Efficient Implementation of Multiuser Detectors for Asynchronous CDMA

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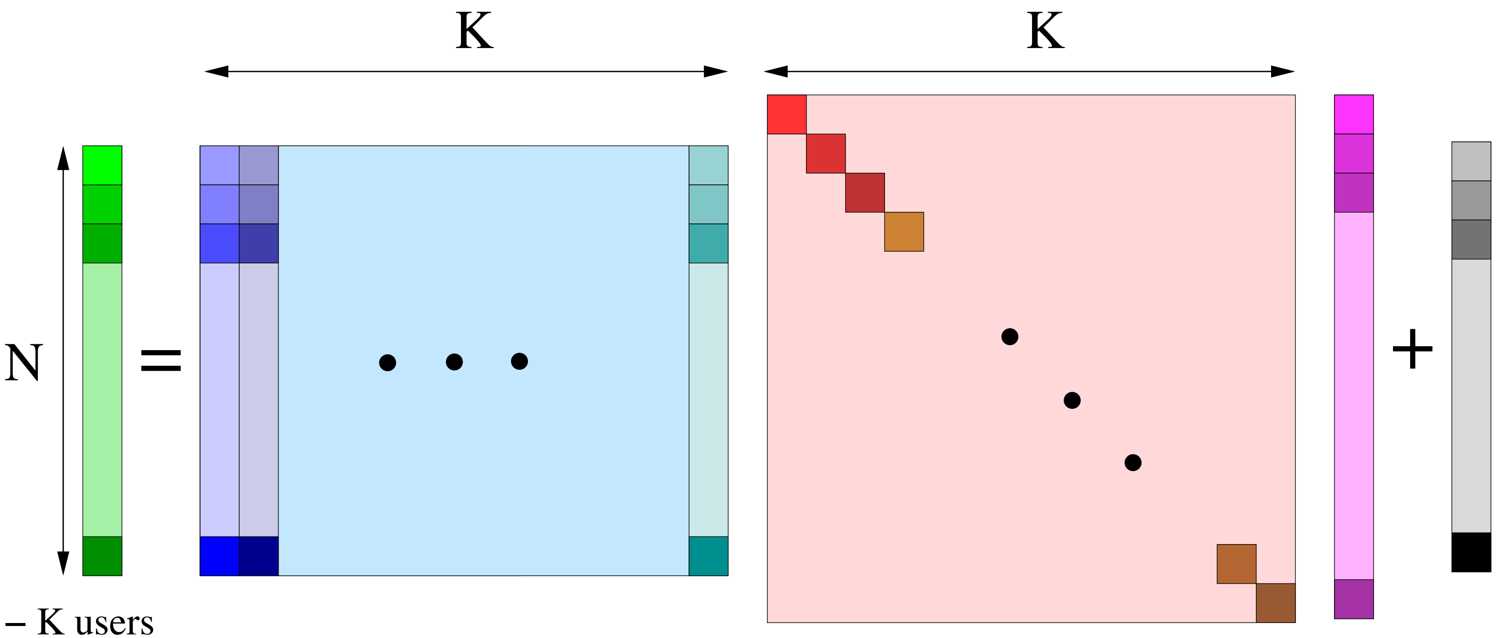
1. Problem Statement
2. Subspace Basis: Effects on Complexity and Performance
3. Multistage Detectors versus LMMSE detectors: Performance Analysis
4. Summary

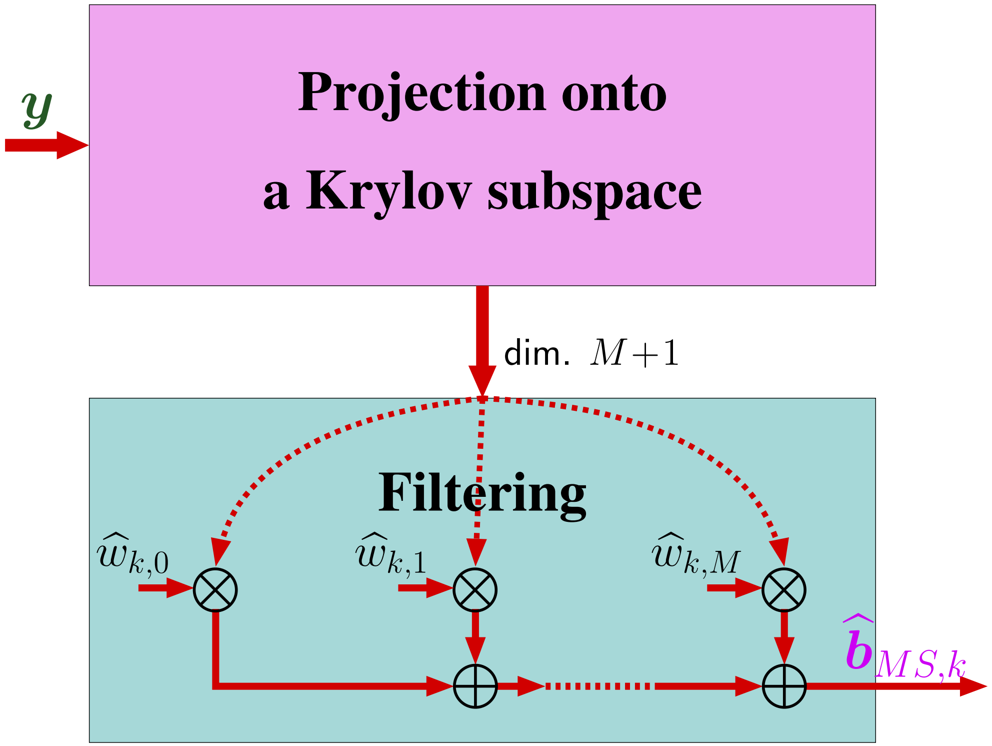
Synchronous CDMA Uplink System Model

$$\boldsymbol{y}(m) = \underbrace{\boldsymbol{S}(m)\boldsymbol{A}(m)}_{\boldsymbol{H}(m)}\boldsymbol{b}(m) + \boldsymbol{n}(m)$$



– K users
– Spreading factor: N

Multistage Detection as Subspace Method



Subspace basis I

$$\{\boldsymbol{h}_k, (\boldsymbol{H}_k\boldsymbol{H}_k^H)\boldsymbol{h}_k, \ldots, (\boldsymbol{H}_k\boldsymbol{H}_k^H)^M\boldsymbol{h}_k\}$$

Subspace basis J

$$\{\boldsymbol{h}_k, (\boldsymbol{H}\boldsymbol{H}^H)\boldsymbol{h}_k, \ldots, (\boldsymbol{H}\boldsymbol{H}^H)^M\boldsymbol{h}_k\}$$

