let the BS probe all the users and select the user which reports the best channel quality at any given time-slot.

• The selected user is given access to either upload or download information.

• The average spectral efficiency (ASE) of the system can be maximized by transmitting with the highest possible rate supported by the selected channel.
• A multiuser access scheme aiming to detect the best user at any given time-slot is conceptually equal to the selection combining (SC) algorithm in a spatial diversity system.

• The SC approach yields the best average rate (ASE for a certain target bit-error-rate (BER)), but it comes at the expense of a high feedback load $N_e$.

• Feedback load $N_e \triangleq$ the number of estimated channels/users per time-slot before channel access.

• For the SC algorithm, $N_e$ will be deterministic and equal to the number of users connected to the BS.
• In an attempt to simplify the multiuser selection procedure and reduce the feedback load, a set of switched multiuser access schemes are proposed based on switched diversity algorithms originally devised to select between antennas in a spatial diversity system.

• The proposed multiuser access schemes are performed in a sequential manner, looking not for the best user but for an acceptable user.

• A user qualifies as an acceptable user and is selected by the base station when the reported channel quality is above a predefined switching threshold.
System model

- A TDM system is considered.
- Only one user has channel access (for uplink or downlink) per time-slot.
- It is assumed that a time-slot is roughly equal to the channel coherence time.
- A single time-slot is divided into a guard time and an information transmission time. During the guard time, the BS selects the user who will access the channel in the subsequent transmission time.
- The data burst is assumed to experience the same fading conditions as the preceding guard period (block fading).
- Fading across users are assumed to be independent.
- The individual users and the base station are all equipped with just a single antenna.
• A rate-adaptive coding scheme using $N = 8$ multidimensional trellis codes originally designed for additive white Gaussian noise (AWGN) channels is utilized on each selected link to ensure a high ASE of the system.

• The codes are based on quadrature amplitude modulation (QAM) signal constellations of growing size $\{M_n\} = \{4, 8, 16, 32, 64, 128, 256, 512\}$.

• Rate adaption is performed by splitting the signal-to-noise ratio (SNR) range into $N+1$ fading regions (bins), and the separate fading regions are defined by the SNR thresholds $0 < \gamma_1 < \gamma_2 < \ldots < \gamma_N < \gamma_{N+1} = \infty$.

\[
\begin{array}{cccccc}
\text{Outage} & R_1 & R_2 & \ldots & R_{N-1} & R_N \\
0 & \gamma_1 & \gamma_2 & \gamma_3 & \gamma_{N-1} & \gamma_N & \infty
\end{array}
\]

• The lower limit $\gamma_n$ of each fading region $[\gamma_n, \gamma_{n+1})$ is equal to the smallest SNR which guarantees that a predefined target BER (BER$_0$) is achieved by code $n$. 
The ASE of the system is obtained as a sum of the spectral efficiencies $R_n$ for the individual codes, weighted by the probability $P_n$ that the instantaneous SNR reported to the BS falls in the $n$th fading region:

$$\text{ASE} = \sum_{n=1}^{N} R_n \cdot P_n,$$

where

$$P_n = \int_{\gamma_n}^{\gamma_{n+1}} p_{\gamma_{BS}}(\gamma) d\gamma.$$  

The function $p_{\gamma_{BS}}(\gamma)$ denotes the probability density function (PDF) of the instantaneous SNR as observed by the BS. The shape of this PDF will depend on the mode of operation of the selected multiuser access scheme.
• The BER when averaged over all codes and SNRs is given as the average number of bits in error divided by the average number of bits transmitted:

\[
\overline{\text{BER}} = \frac{\sum_{n=1}^{N} R_n \cdot \overline{\text{BER}}_n}{\sum_{n=1}^{N} R_n \cdot P_n},
\]

(3)

where \( \overline{\text{BER}}_n \) is the average BER (averaged over all SNRs) experienced when code \( n \) is applied.

• An expression for \( \overline{\text{BER}}_n \) is obtained by utilizing the exponential approximation \( \overline{\text{BER}}_n \approx a_n \cdot e^{-b_n \gamma/M_n} \):

\[
\overline{\text{BER}}_n = \int_{\gamma_n}^{\gamma_{n+1}} a_n \cdot e^{-\frac{b_n \gamma}{M_n}} p_{\gamma_{BS}}(\gamma) d\gamma,
\]

(4)

where \( a_n \) and \( b_n \) are code-dependent constants found by least-square fitting to simulated data on AWGN channels.

