Multiuser Equalization for Random Spreading: Limits of Decorrelation with and without Decision–Feedback*

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Abstract

Synchronous code–division multiple–access (CDMA) communication with randomly chosen spherical spreading sequences and capacity achieving error correction coding is analyzed. Emphasis is put on the penalties to be paid by applying single user coding in conjuction with decorrelation as pre–equalization. In contrast to previous work on this topic, the analysis also extends to re–encoded decision feedback structures where an equal rate and and an equal power case are distinguished. The results are nonasymptotic in the number of users and show that fluctuations of the signal–to–noise ratio due to the random nature of the sequences has no deleterious effect onto capacity for the spherical random sequence model.

1 Introduction

From founding the area of multiuser detection in [1] up to now a nearly uncountable number of papers appeared proposing and analyzing the performance of suboptimal multiuser detectors, see e.g. [2] and references therein. Unfortunately, comparing multiuser detectors involves many different parameters like sequence design, power control, number of users, spreading factor, signal–to–noise ratio, and design of channel coding (if used). This makes a quantitative comparison of these various results quite difficult.

In this paper the performances of the decorrelator and the decorrelating decision–feedback equalizer, both in conjunction with single user channel coding, are analyzed and compared in a more general manner by deriving their position in the power–bandwidth plane under the spherical random sequence model. This allows to optimize the coding spreading trade–off as well as the system load.

In Sections 2 and 3 the decorrelating and decorrelating decision–feedback detector are analyzed in the power–bandwidth plane, respectively, while Section 4 points out the conclusions.

1.1 Spherical Random Sequence Model

In this work each overall spreading sequence $\boldsymbol{b}$ is modelled by an L dimensional complex Gaussian random vector $[g_1, g_2, \ldots, g_L]^{\mathrm{T}}$ which is normalized to unit Euclidian norm with

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L denoting the spreading factor

$$\boldsymbol{b} = [g_1, g_2, \dots, g_L]^{\mathrm{T}} \Big/ \sqrt{\sum_{i=1}^{L} |g_i|^2}. \tag{1}$$

This normalization keeps the average energy per symbol independent of the signature sequences. Moreover, the random variables $g_i, 1 \le i \le L$, are assumed to be independent zero–mean complex Gaussian distributed with equal variance. Thus, the spreading vectors $\boldsymbol{b}$ are uniformly distributed on the $L-1$ dimensional surface of an L dimensional sphere. The advantage of this model is that it allows analytical tractability of many multiuser detection schemes. For details on this random sequence model the reader is referred to [2, 3].

1.2 Power–Bandwidth Plane

From Shannon's [4] celebrated formula

$$C_{\mathrm{T}} = B \log_2 \left(1 + \frac{P}{\sigma^2}\right) \tag{2}$$

for the capacity of the channel corrupted by additive white Gaussian noise (AWGN) of power σ^2, limited to bandwidth B, and constrained to signal power P it is staightforward to obtain

$$\frac{E_{\mathrm{b}}}{N_0} = \frac{2^C - 1}{C} \tag{3}$$

with the capacity normalized to bandwidth, the energy per bit, and the noise power spectral density defined by $C \triangleq C_{\mathrm{T}}/B, E_{\mathrm{b}} \triangleq P/C_{\mathrm{T}}, N_0 \triangleq \sigma^2/B$, respectively. Eq. (3) establishes a fundamental relationship between power efficiency N_0/E_{b} and bandwidth–efficiency C of the AWGN channel and shows that one can only be obtained at the expense of the other.