Multistage detectors

Basis I

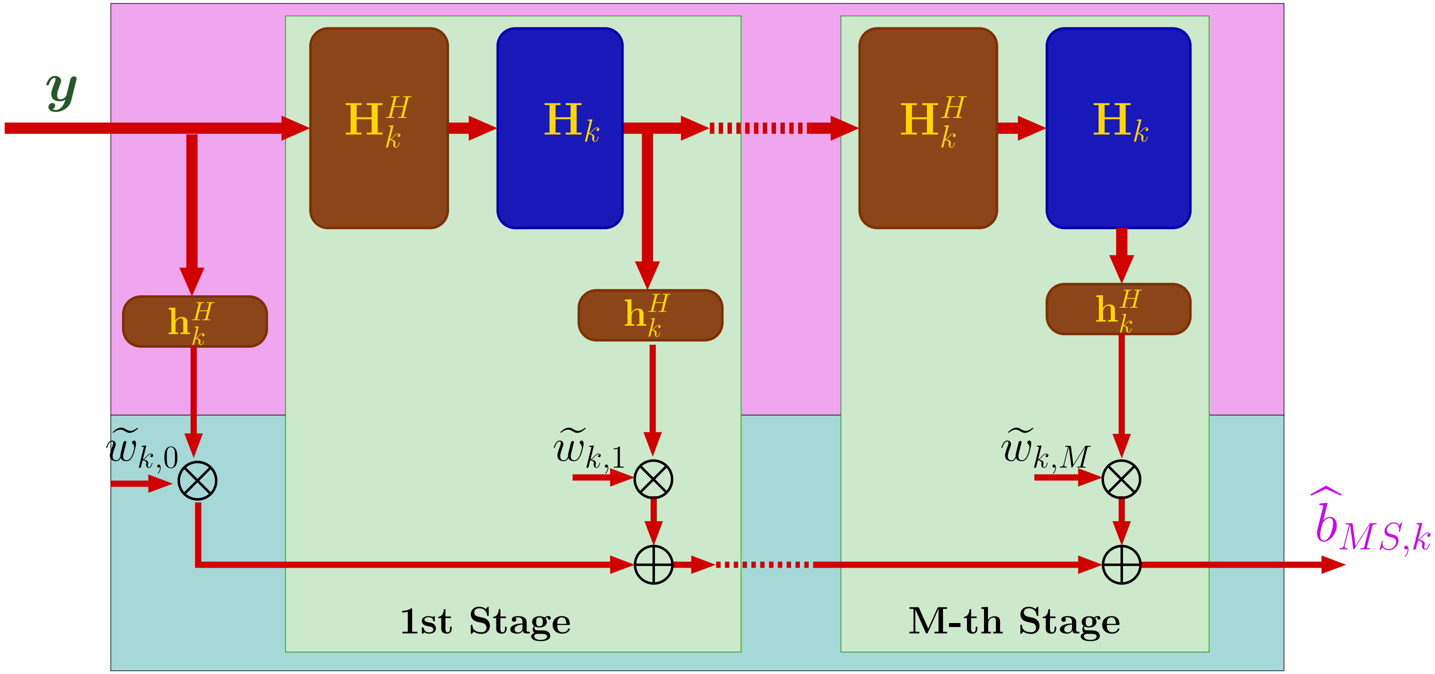
$$\widehat{\boldsymbol{b}}_{MS,k} = \sum_{m=0}^{M} \widetilde{w}_{k,m}\boldsymbol{h}_k^H(\boldsymbol{H}_k\boldsymbol{H}_k^H)^m\boldsymbol{y}$$

Basis J

$$\widehat{\boldsymbol{b}}_{MS,k} = \sum_{m=0}^{M} w_{k,m}\boldsymbol{h}_k^H(\boldsymbol{H}\boldsymbol{H}^H)^m\boldsymbol{y}$$

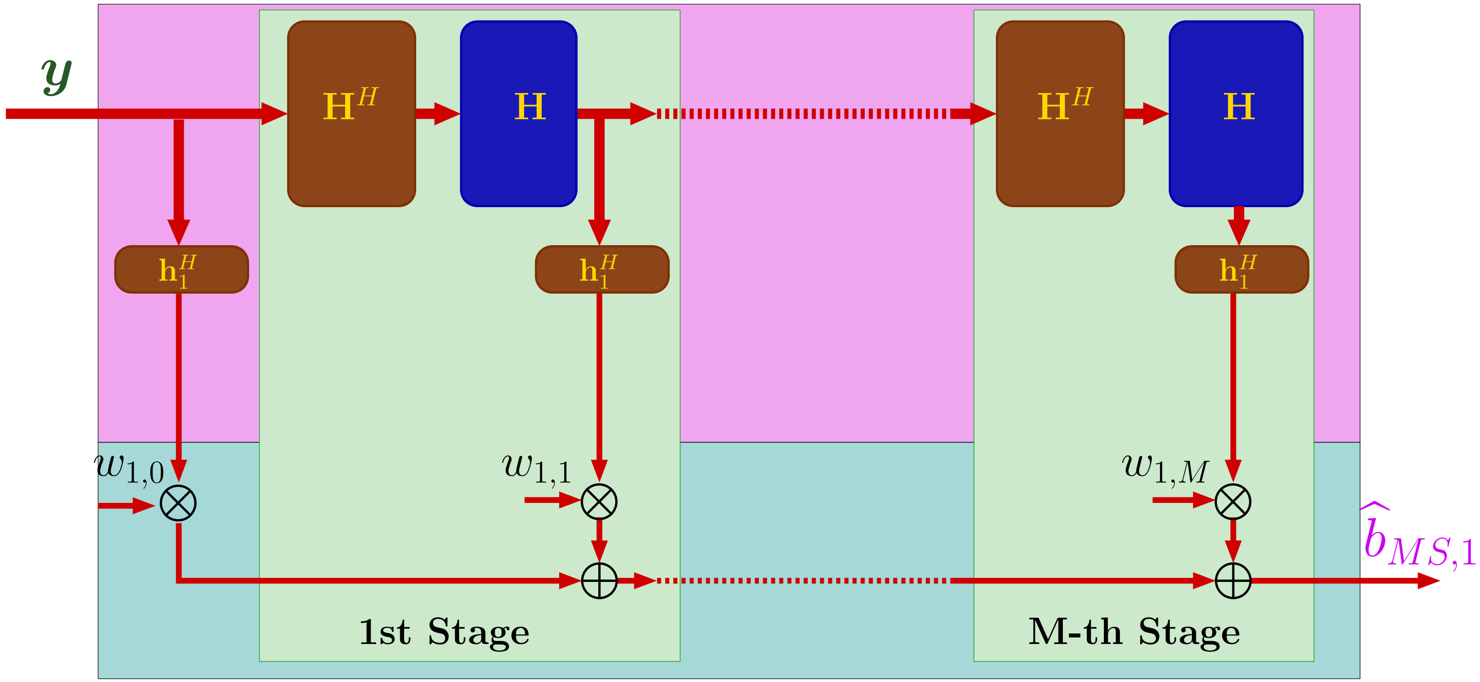
k : user of interest
$\boldsymbol{h}_k$: spreading of user k
$\boldsymbol{H}_k$: obtained from $\boldsymbol{H}$ suppressing $\boldsymbol{h}_k$

