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Letter

Successive vs. Joint Decoding under Complexity and Performance Constraints*

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Abstract **Helpful guidelines to efficient design of multiuser systems based on successive decoding under a complexity constraint are given. It is shown that in practical systems complexity saved by chosing shorter codes can often be successfully invested in joint-multiuser decoding in order to achieve a better performance with the same overall complexity.**

Keywords **Information theory, Communication theory, Successive decoding**

1. Introduction

The complexity for decoding a multiuser code employed on the asynchronous multiple-access channel is exponential in the product of the number of users K and the code word length ℓ, in general [1]. Under some circumstances, i.e. the users' individual rates form a vertex of the channel's capacity region, information theory allows for methods to break up the multiuser decoding problem into K independent single-user decoding problems [2, 3]. This reduces the complexity from $\mathrm{O}(|\mathcal{A}|^{\ell K})$ to $\mathrm{O}(K \cdot |\mathcal{A}|^{\ell})$ with $\mathcal{A}$ denoting the code's alphabet, i.e. $\mathcal{A} = \{+1, -1\}$ for binary codes.

All such simplifying methods which are known up to now rely on the principle of successive decoding: That user which is decoded currently does not assume any knowledge about the statistics of the signals of the users which are to be decoded subsequently. After being decoded, the current user's signal is reconstructed perfectly and eliminated from the signals the users to be decoded subsequently operate with. This principle has been frequently used in multiuser information theory in order to prove the achievability of the capacity region of a variety of multiple-access channels [2].

While successive decoding does not affect the individual users' channel capacities which are achieved only for infinite code word length, it was already pointed out in [4] using bounds on the error probability of random codes that for finite code word length the situation is different, in general. Thus, multiuser coding should be more powerful than single-user coding combined with successive decoding, in general, but it also involves much larger complexity. Hereby, the natural question arises whether multiuser decoding can also be favourable if both schemes are compared with respect to identical complexity. This question will be answered for communication over the Gaussian multiple-access channel – the multiuser equivalent to the additive white Gaussian noise (AWGN) channel – under some simplifying, but practically accurate assumptions. The presented analysis will also give practical guidelines how to optimally partition a set of users into subsets of users that shall be decoded jointly within the sets and successively among the sets.

2. Successive vs. joint decoding

The information rate to be transmitted arbitrarily reliable over an AWGN channel is well-known [2] to be

$$R \leq \frac{1}{2} \log_2 \left(1 + \frac{P}{N}\right) \tag{1}$$

with P and N denoting the power of the user's signal and the AWGN, respectively. Equality in (1) holds only for infinite code word length. In practice, the allowable rate at a given target error probability $P_\mathrm{e} \ll 1$ will be smaller than channel capacity or corollary the signal power required to support capacity rate will be higher. Following the second approach to describe the point enables to write

$$P(N) = VN\left(4^R - 1\right) \tag{2}$$

where $V > 1$ denotes the power gap between infinite and finite code word length for a given target error probability. For sake of simplicity, the power penalty V is assumed to be identical for all users. Though an approximation, this assumption is well-justified by the results in [5] showing that low-rate codes can be used as constituent codes in multilevel codes in such a way that the power penalty is almost preserved for all rates up to infinity.

Successive decoding means to treat the signals of the users to be decoded subsequently as AWGN although code laws provide statistical dependencies between subsequent symbols of the same user. Neglecting error propagation, the sum power of any pair of users which are neighbouring each other in the successive decoding chain satisfies

$$P_\mathrm{S} = P_1\left(N_\mathrm{S} + P_2(N_\mathrm{S})\right) + P_2(N_\mathrm{S}) \tag{3}$$

where P_1 and P_2 denote the power of the first and the second user's signal, respectively, while N_S denotes the sum

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power of the subsequent users' signals and the AWGN. Denoting the users' corresponding rates with R_1 and R_2, (2) yields

$$P_S = VN_S\left(4^{R_1+R_2}-1\right) + (V-1)VN_S\left(4^{R_1}-1\right)\left(4^{R_2}-1\right). \quad (4)$$

Obviously, the sum power of the two users is not a linear function of the power penalty V.