Multiuser communication systems are described by their capacity region [5] which is given by a set of constraints on the capacities $C_i, 1 \le i \le K$, of the K individual users transmitting at power levels $P_i, 1 \le i \le K$. If the total transmission rate is maximum, for the AWGN channel the constraint

$$\sum_{i=1}^{K} C_i = \log_2 \left(1 + \sum_{i=1}^{K} \frac{P_i}{N}\right) \tag{4}$$

bites [5]. Assuming that all users want to transmit the same amount of data within the same time and bandwidth, i.e. $C_i = C, \forall i$, yields

$$\frac{2^{KC} - 1}{KC} = \frac{1}{K} \sum_{i=1}^{K} \frac{E_{\mathrm{b}i}}{N_0} \triangleq \frac{\overline{E_{\mathrm{b}}}}{N_0}, \tag{5}$$

with implicit definition of the average energy per bit $\overline{E_{\mathrm{b}}}$.

Considering CDMA, there is an inherent bandwidth extension by the spreading factor L. Thus, spectral efficiency (total capacity per chip) relates to single–user capacity C (capacity per user and symbol) like

$$\varGamma = \frac{K}{L} C \triangleq \zeta C, \tag{6}$$

cf. [6], with ζ denoting the *load* of the system. In order to illustrate the performance of CDMA in the power–bandwidth plane we need to find a relationship between spectral efficiency Γ and power efficiency $N_0/\overline{E}_\mathrm{b}$. Spectral efficiency of the optimum multiuser decoder with optimum sequence design is given by Eq. (5). It serves as an upper bound on performance of the suboptimum detection schemes analyzed in the following.

2 Analysis of Decorrelation

Spectral efficiency of zero–forcing[1] multiuser equalization with random speading sequences has been addressed in [7] and [8] by using bounds on the average signal–to–noise ratio and Monte–Carlo methods. Asymptotic results, i.e. number of users K and spreading factor L grow over all bounds, have recently been given in [3]. For related work on performance of linear multiuser receivers in random environments, see also [9, 10]. In this paper, we also examine the case of finite K and L.

Using random signature sequences the signal–to–noise ratio at decorrelator output is a random variable. Setting $P_i, \forall i$, without loss of generality and for ease of notation, its probability density function was derived for spherical random sequences as [11, 12, 13]

$$\mathrm{p}_{\gamma_\mathrm{ZF}}(y) = \begin{cases} \sigma^2 \dfrac{(\sigma^2 y)^{L-K}(1-\sigma^2 y)^{K-2}}{\mathrm{B}(L-K+1, K-1)} & 0 \le y \le \sigma_n^{-2} \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

with $\mathrm{B}(a,b) \triangleq \int_0^1 t^{a-1}(1-t)^{b-1}\mathrm{d}t$ denoting the beta function.

Asymptotically, i.e. for $K = \zeta L \to \infty, \zeta \in (0;1]$, the variance of the signal–to–noise ratio vanishes

$$\lim_{K=\zeta L \to \infty} \mathrm{Var}\left\{\gamma_\mathrm{ZF}\right\} = 0. \tag{8}$$

yielding that the signal–to–noise ratio becomes a deterministic value.

These properties of the signal–to–noise ratio of the decorrelator's output signal are used in the following to give insight into the trade–off between power and bandwidth efficiency of zero–forcing multiuser equalization.

Proposition 1 *The average channel capacity of a CDMA–channel with K users with unit powers which is disturbed by complex AWGN of variance σ^2 and equalized according to the zero–forcing criterion is given by*

$$\overline{C}_\mathrm{ZF} = \log_2\left(1 + \frac{1}{\sigma^2}\right) - \frac{1}{\ln(2)} \sum_{n=1}^{\infty} \frac{(K-1)_n}{n(L)_n} \left(\frac{1}{1+\sigma^2}\right)^n \tag{9}$$

with $(x)_n \triangleq \prod_{i=0}^{n-1}(x+i)$ denoting the Pochhammer polynomials, and bounded by

$$\log_2\left(1 + \frac{L-K}{\sigma^2 L}\right) < \overline{C}_\mathrm{ZF} < \log_2\left(1 + \frac{L-K+1}{\sigma^2 L}\right) \tag{10}$$

if the spreading sequences are uniformly distributed on the $(L-1)$–dimensional surface of an L–sphere.

