Evaluating the Information Efficiency of Multi-Hop Networks with Carrier Sensing Capability

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Abstract—In this contribution, we consider the performance of the CSMA MAC protocol in multi-hop ad hoc networks in terms of “aggregate multi-hop information efficiency”. Our model consists of a wireless network where transmitter nodes are distributed according to a homogeneous 2-D Poisson point process, and packets are generated following a Poisson distribution. Each packet is forwarded to its destination a fixed distance away from the source through an arbitrary number of hops. Approximate analytical expressions are derived for the outage probability of CSMA in its various incarnations, considering different values for the sensing threshold (related to the backoff decision) and the required communication threshold (which determines a correct packet reception). The aggregate multi-hop information efficiency is evaluated as a function of the transmission density, the communication rate, the maximum number of permitted backoffs and retransmissions, and the number of hops. Our results indicate the existence of optimal operating points for achieving maximal efficiency, and a basis is thus established for the optimization of the system parameters in order to improve the performance of multi-hop ad hoc networks.

I. INTRODUCTION

Allowing for carrier sensing (CS) capability in wireless networks has been shown to bring about considerable performance gain. This is because, based on the information obtained about the channel condition, precautions measures may be taken in order to manage the destructive impact of interference caused by other simultaneous transmissions in the network. CS is particularly of interest in ad hoc networks, as they are self-configuring and can be rapidly formed and deployed from whatever nodes available, making the channel condition more unpredictable [1]. The medium access control (MAC) protocol Carrier Sensing Multiple Access (CSMA) is one of the most popular algorithms applied in today’s networks for efficiently managing the issue of interference.

In this study, we consider a multi-hop ad hoc network that employs the CSMA protocol, and we investigate the system performance in terms of aggregate multi-hop information efficiency (AMIE), introduced in [2], [3] to capture the impact of the progress of information in space, time, and frequency, considering among others, the quality of the links and the number of hops between a source and its destination. AMIE is closely related to the well-known transmission capacity (TmC) [4]–[6], which is defined as the product of the density of successful active links and their communication rate. TmC was proposed by Weber et al. in [4] to quantify the spatial spectral efficiency of single-hop ad hoc networks.

In all the above-mentioned works, however, only slotted ALOHA has been assumed. This brings us to the natural extension to consider the CSMA protocol. In the recent work of [7], the authors analyze the performance of the ALOHA and CSMA MAC protocols in single-hop ad hoc networks in terms of outage probability (OP). OP is defined as the probability that a packet is not able to reach its destination either because it is dropped after a given number of backoffs or it is received in error and cannot be decoded correctly after a given number of retransmission attempts. The system model considered in [7] resembles our model, with the difference that we allow for multi-hop communication. As a consequence, the performance metric used in our work has been revised to track changes in more system parameters than the ones considered in the aforementioned works.

This work serves as an extension to prior analysis performed on the performance of the ALOHA and CSMA protocols. The novelty of this work lies in two aspects: 1) The model is extended to involve multi-hop networks, allowing for arbitrary number of hops, and 2) the metric used for analysis, namely AMIE, is a more generalized version of other metrics, taking into account traffic conditions, quality of service requirements, and the routing strategy of the multi-hop environment.

The remainder of this manuscript is organized as follows: In Section II, we present the network model used for the system analysis. Section III introduces and explains the AMIE performance metric. In Section IV, we explain the technique used for analysis and we derive approximate expressions for the OP of CSMA. The numerical results and related discussions are provided in Section V, while Section VI concludes this paper.