• BER\(_n\) is invertible, so the smallest SNR required to achieve a given target BER (BER\(_0\)) can be identified as \( \gamma_n = (M_n/b_n) \ln(a_n/BER_0) \).
Multiuser access schemes

- In the following, 3 switched multiuser access schemes will be presented.

- The SC algorithm mentioned in the introduction will be used as a benchmark multiuser access scheme, denoted from now on as selection combining transmission (SCT).

- The letter $K$ is used to denote the total number of users connected to the BS.

- Independent and identically distributed (i.i.d.) Rayleigh fading channels across the different users are assumed.

- For later reference, the PDF and CDF of the instantaneous SNR $\gamma$ for a single-input single-output Rayleigh fading channel is equal to (and denoted in these slides as) $p_{\gamma}(\gamma) = \frac{1}{\bar{\gamma}} \cdot e^{-\gamma/\bar{\gamma}}$ and $P_{\gamma}(\gamma) = 1 - e^{-\gamma/\bar{\gamma}}$, respectively.

- The symbol $\bar{\gamma}$ denotes the average SNR, equal on all channels.
Selection combining transmission (SCT)

- PDF of $\gamma$ [1]:
  
  $$p_{\gamma_{bs}}(\gamma) = p_{\gamma_{SCT}}(\gamma) = K p_\gamma(\gamma) p_\gamma(\gamma).$$  
  \hspace{1cm} (5)

- ASE:
  
  $$ASE_{SCT} = \sum_{n=1}^{N} R_n \cdot ([P_\gamma(\gamma_{n+1})]^K - [P_\gamma(\gamma_n)]^K).$$  
  \hspace{1cm} (6)

- Average BER:
  
  $$\overline{BER}_{SCT} = \frac{K \sum_{n=1}^{N} R_n a_n (B_{a_n+1}(K, \beta_n) - B_{a_n}(K, \beta_n))}{ASE_{SCT}},$$  
  \hspace{1cm} (7)

  where $\beta_n = 1 + \frac{b_n \gamma}{M_n}$ and $B_z(x, y)$ denotes the incomplete beta function$^*$. 


\hspace{1cm} $^*B_z(x, y) = \int_0^z u^{x-1}(1 - u)^{y-1} du.$
Switch-and-examine transmission (SET)

- The switch-and-examine transmission scheme is conceptually equivalent to the traditional switch-and-examine combining (SEC) scheme [2].

- If no users are above the SNR threshold $\gamma_T$ after a sequential search, the last probed user is granted access to the channel. For this scheme, the PDF of the output SNR is given by [2]

$$p_{\gamma_{BS}}(\gamma) = p_{\gamma_{SET}}(\gamma) = \begin{cases} P_{\gamma}(\gamma_T)^{K-1}p_{\gamma}(\gamma) & \gamma < \gamma_T \\ \sum_{k=0}^{K-1}[P_{\gamma}(\gamma_T)]^kp_{\gamma}(\gamma) & \gamma \geq \gamma_T \end{cases}$$  

(8)

Multiuser access schemes cont’d

- ASE:

\[
ASE_{SET} = p^{K-1} \sum_{n=1}^{N} R_n \delta_n + \left( R_q \nu_q + \sum_{n=q+1}^{N} R_n \delta_n \right) \left( \sum_{k=0}^{K-2} p^k \right),
\]

where \( p = P_\gamma(\gamma_T) \), \( \nu_q = (e^{-\gamma T/\gamma} - e^{-\gamma_{q+1}/\gamma}) \) and \( \delta_n = (e^{-\gamma_n/\gamma} - e^{-\gamma_{n+1}/\gamma}) \).

- \( ASE_{SET} \) is a function of the letter \( q \in [1, 2, \ldots, N] \), which for all the switched multiuser access schemes are used to denote in which fading region the SNR threshold \( \gamma_T \) is placed.

- An optimal fading region \( q_{opt} \) in which to place the switching threshold \( \gamma_T \) can be determined based on selecting the solution that maximizes \( ASE_{SET} \).
Multiuser access schemes cont’d

- Average BER:

\[
\overline{\text{BER}}_{\text{SET}} = \frac{p^{K-1} \sum_{n=1}^{N} R_n \Delta_n + \left( R_q \gamma_q + \sum_{n=q+1}^{N} R_n \Delta_n \right) \sum_{k=0}^{K-2} p^k }{\text{ASE}_{\text{SET}}},
\]

where \( \Delta_n = \frac{a_n}{\mu_n \gamma} (e^{-\mu_n \gamma_n} - e^{-\mu_n \gamma_{n+1}}) \), \( \mu_n = \frac{b_n \gamma + M_n}{M_n \gamma} \), and \( \gamma_q = \frac{a_q}{\mu_q \gamma} (e^{-\mu_q \gamma_T} - e^{-\mu_q \gamma_{q+1}}) \).