Projection: Basis I

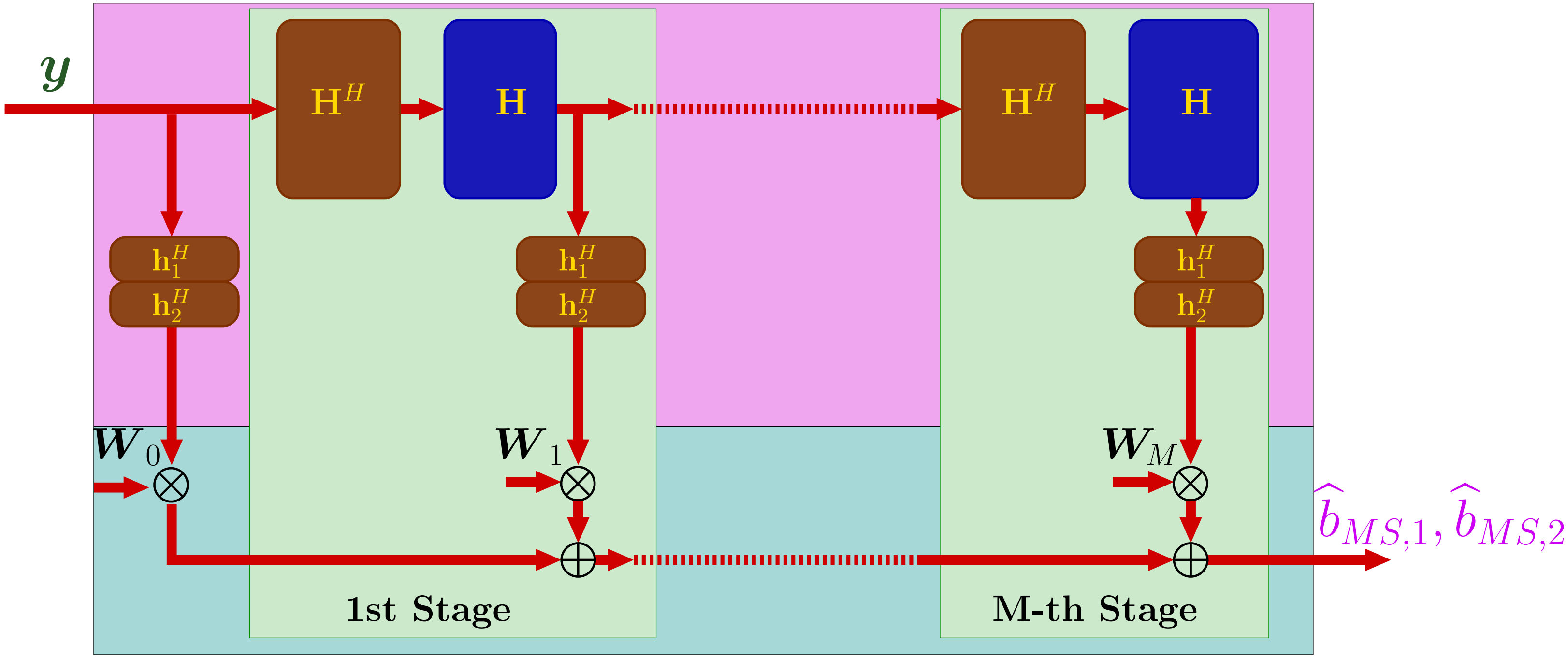


Projection: quadratic complexity per bit.

Projection: Basis J

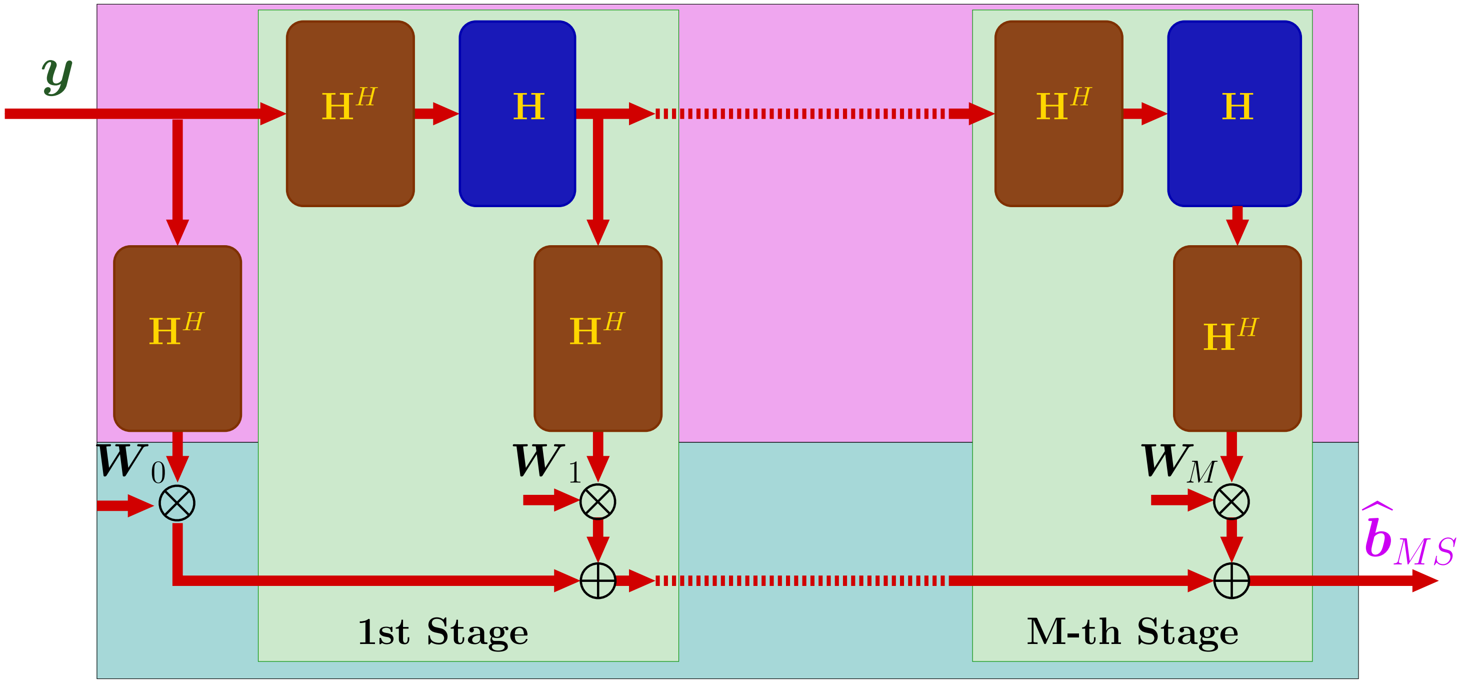


Projection: Basis J



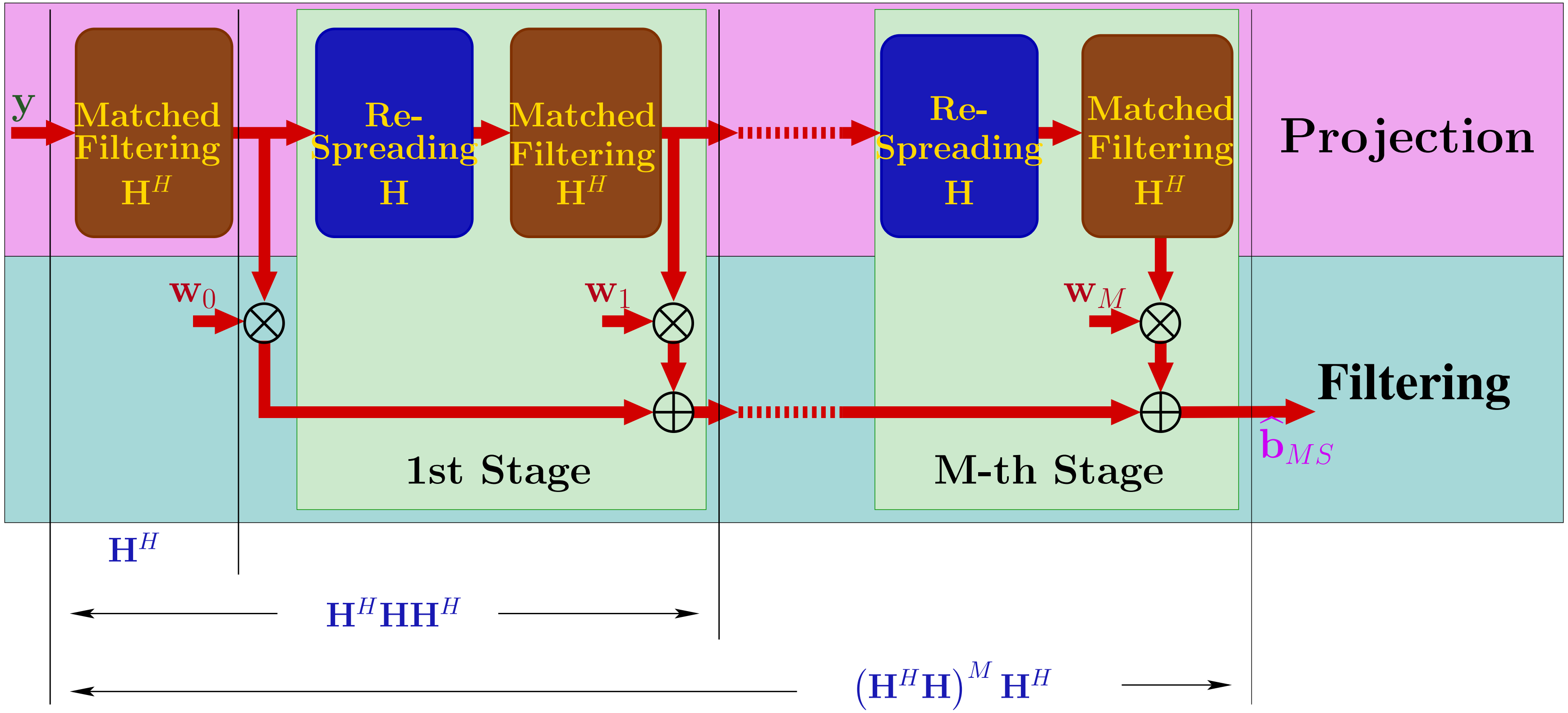
Joint Projection!