The previous assumption that the power penalty is independent of the code rate yields for the power penalty of joint multiuser decoding

$$P_M = VN_S\left(4^{R_1+R_2}-1\right). \quad (5)$$

An additional power penalty

$$V_S = \frac{P_S}{P_M} = 1+(V-1)\frac{(4^{R_1}-1)(4^{R_2}-1)}{4^{R_1+R_2}-1} \quad (6)$$

occurs for successive decoding in comparison to joint multiuser decoding, if $V > 1$. Note that in a successive cancellation chain, the users do not only suffer from their own power penalty, but also suffer from *additional* interference due to the power penalties of the interfering users in extend to the normal multiuser interference which is present when successive decoding is applied.

While the sum power for successive decoding in (4) is certainly higher than the power required for joint decoding in (5), the latter method also involves much higher complexity. In order to base the comparison at similar complexity, we have to reduce the code word length* by a factor of two in the joint decoding approach. As shorter codes show worse performance, this will result in an additional power penalty $V_M > 1$ such that the total power penalty of the joint decoding approach becomes $V_M V$.

In order to illustrate this result, consider the case $R_1 = R_2 = R$ which leads to the simplified result

$$V_S = 1+(V-1)\tanh\left(R\ln 2\right). \quad (7)$$

Let both users employ the rate $R = \frac{1}{n}$ convolutional codes with Z states that were reported best in [6, 7] for a target bit error rate of $P_e = 10^{-6}$. Their power penalties in comparison to the capacity limit $(E_b/N_0)_{min} = (4^R-1)/(2R)$, the additional power penalties V_M to the respective best codes in [6, 7] with $\sqrt{Z}$ states, and the additional power penalties V_S, cf. (7), are given in Tables 1 to 3, respectively.

Comparing the additional power penalities in Tables 2 and 3, successive decoding is found to be favourable for long codes while joint decoding shows superior performance for short codes. The crossover point between the two approaches depends on the code rate. Hereby, the lower-rate code supports successive decoding and the higher-rate code favours joint decoding.

* For convolutional codes, code word length corresponds to code memory, in this context.

Table 1. Power penalties to capacity limit

V [dB]	$Z=64$	$Z=256$	$Z=1024$	$Z=4096$
$R=\frac{1}{2}$	5.238	4.462	3.905	3.503
$R=\frac{1}{3}$	5.468	4.665	4.139	3.711

Table 2. Additional power penalties for codes with $\sqrt{Z}$ states

V_M [dB]	$Z=64$	$Z=256$	$Z=1024$	$Z=4096$
$R=\frac{1}{2}$	1.468	1.718	1.840	1.735
$R=\frac{1}{3}$	1.818	1.843	1.806	1.757

Table 3. Additional power penalties for successive decoding

V_S [dB]	$Z=64$	$Z=256$	$Z=1024$	$Z=4096$
$R=\frac{1}{2}$	2.505	2.036	1.720	1.503
$R=\frac{1}{3}$	1.966	1.576	1.341	1.161

The previous considerations were based on two users only. Assuming that the total number of users within the system is a power of two and the rates of the users are identical, the two user case is no loss of generality, since the results for two users can be recursively applied to two sets of two users, two sets of two sets of two users, etc. This means, if it turns out favourable to decode two users jointly, one has to check whether it is also favourable to decode two sets of two users jointly, and so on until successive decoding becomes favourable. For general m-ary splitting-trees, one has to compare the additional power penalties for successive decoding with m users against the additional power penalties for codes with $Z^{1/m}$ states.

3. Prospect

The comparison between successive and joint decoding conditioned on comparable complexity has been applied to the standard Gaussian multiple-access channel. In practice, at least two important additional effects have to be taken into account: On the one hand, there may occur fading which causes additional trouble for successive decoding as it prohibits perfect cancellation of the interfering signals. On the other hand, symbol waveforms are rarely identical, but only correlated to some extend like in code-division multiple-access (CDMA) systems. The latter case significantly complicates the problem, and analytical analysis, if possible, will hardly provide illustrative hints for system design. Nevertheless, the principles outlined in this paper remain valid. For some results applying to CDMA with random spreading, the reader is refered to [8].

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