- There is no point of minimizing \( \overline{\text{BER}}_{\text{SET}} \) as a function of \( q \), since \( \text{BER}_0 \) will be achieved regardless of the selected fading region.
Switch-and-examine transmission with post-selection (SETps)

- The same selection procedure as SET, except that if no acceptable link has been found, the best one of all the probed users is selected at the end instead of just picking the last one for simplicity [3].

- PDF of $\gamma$ [3]:

$$p_{\gamma_{BS}}(\gamma) = p_{\gamma_{SETps}}(\gamma) = \begin{cases} KP_{\gamma}(\gamma)p_{\gamma}(\gamma) & \gamma < \gamma_T \\ \sum_{k=0}^{K-1} [P_{\gamma}(\gamma_T)]^k p_{\gamma}(\gamma) & \gamma \geq \gamma_T \end{cases} \tag{11}$$

Scan-and-wait transmission (SWT)

- Instead of selecting a user at the end even if no user is above the switching threshold, an option is to wait a period longer than the channel coherence time, and try again [4].

- PDF of $\gamma$ [4]:

$$p_{\gamma_{BS}}(\gamma) = p_{\gamma_{SWT}}(\gamma) = \begin{cases} \frac{p_{\gamma}(\gamma)}{1-P_{\gamma}(\gamma)} & \gamma \geq \gamma_T \\ 0 & \text{otherwise} \end{cases} \quad (12)$$

Multiuser access schemes cont’d 25/05/04

• Given the mode of operation for the SET, SETps and SWT access schemes, the number of estimated paths before channel access \( N_e \) will no longer be deterministic.

• \( N_e \) will be a discrete random variable (RV) described by a probability mass function (PMF).

• Statistical measures are needed to quantify the feedback load.

In the following, note that:

• \( p = P_\gamma(\gamma_T) \) equals the probability of being below \( \gamma_T \).

• \( 1 - p \) equals the probability of being above \( \gamma_T \).
Multiuser access schemes cont’d

SET and SETps

- PMF of $N_e$:

$$P[N_e = k] = \begin{cases} 
p^{k-1} \cdot (1 - p) & k = 1, 2, \ldots, K - 1 \\
p^{K-1} & k = K \\
0 & \text{otherwise}
\end{cases}$$  \hspace{1cm} (13)

- Average feedback load ($\bar{N}_e \triangleq \mu_{N_e}$):

$$\mu_{N_e} = (1 - p^K)/(1 - p)$$  \hspace{1cm} (14)

- Variance:

$$\sigma^2_{N_e} = \frac{p + (p^{K+1} - p^K)(3K^2 + 2K - 1) + (p^{K-1} - p^{K+2})K^2 - p^{2K}}{(1 - p)^2}$$  \hspace{1cm} (15)

- Excess probability:

$$P_e = \text{Prob}[N_e > N_{th}] = \begin{cases} 
p^{N_{th}} & 1 \leq N_{th} \leq K - 1 \\
0 & \text{otherwise}
\end{cases}$$  \hspace{1cm} (16)
**Multiuser access schemes** cont’d

**SWT**

- **PMF of** $N_e$:

  \[
P[N_e = lK + k] = p^{Kl}p^{k-1}(1 - p),
  \]
  
  for $l = 0, 1, \ldots$ and $k = 1, 2, \ldots, K$.

- **Average feedback load**:

  \[
  \mu_{N_e} = 1/(1 - p)
  \]
  
  (18)

- **Variance**:

  \[
  \sigma^2_{N_e} = p/(1 - p)^2
  \]
  
  (19)

- **Excess probabilities**:

  \[
  P_e = \text{Prob}[N_e > N_{th}] = p^{N_{th}}
  \]
  
  (20)

  \[
  P_K = \text{Prob}[N_e < K] = 1 - p^{K-1}
  \]
  
  (21)
A specific feature of the SWT access scheme is that no user is selected if none of the $K$ users succeeded to be above threshold.

It is of interest to know some statistics of the number of coherence times $N_c$ the BS has to wait before an acceptable user is found. The number $N_c$ will be a discrete RV with PMF:

$$P[N_c = t] = p^K t (1 - p^K),$$

for $t = 0, 1, \ldots$.