II. NETWORK MODEL

Our system model considers an ad hoc network where transmitter (TX) nodes are located on an infinite 2-dimensional plane according to a homogeneous Poisson point process (PPP) [8] with spatial density $\lambda_s$ [nodes/m²]. Each TX has packets of constant length $T$ arriving in time according to an independent 1-dimensional PPP with density $\lambda_{pkt}$ [packets/sec/node]. Applying the same approach described in [7], we find the spatial density $\lambda$ [packets/m²] of generated packets in a given time period $T$ to be

$$\lambda = \lambda_s \times \lambda_{pkt} \times T. \quad (1)$$
Upon the formation of each packet, it is transmitted with a constant power $p$ to its receiver (RX) in the next hop, which is assumed to be positioned a distance $d_{sh}$ away. The multi-hop distance, i.e., the distance from each source to its final destination, is denoted by $d_{mh}$. We assume that $d_{mh}$ is fixed and that we have a perfect routing protocol (i.e., packets travel on a straight line from their sources to their destinations). Each packet goes through $h$ hops to reach its destination;

$$h = \frac{d_{mh}}{d_{sh}} \quad (2)$$

For the channel model, only path loss attenuation effects (with exponent $\alpha > 2$) are considered, i.e., channel effects such as shadowing and fast fading are ignored. Each RX potentially sees interference from all active TXs, and these independent interference powers are added to the channel noise power $\sigma^2$, resulting in a signal to interference plus noise ratio (SINR) of:

$$\text{SINR} = \frac{\rho G_{d_{sh}} d_{sh}^{-\alpha}}{\sigma^2 + \sum_{i \in I} \rho G_{d_{i}} d_{i}^{-\alpha}} = \frac{\rho d_{sh}^{-\alpha}}{\sigma^2 + \sum_{i \in I} \rho d_{i}^{-\alpha}} \quad (3)$$

where $I$ is the random set of interferers on the network area, $d_i$ is the distance between the $i$-th interferer and the reference receptor, and $G_{d}$ is a constant gain at a unit distance away.

Let us assume that the packets are transmitted with a required rate $R$ over a fixed bandwidth $W$. So the spectral efficiency related to $R$ and $W$, hereafter dubbed required spectral efficiency and denoted by $\eta$, can be obtained as $R/W$.

Now, neglecting the thermal noise and considering the power of the aggregate interference as a Gaussian process [6], the signal-to-interference ratio (SIR) threshold required for a packet to be received successfully at each hop, $\beta_{req}$, can be obtained by applying Shannon’s capacity formula, which yields

$$\beta_{req} = 2^\eta - 1 \quad (4)$$

If the experienced SIR at the receiver node of each single hop falls below $\beta_{req}$ at any time during a packet transmission, the packet is then received in error. The probability of this is denoted by $P_{\text{error}}$:

$$P_{\text{error}} = \Pr \left\{ \text{SIR} = \frac{d_{sh}^{-\alpha}}{\sum_{i \in I} d_{i}^{-\alpha}} \leq \beta_{req} \right\} \quad (5)$$

The packet transmissions occur according to the carrier sensing multiple access (CSMA) protocol. In this protocol, the decision-making node (which is the TX in CSMA$_{TX}$ and the RX in CSMA$_X$) senses the channel prior to transmission. If the measured SIR is above the sensing threshold $\beta_{sens}$, the packet transmission is initiated; otherwise, it is backed off. After $M$ backoffs, the packet is dropped. Since the objective of this work is not to evaluate the backoff scheme, but rather to analyze qualitatively what its impact is on the OP, we simply assume that the backoff times are random, uncorrelated, and exponentially distributed.

Once the transmission is initiated, but the packet is received in error, it is dropped and counted to be in outage, contributing to the total OP, $P_{\text{out}}$. The communication between each TX and its RX is assumed to occur over an orthogonal control channel, and the delay introduced by the feedback is presumed to be insignificant compared to the packet length.

### III. Aggregate Multi-hop Information Efficiency

Aggregate multi-hop information efficiency (AMIE) was proposed in [2], [3] and is our metric for evaluating the efficiency of packet progress in space, time, and frequency. The mathematical definition of AMIE, denoted here by $\epsilon$ and measured in units ([bit-(s/m)/(s-Hz-m$^2$)]), is given by

$$\epsilon = \frac{\eta}{h}$$

where $\eta$ is the multi-hop spectral efficiency, and $\eta_{suc}$ is the density of successful transmissions.