Projection: Basis J



Joint Projection!

Joint Projection: Multistage Detector Structure



Joint projection: Linear complexity per bit as SUMF!

Filtering

Joint Weighting

$$\hat{\boldsymbol{b}}_{\mathrm{MS}} = \sum_{m=0}^{M} w_m \left(\boldsymbol{H}^H \boldsymbol{H}\right)^m \boldsymbol{H}^H \boldsymbol{y}$$

w_m *scalar* minimizing the mean square error

$$\mathrm{E}\left\{\left\|\boldsymbol{b} - \widehat{\boldsymbol{b}}_{\mathrm{MS}}\right\|^2\right\}$$

Individual Weighting

$$\hat{\boldsymbol{b}}_{\mathrm{MS}} = \sum_{m=0}^{M} \boldsymbol{W}_m \left(\boldsymbol{H}^H \boldsymbol{H}\right)^m \boldsymbol{H}^H \boldsymbol{y}$$

$\boldsymbol{W}_m$ *diagonal matrix* minimizing the mean square error

Individual weighting outperforms joint weighting.

With asymptotic weighting approximation the complexity of weighting design is negligible w.r.t. the projection complexity.

Complexity of Multistage Detectors

Taxonomy

	Joint Projection	Individual Projection
Joint Filtering	TYPE I[1]	$\nexists$
Individual Filtering	TYPE II[2]	TYPE III[3]

Complexity order per symbol with asymptotic weighting

Detector	One user's detection	All users' detection
SUMF	$\mathcal{O}(K)$	$\mathcal{O}(K)$
TYPE I	$\mathcal{O}(K^2)$	$\mathcal{O}(K)$
TYPE II	$\mathcal{O}(K^2)$	$\mathcal{O}(K)$
TYPE III	$\mathcal{O}(K^2)$	$\mathcal{O}(K^2)$
LMMSE	$\mathcal{O}(K^3)$	$\mathcal{O}(K^2)$

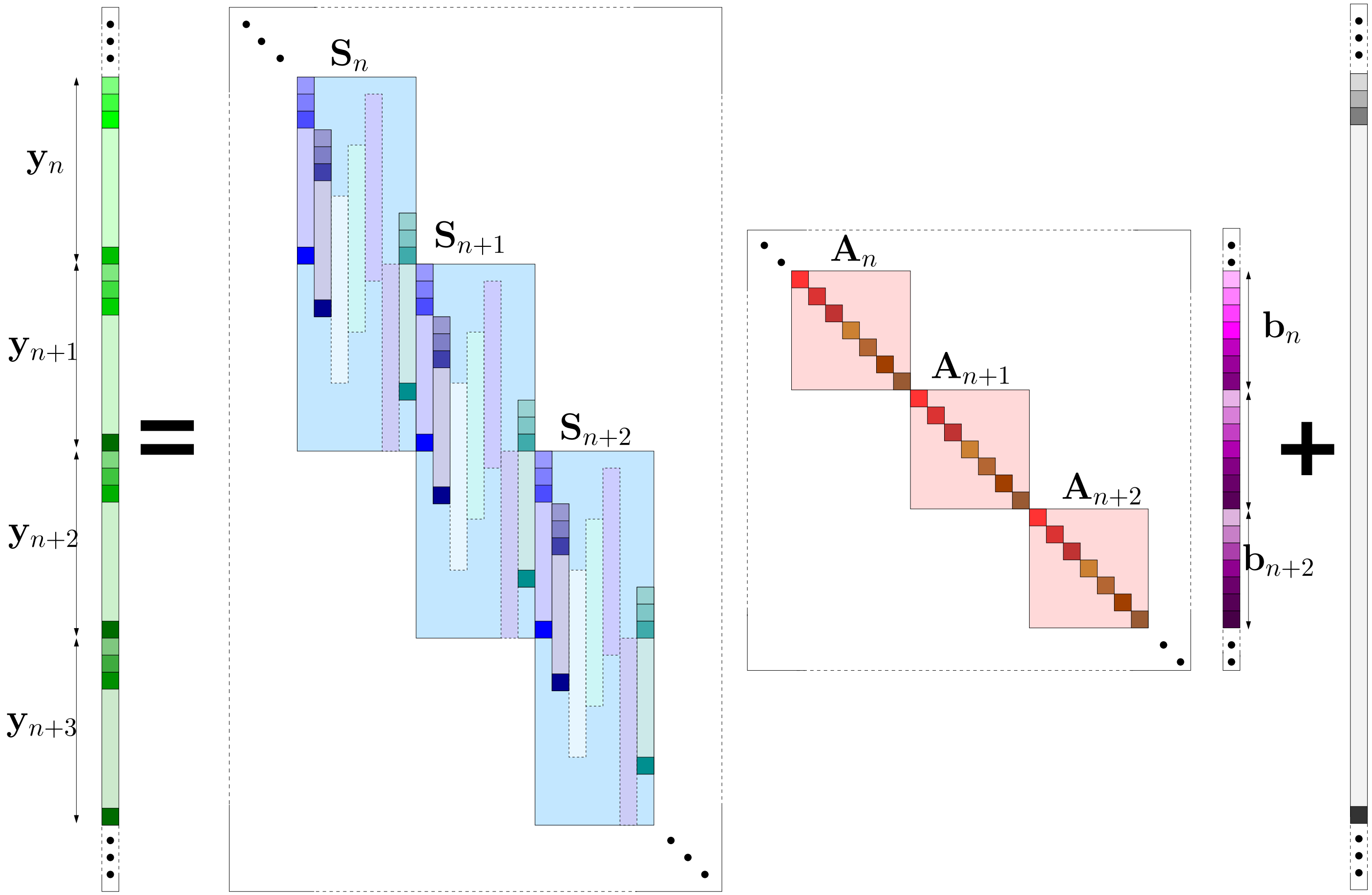
Type II detectors

- **Outperform Type I detectors.**
- **Have lower complexity than Type III detectors.**

