ABSTRACT

This paper presents the evaluation of the multi-hop aggregate information efficiency of the slotted and unslotted ALOHA protocols. We consider a multi-hop wireless network where the nodes are spatially characterized by a Poisson point process and the traffic generation also follows a Poisson distribution. By applying the properties of stochastic geometry, we derive a closed-form lower bound on the outage probability as a function of the required communication rate, the single-hop distance, the number of hops and the maximum number of retransmissions. The results indicate that slotted ALOHA always outperforms its unslotted version, demonstrating the importance of synchronization in distributed networks. In addition, we show that it is always possible to optimize the network efficiency by properly setting the required rate for a given packet density. Finally, in the scenario considered, the use of retransmissions and multiple hops never achieves the best performance if compared to the option of single-hop links without retransmissions.

1. INTRODUCTION

The efficiency of multi-hop networks depends on several factors, such as routing strategy, traffic conditions of the nodes and medium access control (MAC) protocol. In fact, these aspects are intrinsically related to the interference level of the network and, consequently, to the quality of the link.

In a recent group of papers [1–3], the authors analyze the performance of the ALOHA and CSMA MAC protocols in ad hoc networks. Their study considers the outage probability, i.e. the probability that a receiver node is not able to decode the transmitted packet correctly. Based on this performance figure, their results show that it is possible to optimize the system via the design of the MAC protocol.

In [4], Weber et al proposed the transmission capacity, defined as the product of density of successful active links and their communication rate, to evaluate the spatial spectral efficiency of ad hoc networks. By using this metric, it is possible to evaluate the performance of the wireless network as a whole, in which the outage probability is used to characterize the successful links.

Nevertheless, all aforementioned works simply consider single-hop systems and thus only give insights on multi-hop scenarios. So as to capture the details of multi-hop networks, in [5], the authors proposed the multi-hop aggregate information efficiency to evaluate the efficiency of spatial progress of the information bits progress in time, frequency and area domains, considering transmissions over multiple hops.

The goal of this paper is to apply such metric, defined in Section 2, to evaluate the performance of the slotted and unslotted versions of the ALOHA access protocol. Our study will be based on the network modeling detailed in Section 3 and an analytical lower bound on the outage probability of ALOHA networks is derived in Section 4. The numerical results of this paper are given in Section 5, followed by the conclusions and future directions in Section 6.

2. MULTI-HOP AGGREGATE INFORMATION EFFICIENCY

The multi-hop aggregate information efficiency (MIEA) was proposed in [5] to evaluate the efficiency of the spatial progress of the information bits in the time, frequency and area domains. The mathematical definition of MIEA, measured in [(bits·m)/(s·Hz·m²)], is given by

$$\text{MIE}_A = \frac{d_{mh} \times \eta_{mh} \times \lambda_{suc}}{d_{sh}}, \quad (1)$$

where $d_{mh}$ is the distance between the source and the destination nodes (or multi-hop distance), $\eta_{mh}$ is the multi-hop spectral efficiency and $\lambda_{suc}$ is the density of successful transmissions.

Let us assume that the multi-hop distance $d_{mh}$ is fixed and that we have a perfect routing protocol (i.e. the packets travel in a straight line from their sources to their destinations). Therefore, the number of hops $n_h$ necessary for the packets to finish successfully their transmission can be computed as the ratio between the multi-hop and the single hop distances, which yields

$$n_h = \frac{d_{mh}}{d_{sh}}, \quad (2)$$

where $d_{sh}$ is the distance between the transmitter and the receiver (or single-hop distance).
Intuitively, when an $n_h$-hop link is considered, a packet requires $n_h$ times as many channel uses as a single-hop link to reach its destination. Then, the overall spectral efficiency $\eta_{nh}$ of a multi-hop link can be computed as

$$\eta_{nh} = \frac{\eta_{sh}}{n_h}, \quad (3)$$

where $\eta_{sh}$ is the single-hop spectral efficiency.

Let $\lambda$ and $P_{sh}$ be the density of packets generated in a given period of time and the probability that a packet is not successfully received in a single-hop link after all allowed retransmissions, respectively. Assuming independence between the hops\(^1\), the density of successful multi-hop links $\lambda_{muc}$ can be evaluated as

$$\lambda_{muc} = \lambda \times (1 - P_{sh})^{n_h}. \quad (4)$$

Now, inserting the equations (2), (3) and (4) into (1), the MIE\(_A\) formulation can be rewritten as

$$\text{MIE}_A = d_{sh} \times \eta_{sh} \times \lambda \times (1 - P_{sh})^{n_h}. \quad (5)$$

In the following section, we will characterize the parameters used to evaluate MIE\(_A\) together with the definition of the network modeling.