Now, considering an $h$-hop link, a packet requires $h$ times as many channel uses to reach its destinations as compared to a single-hop link. This means that the overall spectral efficiency $\eta_{mh}$ of a multi-hop link can be computed as

$$\eta_{mh} = \frac{\eta}{h} \quad (7)$$

Moreover, assuming independence between the hops, the density of successful transmissions $\lambda_{suc}$ can be evaluated as

$$\lambda_{suc} = \lambda \times (1 - P_{\text{out}})^h \quad (8)$$

Inserting Eqs. (2), (7), and (8) into Eq. (6), the AMIE formulation may be rewritten as

$$\epsilon = d_{sh} \times \eta \times \lambda \times (1 - P_{\text{out}})^h \quad (9)$$

As the only unknown parameter in our AMIE metric is $P_{\text{out}}$, the following section is devoted to explaining the technique used for the OP derivation, and the results obtained in [7].

### IV. Outage Probability

In the CSMA protocol, nodes perform channel sensing prior to each transmission$^1$. If the measured or estimated received SIR at the start of the packet is greater than $\beta_{sens}$, the transmission is initiated immediately. Otherwise, with a probability of $P_b$, the packet is backed off a random time, at the end of which a new channel sensing is performed and a new decision is made. If the packet transmission is initiated, it has a probability $P_{\text{during}}$ of being received in error and thus counted to be in outage.

Based on this, the density of packets attempting to access the channel, given each link consist of $h$ hops, is given by

$$\lambda_{csma} = h\lambda \times \sum_{m=0}^{M-1} P_{b}^m = h\lambda \times \frac{1 - P_{b}^M}{1 - P_{b}} \quad (10)$$

The total OP of CSMA protocol may be expressed as

$$P_{\text{out}}(\text{CSMA}) = P_{b}^M + (1 - P_{b}^M) \left[ P_{\text{rx}}(\text{active}) + (1 - P_{\text{rx}}(\text{active}))P_{\text{during}} \right] \quad (11)$$

$^1$Channel sensing is not performed in retransmission attempts.
where $P_{bt|active}$ is the probability that the RX is in outage at the start of the packet once it is activated.

In order to derive the OP, we apply the concept of guard zones [9]. Define $s_{req}$ to be the distance between the RX under observation, $RX_0$, and its closest interfering TX that causes the SIR to fall just below the threshold $\beta_{req}$. By manipulation of the SIR expression, $s_{req}$ is derived to be:

$$s_{req} = d_{bh} \times \frac{1}{\alpha}.$$  \hspace{1cm} (12)

Now, consider a circle of radius $s_{req}$ around $RX_0$, and denote this by $B(RX_0, s_{req})$. There are two events that can cause error in the received packet of $RX_0$: (1) If the accumulation of powers from all the interferers outside $B(RX_0, s_{req})$ results in the SIR at $RX_0$ to fall below the threshold $\beta_{req}$, and (2) if at least one active TX, other than $RX_0$’s own TX, $TX_0$, falls inside $B(RX_0, s_{req})$ at any time during the packet transmission. The latter event yields a lower bound to the OP, and is shown in [4] to be asymptotically tight. Hence, we only focus on this bound in our analysis.

In the following, we will present the OP for the CSMA$_{TX}$ and CSMA$_{RX}$ protocols, considering different values of $\beta_{sens}$ and $\beta_{req}$ (translating to $s_{sens}$ and $s_{req}$ through Eq. (12)). For the sake of readability of the following formulas, we denote $s_{sens}$ by $s$. Due to the space constraint, we do not include the proofs of the various expressions in the following theorems, as these are presented in [7].

### A. CSMA with Transmitter Sensing

In CSMA$_{TX}$, the TX senses its channel upon arrival, estimates the SIR at the RX based on its own measured interference, and thereby makes the backoff decision. CSMA$_{TX}$ is the conventional CSMA protocol, and its OP is stated in the following theorem.