Joint projection and individual filtering

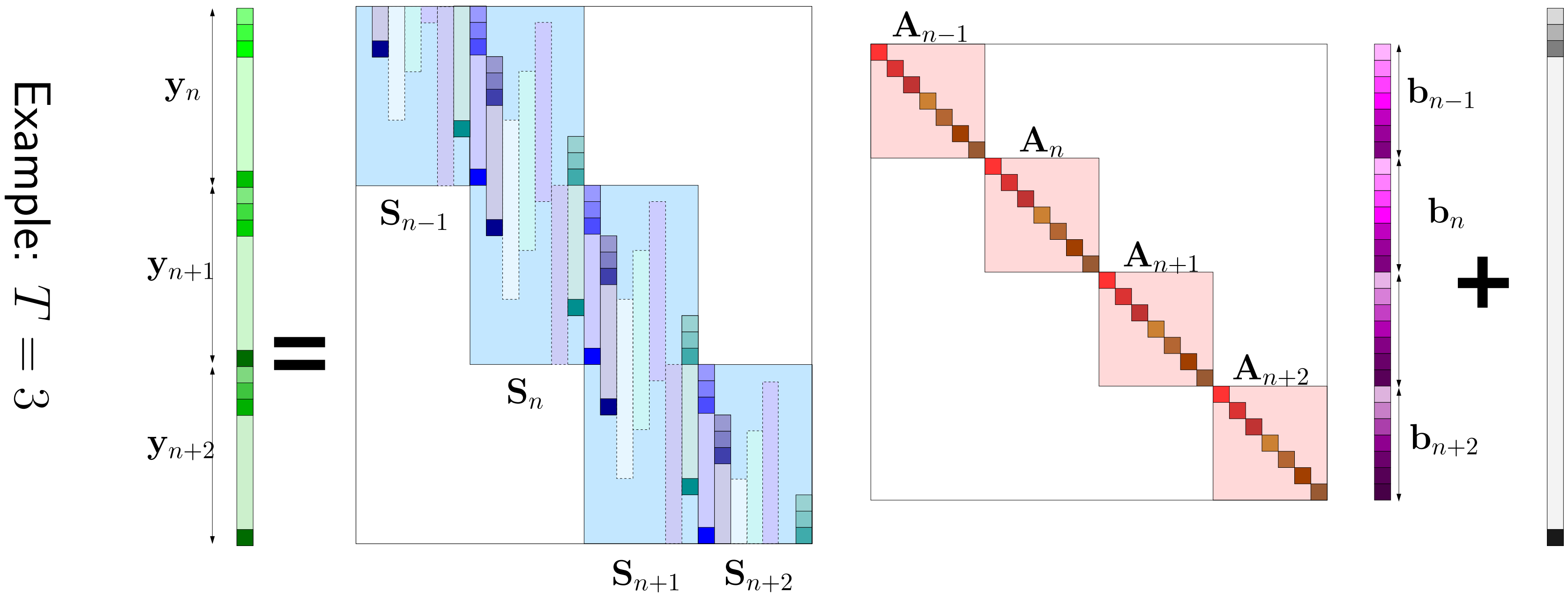
1) Multistage detector by Moshavi et al., '96. Asymptotic weighting: Müller et al. '01, Cottatellucci et al., '02.
2) Individual LMMSE detector in the projection subspace by Cottatellucci et al., '02. Asymptotic weighting: Cottatellucci et al., '02.
3) Multistage Wiener Filter by Goldstein et al., '98. Asymptotic weighting: Li et al., '01, '04.

Symbol-Async. and Chip-Sync. Systems Model



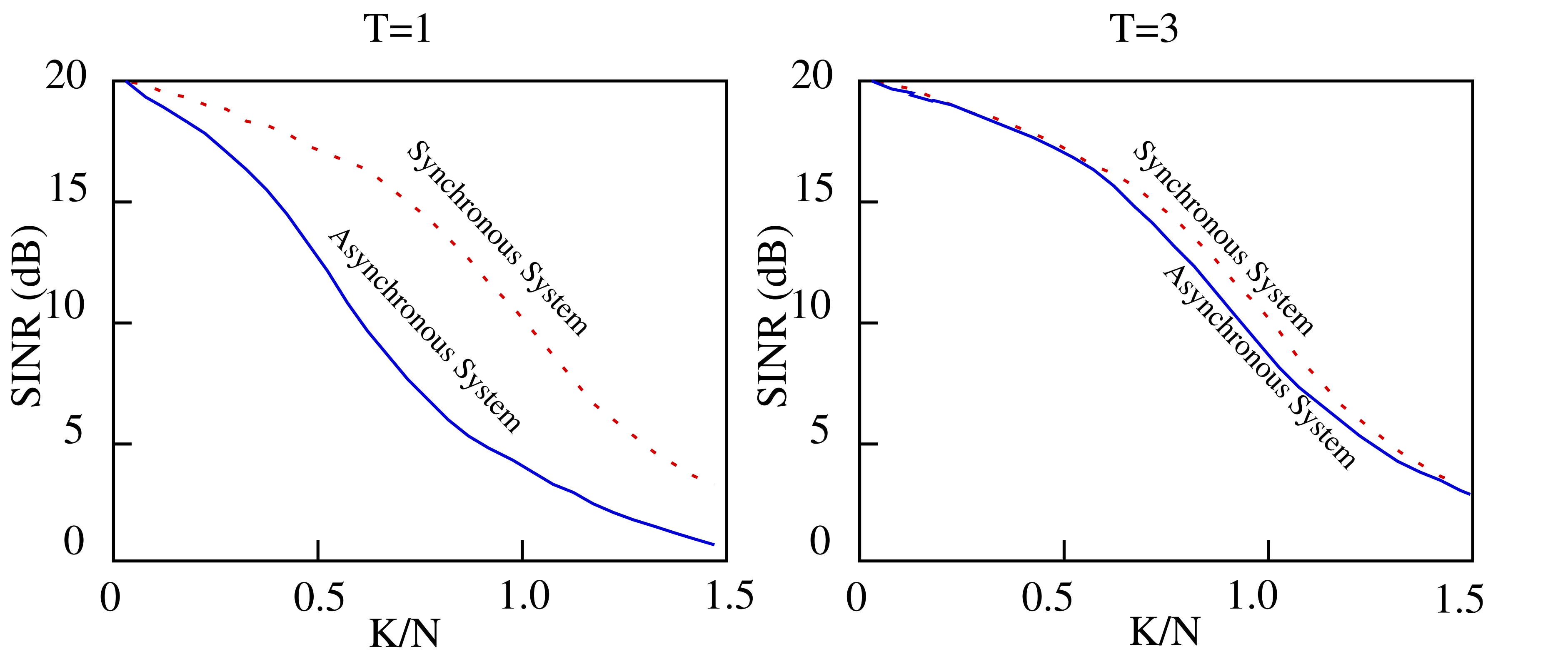
$$\mathcal{Y} = \underbrace{\mathcal{S}\mathcal{A}}_{\mathcal{H}}\mathcal{B} + \mathcal{N}$$

LMMSE Detector: Truncation



$$\mathcal{Y}_T(n) = \underbrace{\mathcal{S}_T(n)\mathcal{A}_T(n)}_{\mathcal{H}_T(n)}\mathcal{B}_T(n) + \mathcal{N}_T(n)$$

LMMSE Detector: Truncation Effects



As the window size $T \to \infty$, the effect of asynchronicity vanishes.