Average waiting time and variance:

$$\mu_{N_c} = p^K / (1 - p^K)$$
$$\sigma^2_{N_c} = p^K / (1 - p^K)^2$$

Dropping probability

$$P_d = \text{Prob}[N_c > N_{th}] = p^K (1 + N_{th})$$
• In an attempt to reduce the average feedback load, a linear constraint is introduced such that the average feedback load do not exceed a certain percentage of the number of users $\mu_{N_e} \leq \alpha K \ (0 < \alpha \leq 1)$.

• Using this constraint, $\gamma_T$ must be chosen such that
  \[ p^K - \alpha K p + \alpha K - 1 \geq 0 \] (SET and SETps)
  \[ \gamma_T \leq \bar{\gamma} \ln(\alpha K) \] (SWT)

• In practice, these equations effectively reduce the acceptable range for $\gamma_T$ by introducing an upper bound.

• An optimal $\gamma_T$ is then obtained by finding the threshold that maximizes the ASE when $\gamma_T$ is selected from the reduced set of acceptable solutions.
Maximum ASE of the SCT, SET, SETps and SWT access schemes when the multiuser system is operating on i.i.d. Rayleigh fading channels with average SNR \( \bar{\gamma} = [5, 10, 15, 20, 25] \) dB.
AFL for unconstrained optimization of the SET and SETps access schemes. Selected linear upper bounds for the AFL are depicted in order to visualize in which user regions the SET scheme will be affected. The multiuser system is operating on i.i.d. Rayleigh fading channels with average SNR $\bar{\gamma} = 15\text{dB}$. 
ASE of the SETps access scheme when the AFL is upper bounded by $\text{AFL} < \alpha K$. The multiuser system is assumed operating on i.i.d. Rayleigh fading channels with average $\text{SNR} \overline{\gamma} = 15\text{dB}$.
Constraint function $F(\alpha, K, p) = p^K - \alpha K p + \alpha K - 1 \geq 0$ as a function of $\gamma_T$ for the SETps access scheme when $K = [30, 40]$ and $\alpha = 0.3$. Regular solid line: $K = 30$, Bold solid line: $K = 40$. The multiuser system is assumed operating on i.i.d. Rayleigh fading channels with average SNR $\bar{\gamma} = [5, 10, 15, 20, 25, 30]$dB.
Numerical results cont’d

Acceptable range for $\gamma_T$ as a function of $\gamma$ for the SETps access scheme when $K = [30, 40]$ and $\alpha = [0.3, 1]$. 
ASE of the SET access scheme when the AFL is upper bounded by $\text{AFL} \leq \alpha K$. The multiuser system is assumed operating on i.i.d. Rayleigh fading channels with average $\text{SNR} \bar{\gamma} = 15\text{dB}$. 
ASE of the SWT access scheme when the AFL is upper bounded by AFL $< \alpha K$. The multiuser system is assumed operating on i.i.d. Rayleigh fading channels with average SNR $\gamma = 15$dB.
Excess probability $P_e = \text{Prob}[N_e > \mu_N + \sigma_{N_e}]$ for the SET, SETps and SWT access schemes when AFL $\leq 0.3K$ and the multiuser system is operating on i.i.d. Rayleigh fading channels with average SNR $\bar{\gamma} = 15\text{dB}$. 
Excess probability $P_K = \text{Prob}[N_e < K]$ for the SET, SETps and SWT access schemes when $\text{AFL} \leq 0.3K$ and the multiuser system is operating on i.i.d. Rayleigh fading channels with average SNR $\overline{\gamma} = 15\text{dB}$. 
Average waiting time (AWT), standard deviation and excess probability 

\[ P_e = \text{Prob}[N_c > \mu_{N_c} + \sigma_{N_c}] \]

for the SWT access scheme when AFL \( \leq 0.3K \) and the multiuser system is operating on i.i.d. Rayleigh fading channels with average SNR \( \overline{\gamma} = 15\text{dB} \).
• A set of switched multiuser access schemes have been proposed for systems operating in a TDM mode.

• The new access schemes are aimed to reduce the average feedback load in multiuser systems relying on feedback from the users to maximize the ASE.

• Numerical results quantifying the trade-off between ASE and average feedback load have been presented, showing that the average feedback load can be reduced significantly compared to the optimal selective diversity scheme without experiencing a big performance loss in ASE.

• The proposed access schemes are quite attractive also from a fairness perspective.