**Theorem 1:** The AMIE of CSMA$_{TX}$ is given by Eq. (9), where $P_{out}$ is given by Eq. (11), with:

- $P_{b} \approx \tilde{P}_{b}$ is given as the (numerical) solution to $\tilde{P}_{b} = 1 - e^{-\lambda h (1 - \tilde{P}^M_{b}) \pi r^2}$. \hspace{1cm} (13)
- $P_{\text{during}} \approx \tilde{P}_{\text{during}}$ is the approximate probability that the activated packet using CSMA$_{TX}$ is received erroneously and it is given by Eq. (14), presented on next page.
- $P_{\text{tx|active}} \approx \tilde{P}_{\text{tx|active}}$ is the approximate probability that the RX is in outage at the start of the packet given it has been activated. This is given by Eq. (15) for $s \leq s_{req} + d_{bh}$ (presented on next page). Otherwise, $\tilde{P}_{\text{tx|active}} = 0$. In addition, $P_{tx} = 1 - e^{-\lambda h (1 - \tilde{P}^M_{b}) \pi r^2_{\text{req}}}$. \hspace{1cm} (17)

### B. CSMA with Receiver Sensing

CSMA$_{RX}$ is the modified version of the conventional CSMA protocol, and was proposed in [7]. In this protocol, the RX senses the channel at the start of each packet and informs its TX over an orthogonal control channel whether or not to back off. The simple feedback channel added in CSMA$_{RX}$ was shown to provide considerable performance gain.

**Theorem 2:** The AMIE of CSMA$_{RX}$ is given by Eq. (9), where $P_{out}$ is given by Eq. (11), with:
- $P_{b} \approx \tilde{P}_{b}$ is given by Eq. (13); $\tilde{P}_{x}$ is given in Theorem 1;
- $P_{\text{during}} \approx \tilde{P}_{\text{during}}$ is the approximate probability that an activated packet is received erroneously, given by Eq. (16) presented on next page. $\lambda_{\text{csma}}$ is given by Eq. (10), and

$$P(\text{active}|d, \phi) = 1 - \frac{1}{\pi} \cos^{-1} \left( \frac{r^2 + 2d_{bh}^2 - s^2 - 2d_{bh}r \cos \phi}{2d_{bh} \sqrt{r^2 + d_{bh}^2 - 2d_{bh}r \cos \phi}} \right). \hspace{1cm} (14)$$

- $P_{\text{tx|active}} \approx \tilde{P}_{\text{tx|active}}$ is the approximate probability that the RX is in outage at the start of the packet once the packet transmission has been activated;

$$\tilde{P}_{\text{rx|active}} = \begin{cases} P_{\text{tx}} \left[ 1 - \frac{2}{s_{\text{req}}} \right], & s < s_{\text{req}} \\ 0, & \text{otherwise} \end{cases}$$  \hspace{1cm} (17)

### V. Numerical Results

This section addresses the numerical evaluation of the AMIE for the CSMA$_{RX}$ and CSMA$_{TX}$ protocols. The results are obtained considering $d_{mn} = 1$ (normalized distance), $T = 10$ s, and $\alpha = 4$.

Firstly, Fig. 1 illustrates how the AMIE behaves as a function of the packet density $\lambda$. Clearly, there exists a density that maximizes the efficiency of the network, independently of $h$. By increasing $\lambda$, more packets are active in the network and, then, more information can be transmitters, which increases AMIE. On the other hand, the higher the $\lambda$, the higher the probability that an outage event occurs. In fact, when a high packet density is considered, the backoff probability increases and, then, more packets will be dropped due to the limited number of access attempts. In the same way, the initialized packets will suffer with the greater number of active links throughout the network and backed off links which may access the network and cause an error event.

Fig. 1 also reveals that the CSMA$_{RX}$ outperforms its TX sensing version in the interesting regions. Since the collisions occur at the RX node, the existence of a silent zone around the RX is more effective than TX sensing to guarantee a successful communication. This explains the higher $e$-value for CSMA$_{RX}$.