Projection Subspace for Asynchronous Systems

$$\left\{\boldsymbol{\mathcal{H}}_T, (\boldsymbol{\mathcal{H}}_T\boldsymbol{\mathcal{H}}_T{}^H)\boldsymbol{\mathcal{H}}_T, \ldots, (\boldsymbol{\mathcal{H}}_T\boldsymbol{\mathcal{H}}_T{}^H)^M\boldsymbol{\mathcal{H}}_T\right\}$$

Performance degradation:

- **Subspace method**
- **Truncation**

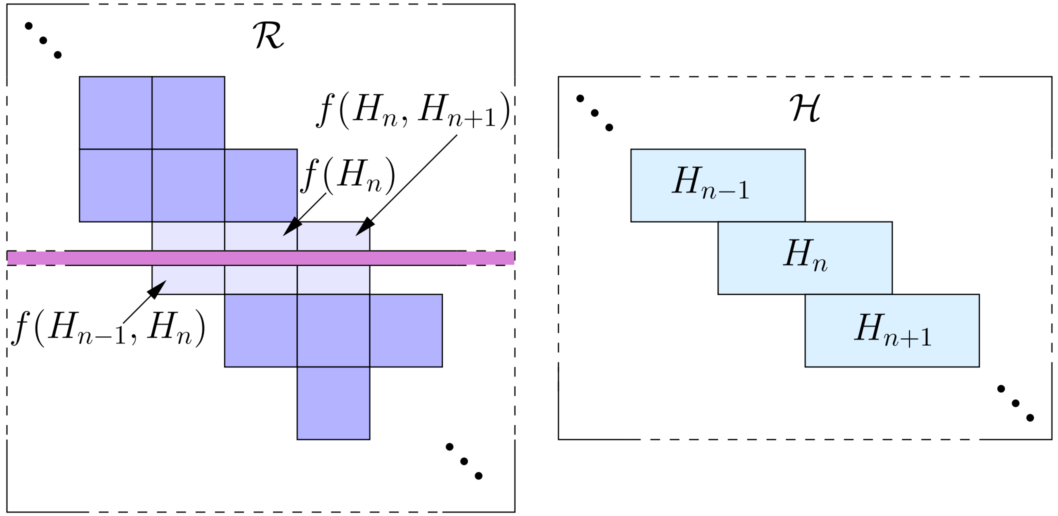
$$\left\{\boldsymbol{\mathcal{H}}, (\boldsymbol{\mathcal{H}}\boldsymbol{\mathcal{H}}^H)\boldsymbol{\mathcal{H}}, \ldots, (\boldsymbol{\mathcal{H}}\boldsymbol{\mathcal{H}}^H)^M\boldsymbol{\mathcal{H}}\right\}$$

No truncation effects, but...

...Is it possible to implement a multistage detector with finite delay?

Multistage Detectors: No Truncation Effects

1-st Stage: $\mathcal{R}\mathcal{H}^H$

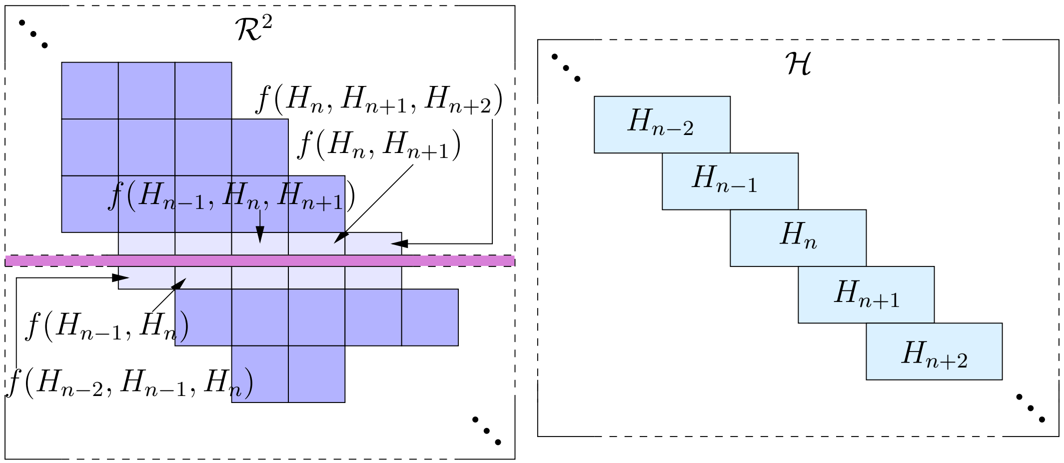


$$\mathcal{R} = \mathcal{H}^H\mathcal{H}$$

The projector onto $\mathcal{R}\mathcal{H}^H$ for the estimation of $b_i(n)$ depends only on H_{n-1}, H_n, H_{n+1}!

Multistage Detectors: No Truncation Effects

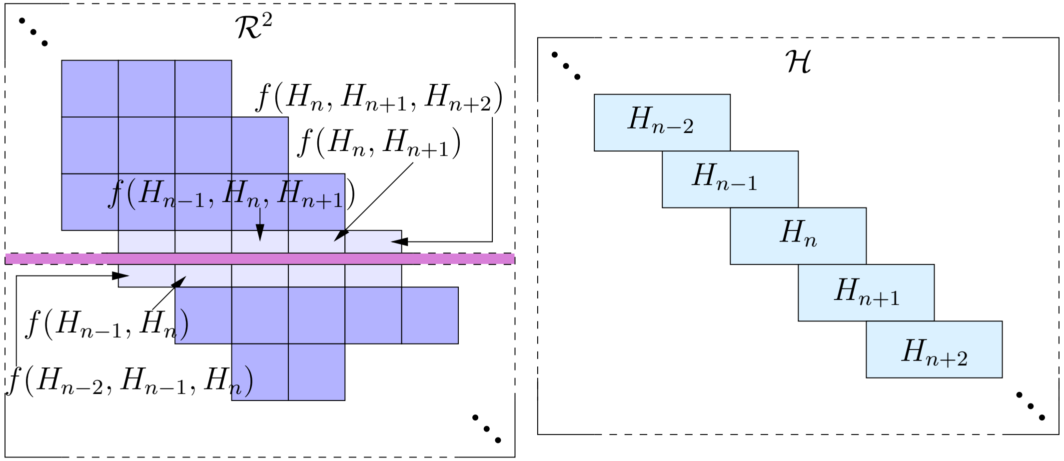
2-nd Stage: $\mathcal{R}^2\mathcal{H}^H$



The projector onto $\mathcal{R}^2\mathcal{H}^H$ for the estimation of $b_i(n)$ depends only on H_{n-2}, H_{n-1}, H_n, H_{n+1}, H_{n+2}!