[1] The terms *decorrelation* and *zero–forcing* are used synonymously throughout this paper.

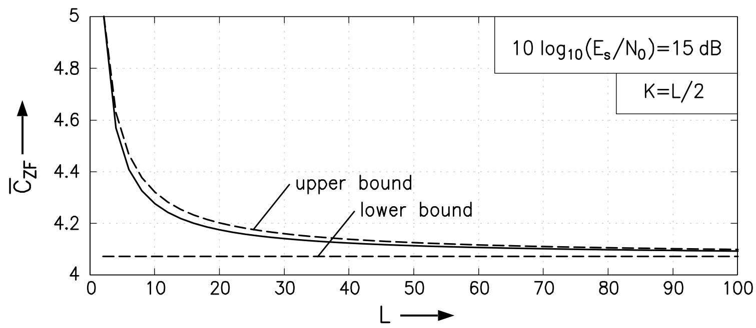

Figure 1: Channel capacity $\overline{C}_{\text{ZF}}$ with zero–forcing equalization versus spreading factor L for $K = L/2$ and $10\log_{10}(E_s/N_0) = 15\,\text{dB}$. The dashed lines refer to the upper and lower bound, respectively.

The proof is omitted in this conference version of the paper. It can be found in [12, 13]. Proposition 1 implies some surprising consequences. Channel capacity is a concave function of the random variable signal–to–noise ratio. Thus, the average capacity is lower than capacity of the average signal–to–noise ratio. Eq. (8) tells that the variance of the signal–to–noise ratio vanishes for infinite spreading factor. This should imply that for infinite L capacity reaches its maximum if K/L is fixed, but surprisingly this statement is disproved by the left hand side of (10). In fact, the average signal–to–noise ratio depends on the $(K-1)/L$ ratio and for fixed $(K-1)/L$ ratio the above statement actually holds, but the small impact by the fact that users do not interfere with themselves overrules the impact of decreasing variance of signal–to–noise ratio. As depicted in Fig. 1 the upper bound is even much closer than the lower bound.

The asymptotic relationship between power and bandwidth efficiency results with (3) and (6)

$$\lim_{K=\zeta L\to\infty}\left(\frac{E_\text{b}}{N_0}\right)_{\text{ZF}} = \frac{2^{\Gamma/\zeta}-1}{\Gamma/\zeta-\Gamma} \tag{11}$$

from Proposition 1 and is illustrated in Fig. 2. Eq. (11) does not only hold for signature sequences which are uniformly distributed on the surface of an L–sphere, but also for independent binary distributed signature sequences with zero mean [3]. Fig. 2 indicates that the load ζ has to be carefully adjusted if high spectral efficiency shall be obtained.

Considering Fig. 2, a large gap between the orthogonal system and one applying CDMA with decorrelation and random sequences occurs. Applying (11) the loss of the decorrelator in comparison to the single–user Shannon bound is given by

$$V_{\text{ZF}}(\Gamma) \triangleq \frac{\min\limits_{\zeta}\left(\frac{E_\text{b}}{N_0}\right)_{\text{ZF}}}{\frac{2^\Gamma-1}{\Gamma}} = \min_{\zeta}\frac{2^{\Gamma/\zeta}-1}{\left(2^\Gamma-1\right)(1/\zeta-1)} = \min_{\zeta}\frac{2^{\Gamma(1/\zeta-1)}}{1/\zeta-1} + \frac{\mathcal{O}\left(2^{\Gamma(1/\zeta-2)}\right)}{1/\zeta-1} \tag{12}$$

This simple optimization problem can be easily solved by first order derivation and gives

$$V_{\text{ZF}}(\Gamma) = \text{e}\,\ln(2)\,\Gamma + \mathcal{O}\left(\Gamma 2^{-\Gamma}\right). \tag{13}$$

Obviously, the asymptotic loss, i.e. $V_{\text{ZF}}(\Gamma), \Gamma\to\infty$, is proportional to spectral efficiency. For $\Gamma > 4$ its difference to the exact loss is quite small. Eq. (13) shows that the loss to the optimum detector grows over all bounds if spectral efficiency does. Nevertheless, this is an important improvement in comparison to the conventional detector whose loss becomes infinite for *finite* spectral efficiency because of interference limitation.