### 3. NETWORK MODELING

Let $\lambda_\text{a}$ [nodes/m\(^2\)] be the density of transmitter nodes randomly distributed over the network area according to a homogeneous Poisson point process (PPP) \(^7\) and $\lambda_{pkt}$ [packets/s/node] be the intensity of the packet arrival process which respects an 1-dimensional Poisson process. We assume that all packets have a fixed duration of $T$ seconds and they are transmitted with a fixed power $\rho$ to the intended receiver, which lies a fixed distance $d_{sh}$ meters away.

Applying the same approach described in \([1–3]\), it is possible to compute the spatial density $\lambda$ [packets/m\(^2\)] of the packets that have been generated in a given period of time $T$ as follows:

$$\lambda = \lambda_\text{a} \times \lambda_{pkt} \times T. \quad (6)$$

Now assuming that the packets are transmitted with a required rate $\eta$ to perform the communication\(^2\), the thermal noise is neglected, the fading effects are ignored and the aggregate interference is Gaussian-distributed\(^3\), the signal-to-interference ratio (SIR) threshold $\beta$ required for a packet to be received successfully can be obtained by applying the Shannon’s capacity formula \([9]\), which yields

$$\beta = 2^n - 1. \quad (7)$$

\(^1\)This assumption is coherent when a cooperative transmission scheduling is used, see \([6]\).

\(^2\)Notice that the rates $\eta$ and $\eta_{sh}$ are different. Their mathematical relation will be stated in the sequel of this section.

\(^3\)It is an accepted assumption in interference-limited networks \([4, 8]\).

In other words, any receiver node that experiences an SIR lower than or equal to the threshold $\beta$ will be in outage. Assuming the distance-dependent path-loss model \([10]\) with exponent $\alpha$, we can then define the probability $P_{\text{out}}$ that a packet is not correctly received during a given attempt as

$$P_{\text{out}} = \Pr \left\{ \text{SIR} \leq \frac{d_{sh}^{-\alpha}}{\sum_{n \in I} d_{i,n}^{-\alpha}} \leq \beta \right\}, \quad (8)$$

where $I$ is the random set of interferers on the network area and the $d_{i,n}$ is the distance between the $n$-th interferer and the reference receptor.

We consider that each packet has a maximum of $m$ retransmission attempts in each hop. Since the objective of this work is not to evaluate the back-off scheme, we simply assume that the back-off times are random, uncorrelated, and exponentially distributed. If the packet is received erroneously after $m$ retransmissions, it will be discarded. Assuming independence between the transmissions, the probability $P_{sh}$ that a packet is lost in a single-hop link can be computed as

$$P_{sh} = P_{\text{out}}^{m+1}. \quad (9)$$

In any case, the retransmissions allow the single-hop links to access the network up to $m + 1$ times, resulting in diminution of the link spectral efficiency. Mathematically, we can compute the single-hop rate $\eta_{sh}$ as the required rate $\eta$ divided by the average number of transmission attempts $\bar{q}$, i.e.

$$\eta_{sh} = \frac{\eta}{\bar{q}}. \quad (10)$$

where $\bar{q}$ is evaluated as

$$\bar{q} = 1 + P_{\text{out}} + P_{\text{out}}^2 + \ldots + P_{\text{out}}^m = \frac{1 - P_{\text{out}}^{m+1}}{1 - P_{\text{out}}}. \quad (11)$$

In the following section, we will present an analytical closed-form lower bound on the outage probability $P_{\text{out}}$ for the slotted and unslotted ALOHA medium access protocols as a function of network topology, the maximum number of retransmissions and the system parameters presented in this section.

### 4. OUTAGE PROBABILITY OF ALOHA

From the concept of the ALOHA protocol, all packets are transmitted regardless of the network condition. The communication between the transmitter and its receiver is assumed to occur over an orthogonal control channel, meaning that there are no interference issues between the control signals and the data packets. Furthermore, the delay introduced by the feedback is assumed to be insignificant compared to the packet length.
4.1. Slotted ALOHA

In slotted ALOHA, transmitters can only start their transmissions at the beginning of the next time slot after each packet has been formed.