Multistage Detectors: No Truncation Effects

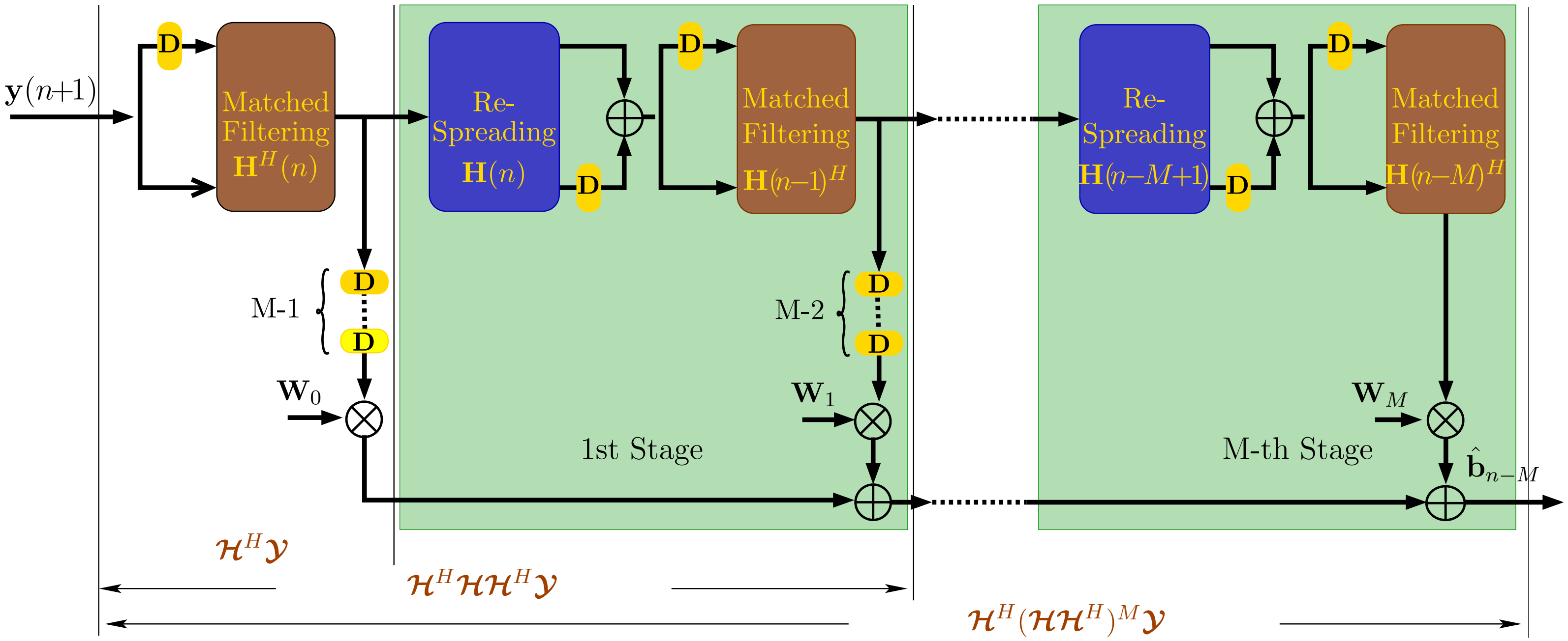
2-nd Stage: $\mathcal{R}^2\mathcal{H}^H$



The projector onto $\mathcal{R}^2\mathcal{H}^H$ for the estimation of $b_i(n)$ depends only on H_{n-2}, H_{n-1}, H_n, H_{n+1}, H_{n+2}!

The signal projection onto $\mathcal{H}^H$, $\mathcal{R}\mathcal{H}^H$, $\mathcal{R}^2\mathcal{H}^H$... can be computed exactly with finite delay!

Asynchronous Detector Structure



1. **No truncation effects.**
2. **Enlargement of the observation window with the number of stages.**
3. **Linear complexity order per bit.**

Linear Detectors: A Comparison

- **Linear MMSE Detector:**
  - Optimization in the full signal space.
  - Truncation effects!
- **Multistage Detector:**
  - Optimization in a subspace.
  - No truncation effect.

WHICH ONE PERFORMS BETTER?

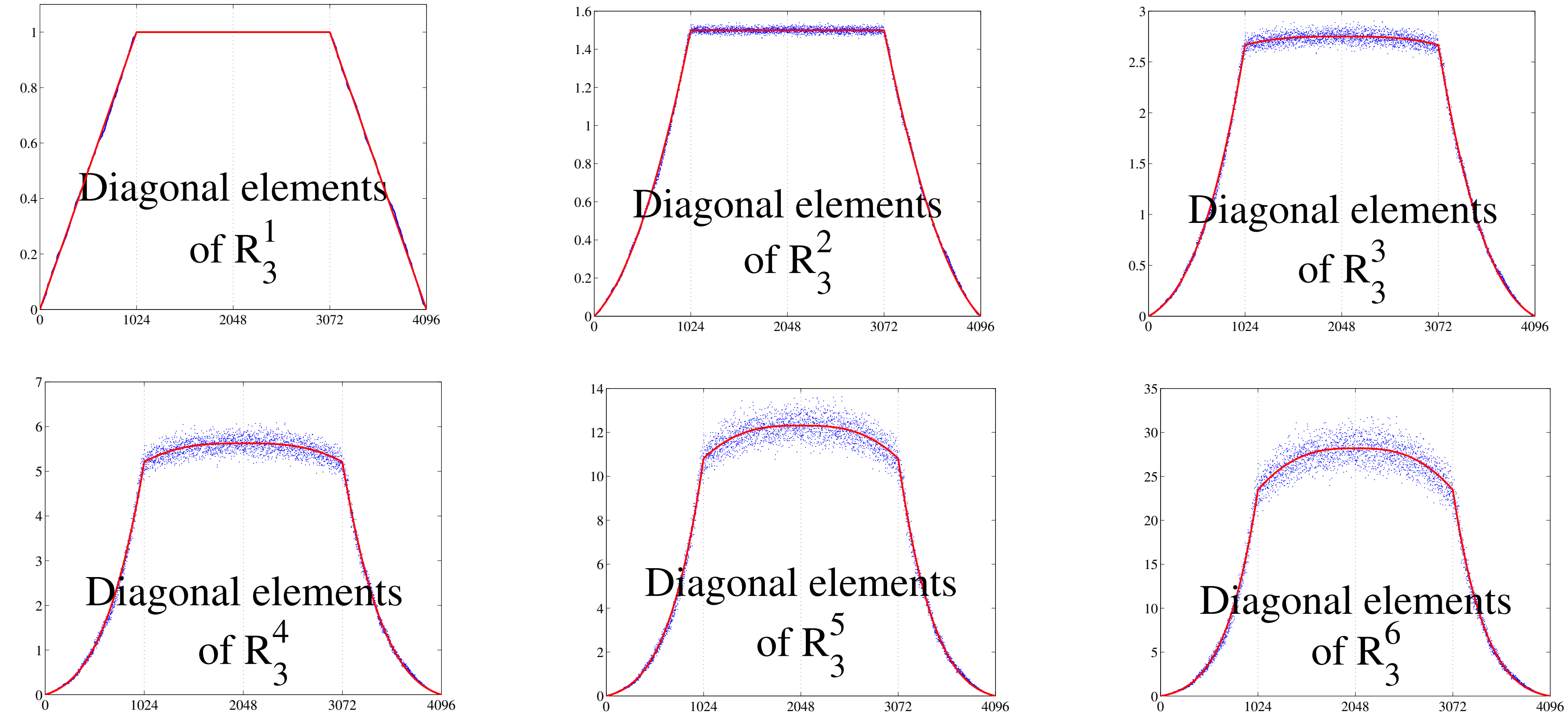
Performance Analysis

$$\mathrm{SINR}[b_k(n)] \text{ depends on } \begin{cases} \mathcal{R}_{kk}(n),\ \mathcal{R}^2_{kk}(n), \ldots, \mathcal{R}^{2M+2}_{kk}(n) & M - \text{stage detector} \\ \mathcal{R}_{T,kk}(n),\ \mathcal{R}^2_{T,kk}(n), \ldots & \text{LMMSE detector} \end{cases}$$

$\mathcal{R}^m_{T,kk}(n)$ converges almost surely to a deterministic constant*
as $K, N \to \infty$ with $\frac{K}{N}$ constant

* see Theorem in the paper.