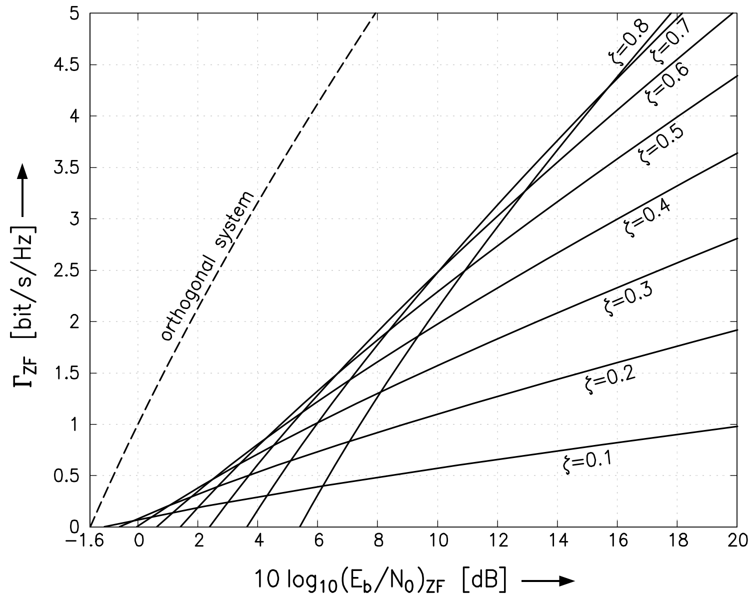

Figure 2: Power–bandwidth–plane for zero–forcing CDMA equalization with the K/L–ratio ζ as parameter. For comparison the dashed line shows an orthogonal system.

3 Decorrelating Decision Feedback Equalization

Combining decorrelating decision–feedback equalizatione with channel coding, there are two main implementations one can think about. The first one, called *direct feedback* in the following, is simply feeding the soft–output signals of the decorrelating decision–feedback detector [2] into the decoders. This method substantially suffers from bit errors. In comparison with uncoded transmission the problem of incorrect decisions is much more severe, as the average energy per symbol is significantly smaller if channel coding is applied. The second one is depicted in Fig. 3 for the two users' case. The cancellation

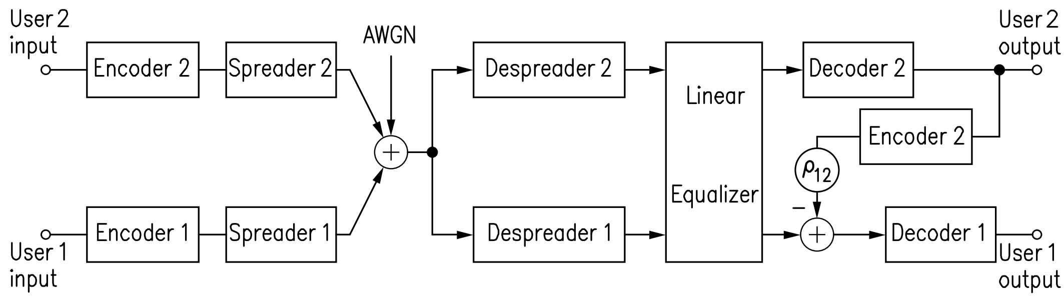

Figure 3: Structure of decision–feedback decoding for two users.

of interfering signals is performed after the decoding process. This provides (almost) error–free feedback.