**Theorem 1** The probability $P_{\text{out}}$ that a packet is erroneously received in a given transmission attempt can be lower-bounded by the solution of the following equation:

$$P_{\text{out}} = 1 - \exp \left( -n_h \frac{1 - P_{\text{out}}^{m+1}}{1 - P_{\text{out}}} \lambda \pi d_{sh}^2 \beta^{2/\alpha} \right).$$  \hspace{0.5cm} (12)

**Outline of Proof:** To derive a lower bound $P_{\text{out}}$ of the slotted ALOHA system, we apply the procedure introduced in [4]. Define $s$ to be the distance between the receiver under observation, $RX_0$, and its closest interfering transmitter that causes the SIR to fall lower or equal to the threshold $\beta$. Considering the noise power negligible, the distance $s$ is derived to be:

$$s = \left( \frac{d_{sh}^{-\alpha}}{\beta} \right)^{1/\alpha} = d_{sh}^{1/\alpha}. \hspace{0.5cm} (13)$$

Consider a circle of radius $s$ around $RX_0$, and denote this by $B(RX_0,s)$. There are two events that can cause error in the received packet of $RX_0$: (1) If the accumulation of powers from all the interferers outside $B(RX_0,s)$ results in the SIR at $RX_0$ to fall below the threshold $\beta$, and (2) if at least one active transmitter, other than $RX_0$’s own transmitter $TX_0$, falls inside $B(RX_0,s)$ at any time during the packet transmission (i.e. $d_{iT} \leq s$). The latter event yields a lower bound to the probability $P_{\text{out}}$ [4], which is shown to be tight at low densities and losing its tightness for higher values of $P_{\text{out}}$.

Now, consider the communication link of $TX_0$-$RX_0$. Due to the slotted time, only packets arriving during the last $T$ seconds start simultaneously with the one generated by $TX_0$, and have thus the potential to result in an erroneous packet reception at $RX_0$. So the expected number of interferences for $RX_0$ is given by:

$$\mathbb{E} \{ \text{# of interferers inside } B(RX_0,s) \text{ at some } t \in (-T,0] \} = \lambda_1 \pi s^2 = \lambda_1 \pi d_{sh}^2 \beta^{2/\alpha}, \hspace{0.5cm} (14)$$

where $\lambda_1$ is the density of active packets on the plane.

Allowing for retransmissions and multiple hops increase the number of packets that attempt to access the channel. Knowing that the average number of access attempts that a packet has is $\bar{q}$ and the number of hops is $n_h$, we have that the density of active packets in the channel at each time instant is

$$\lambda_3 = n_h \times \bar{q} \times \lambda. \hspace{0.5cm} (15)$$

Finally, the probability $P_{\text{out}}$ that a packet is erroneously received during a given attempt can be lower bounded in Poisson distributed networks as the solution of equation $P_{\text{out}} = 1 - e^{-\mathbb{E} \{ \text{# of interferers} \}}$, which is equation (12).

4.2. Unslotted ALOHA

In unslotted ALOHA, packets are transmitted as soon as they are formed. Continuous-time protocols are particularly of interest at systems that have no synchronization abilities.

**Theorem 2** The probability $P_{\text{out}}$ that a packet is erroneously received in a given transmission attempt can be lower-bounded by the solution of the following equation:

$$P_{\text{out}} = 1 - \exp \left( -2 n_h \frac{1 - P_{\text{out}}^{m+1}}{1 - P_{\text{out}}} \lambda \pi d_{sh}^2 \beta^{2/\alpha} \right). \hspace{0.5cm} (16)$$

**Outline of Proof:** This proof is similar to that of Theorem 1, with the difference that we now have to consider packet arrivals during $[-T,T]$, in order to also account for the partial overlap of packets. This results in the factor 2 in the exponent of the $\exp(\cdot)$-expression.

5. NUMERICAL RESULTS

In this section, we evaluate the multi-hop aggregate information efficiency of ALOHA wireless networks, presented in equation (5). To perform the numerical results, we consider a multi-hop distance $d_{sh} = 1$ meter, a packet duration $T = 10$ seconds and a path-loss exponent $\alpha = 4$.