Finally, regarding the number of hops, Fig. 1 shows that the lower the number of hops, the higher the AMIE. The discussion of the effect of $h$ will be provided later in this section.

Fig. 2 presents the AMIE as a function of the required spectral efficiency $\eta$ for the CSMA$_{TX}$ and CSMA$_{RX}$ protocols. The required spectral efficiency $\eta$ has a two-fold effect on the network performance. On one hand, when a high $\eta$ is required, the AMIE directly increases. On the other hand, a high $\eta$ corresponds to a large SIR threshold $\beta_{req}$, which leads
evinces that the backoff mechanism at RX is more effective and thus a lower AMIE. The results reveal that there exists a spectral efficiency $\eta$ that maximizes $\epsilon$, thus representing the best trade-off between interference robustness and the link spectral efficiency.

Interestingly, Fig. 2 reveals that, for lower requirement of $\eta$, CSMA\textsubscript{RX} outperforms CSMA\textsubscript{TX}, while the opposity is noticed for higher $\eta$. This is explained by the contention characteristic of both protocols. In the former, a higher AMIE evinces that the backoff mechanism at RX is more effective to avoid destructive interference considering a lower threshold $\beta_{\text{req}}$ (lower $\eta$). However, requiring a higher $\eta$, the combined effect between active packet collisions and backoff errors leads to advantages to CSMA\textsubscript{TX}.

To study the effect of the maximum number of backoffs on the network efficiency, we consider the aggregate multi-hop information efficiency versus $M$ in Fig. 3. By increasing $M$, links are given more attempts to get their packets successfully across, thus reducing the OP. However, a higher $M$ also increases the number of active links in the network, which would result in an increase in the OP. Fig. 3 shows that considerable gain can be obtained in this example even by allowing for only one backoff, i.e., for $M = 2$.

Fig. 4 shows the effects of different sensing ranges in the network efficiency. Setting a larger sensing range $s$, the higher the probability of a link be backed off, which increases the outage probability. On the other hand, considering the same large $s$, the links which succeed to access the network will have low reception error probability. So the optimal AMIE reflects the best trade-off between the aforementioned effect. It is also worth to note that after a certain value ($s > 2d_{\text{sh}} + s_{\text{req}}$) the probability that initialized packets are received in error approaches 0.
Fig. 3. AMIE ($\epsilon$) of CSMA$_{TX}$ and CSMA$_{RX}$ versus $M$, considering $s = d_{sh}$, $\eta = 2$, $\lambda = 0.1$ for different number of hops.

Fig. 4. AMIE ($\epsilon$) of CSMA$_{TX}$ and CSMA$_{RX}$ versus $s$, considering $M = 2$, $\eta = 2$, $\lambda = 0.1$ for different number of hops.

Fig. 5 presents the AMIE as a function of the number of hops $h$. We consider a fixed multi-hop distance $d_{mh}$ and vary the single-hop distance $d_{sh}$ in order to control the number of hops $h$. If a small $d_{sh}$ is considered, the multi-hop link will be composed of more robust single-hop links (due to the distance-dependent path loss). However, as $h$ would be higher, the traffic in the network also increases. Our results shows that the degradation in the network performance due to worsened traffic conditions is the preponderant factor in the aggregate multi-hop information efficiency of the network.

VI. CONCLUSIONS

In this paper, we evaluate the AMIE of ad hoc networks using the CSMA MAC protocol. AMIE captures the effect of traffic conditions, the quality of service requirements, and the routing strategy of the multi-hop network. For the network modeling, we assume that transmitter nodes are distributed in space according to a 2-D PPP and that packet are generated in time also following a Poisson process. Each packet travels a fixed distance to reach its destination through an arbitrary number of hops. Based on analytical approximations, the AMIE of the different flavors of CSMA are derived, and validated with Monte Carlo simulations. Our results indicate, among others, that there exist optimal operating points, where the efficiency of the network can be maximized.

As a natural future work, we plan to analytically obtain the best operational points including the possibility of retransmissions, given a set of quality requirements.

REFERENCES