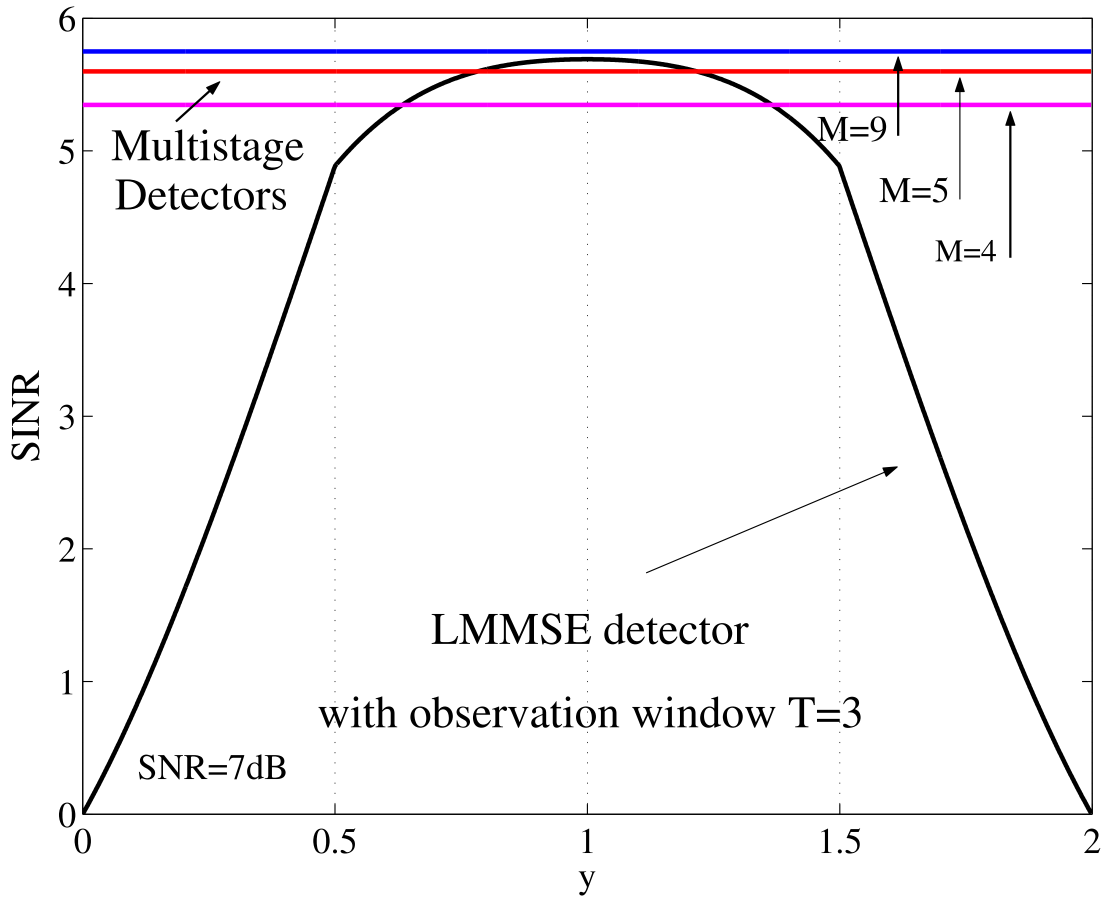
$\mathcal{R}^m_{T,kk}(n)$: Asymptotic versus Finite Systems



$\boldsymbol{A} = \boldsymbol{I}$ $\quad T = 3 \quad N = 2048 \quad K = 1024$

LMMSE versus Multistage Detectors: Asymptotic Performance

- Channel matrix $\boldsymbol{A} = \boldsymbol{I}$.
- Observation window $T = 3$.
- $\mathrm{SNR} = 7$ dB.
- $\frac{K}{N} = \frac{1}{2}$.



$\mathcal{R}^{m}_{T,kk}(n)$ as $T \to \infty$: Effect on Multistage Detectors

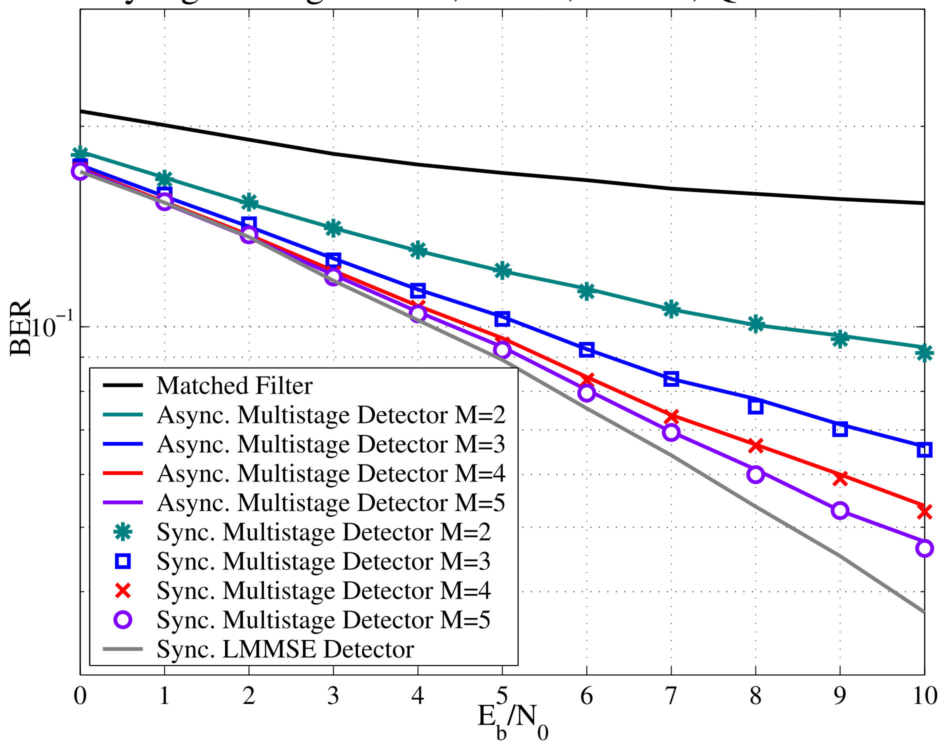
As $T \to \infty$ the diagonal elements of $\mathcal{R}_T$ equal the diagonal elements of $\mathcal{R}$ for synchronous systems.

Multistage detectors for large synchronous and asynchronous systems have:

– Same performance.

– Same weights.

Flat Rayleigh fading channel, K=64, N=128, QPSK modulation.



Summary

- **A multistage detector structure for chip synchronous but symbol-asynchronous systems.**
- **An algorithm to determine the diagonal elements of $\mathcal{R}_T^m$ $\Rrightarrow$ large system performance.**

In contrast to LMMSE detectors, multistage detectors have:

No truncation effects!

Same performance as for synchronous systems.

SINR independent of the time shift of the detected symbol.

Same complexity order per user as the matched filter.

For a sufficient large number of stages they can outperform the MMSE detectors with fixed observation window.

Remarks

Effect of chip asynchronicity:

Multistage detectors:

Laura Cottatellucci and Ralf R. Müller. Multistage detectors for asynchronous CDMA. *International Zurich Seminar on Communications (IZS)*, Zurich, Feb. 2004.

Linear MMSE detector:

Laura Cottatellucci and Ralf R. Müller. A generalized resource pooling result for correlated antennas with applications to asynchronous CDMA. *Proc. of International Symposium on Information Theory and its Applications (ISITA)*, Parma, Oct. 2004.