Firstly, we present in Fig. 1 a comparison between the MIEA analytical upper bounds values and its actual value obtained via Monte-Carlo simulation. One can notice that the upper bound can be a good approximation to the actual MIEA for low density of packets. On the other hand, the upper bound becomes looser as the density increases. This is explained by the tightness of the lower bound on the error probability $P_{\text{out}}$, given by (12) and (16); by construction, this lower bound does not consider the events where the aggregate power of the interferers can cause an error event. When the density of the active packets increases, these events become more frequent and then the $P_{\text{out}}$ lower bound gives a poor result, explaining the MIEA upper bound behavior. Nevertheless, such bounds work reasonably in the cases of interest (lower outage probabilities).

In addition, Fig. 1 reveals that there is a value of $\lambda$ that maximizes the MIEA. Increasing the packet density, there will be more information transmitted in the network, which increases the MIEA. However, the more active packets, the higher the interference power. Therefore, the outage probability also increases, which degrades the network performance. This optimal value of MIEA reflects the best trade-off between these aspects.

Fig. 1 also shows that the slotted ALOHA always outperforms its asynchronous option, namely unslotted ALOHA. This fact indicates the advantage to have a synchronous transmissions in distributed networks.
Multi-hop aggregate information efficiency versus the packet density $\lambda$, considering $m = 0$, $\eta = 1 \text{ bit/s-Hz}$, $\alpha = 4$, $d_{sh} = 1 \text{ m}$ and $d_{inh} = 1 \text{ m}$ for slotted and unslotted ALOHA. Analytical upper bound and simulation results.

Moreover, Fig. 2 also shows that the best operational point of $\eta$ depends on the density of packets $\lambda$. Increasing the density $\lambda$, the rate $\eta$ that maximizes MIE$_A$ will be lower. This fact indicates that, in networks with higher interference levels (higher $\lambda$), the rate $\eta$ required to perform the transmission must be lower to guarantee a reliable communication. In any case, this result points out the dependence of the activity and the interference robustness to design an efficient network.\footnote{The results of unslotted ALOHA have a similar behavior, although with lower MIE$_A$.}

Now we study the effects of retransmission on the network performance. On one side, the higher the number of retransmissions, the lower the outage probability. On the other side, increasing the number of retransmissions, the traffic of active packets in the network will be higher and the spectral efficiency of the links will be lower. Fig. 3 shows that no allowed retransmissions maximizes the MIE$_A$, reflecting that their negative side has the dominant effect on the network efficiency. Fig. 4 presents the multi-hop aggregate information efficiency as a function of the number of hops $n_h$. Considering a fixed multi-hop distance $d_{inh}$, we vary the fixed single-hop distance $d_{sh}$ so as to control the number of hops. If small $d_{sh}$ are considered, the multi-hop link will be composed by several more robust single-hop links (distance-dependent path-loss). However, as the number of hops increases, the activity of the network also increases. Our results shows that the degradation of the network performance due to the higher activity is the preponderant factor in the multi-hop aggregate information efficiency of the network regardless of the packet density $\lambda$.\footnote{The results of unslotted ALOHA have a similar behavior, although with lower MIE$_A$.}
Fig. 4. Multi-hop aggregate information efficiency of slotted ALOHA (upper bound) versus the number of hops $n_h$ considering $m = 0$, $\eta = 1$ bit/s Hz, $\alpha = 4$, $d_{mh} = 1$ m for several packet densities $\lambda$.

6. FINAL REMARKS

In this contribution we evaluate the multi-hop aggregate information efficiency of the slotted and unslotted ALOHA medium access protocols. Based on a Poisson distributed network with Poisson packet arrivals, we characterize a lower bound on the probability that a packet is not successfully received as a function of network topology (density of active nodes, single-hop distance and number of hops) and the system parameters (required rate and maximum number of retransmissions). Our results show that:

(i) the slotted ALOHA outperforms the unslotted one (by approximately a factor of 2 at their peaks);

(ii) there exists a given rate $\eta$ that optimize the $MIE_A$ for each density of active packets;

(iii) 0 allowed retransmissions yields the highest $MIE_A$.

(iv) single-hop transmissions outperform the multi-hop option in terms of $MIE_A$.

These results indicate that unslotted ALOHA is not the right choice to perform asynchronous transmissions in multi-hop wireless networks. For this reason, we plan to evaluate the $MIE_A$ of the CSMA/CA protocol as future work. Furthermore, we also intend to study the optimal operation points of multi-hop wireless networks, in terms of $MIE_A$, subject to certain quality of service requirements.

7. REFERENCES