In this context, that is the evaluation of maximum spectral efficiency, direct feedback is not considered further on, as there are several reasons limiting its sense. The only advantages of direct feedback are its obviously lower complexity, as re–encoding is not required, and the shorter processing delay. In contrast to processing delay, algorithmic delay does not differ in both cases if transmission is at least block–synchronous[2]. This is, as no user can be decoded until the whole codeword has been received. Then, all users

[2]That is, the time–shifts between the codewords differ only by few symbol periods.

can be decoded in arbitrarily short time if processing is arbitrarily fast. As processing may not be fast enough for high speed data transmission and/or a high number of users, the processing delay can be a real advantage of direct feedback. However, for codes operating close to channel capacity the algorithmic delay is expected to dominate the overall data delay. The gain in processing delay by reduced complexity is even negligible, as reencoding is very simple compared to the decoding procedure.

In contrast to zero–forcing equalization, decorrelating decision–feedback is asymetric in user direction. That is, the users do not operate under the same conditions. Some users are more and some are less disturbed by multiuser interference *in the average*. This fact has severe impact on cellular communications design, see e.g. [13, 14]. It yields that the signals of users transmitting at identical signal–to–noise ratios may be characterized by different signal–to–distortion ratios after multiuser equalization. This implies different average channel capacities among the users. Vice versa, users with equal average channel capacities require different signal–to–noise ratios. Although there are many cases with different joint rate *and* signal–to–noise ratio distributions among the users, we distinguish only the two cases previously mentioned in this paragraph, i.e. the equal rate and the equal power case.

3.1 Equal Powers

In the following we assume that all users are received with identical signal–to–noise ratios, i.e. their ratio of energy per symbol to noise power density is fixed. This enables to obtain the average capacity of decorrelating decision–feedback equalization from averaging the average capacity of the decorrelating detector with k users given by (9), see Proposition 1, over a discrete uniform distribution of users $1 \leq k \leq K$.

Proposition 2 *The average channel capacity of a CDMA–channel with K users of unit power which is disturbed by complex AWGN of variance σ^2 and equalized according to the zero–forcing decision–feedback criterion is given by*

$$\overline{C}_{\text{ZFDF1}} = \log_2\left(1+\frac{1}{\sigma^2}\right) - \frac{K-1}{K\ln(2)}\sum_{n=1}^{\infty}\frac{(K)_n}{n(n+1)(L)_n}\left(\frac{1}{1+\sigma^2}\right)^n \tag{14}$$

and bounded by

$$\frac{K-1}{K}\,\mathrm{f}\left(\frac{K}{L\,(1+\sigma^2)}\right) > \overline{C}_{\text{ZFDF1}} - \log_2\left(1+\frac{1}{\sigma^2}\right) > \frac{K-1}{K}\,\mathrm{f}\left(\frac{K+1}{L\,(1+\sigma^2)}\right) \tag{15}$$

with

$$\mathrm{f}(x) \triangleq \left(1-\frac{1}{x}\right)\log_2(1-x) - \frac{1}{\ln(2)} \tag{16}$$

if the spreading sequences are uniformly distributed on the $(L-1)$–dimensional surface of an L–sphere.

The proof is omitted in this conference version of the paper. It can be found in [12, 13].

Unfortunately, (14) does not illustrate the influence of the parameters K, L, σ^2 denoting number of users, spreading factor, and noise variance, respectively, very well. For this purpose, it is more helpful to examine the asymptotic behavior, i.e. the number of users grows over all bounds, but the load $\zeta \triangleq K/L$ remains constant. Asymptotically, both bounds given in Proposition 2 coincide yielding

$$\lim_{K=\zeta L\to\infty} \overline{C}_{\text{ZFDF1}} = \log_2\left(1+\frac{1-\zeta}{\sigma^2}\right) - \frac{1+\sigma^2}{\zeta}\log_2\left(1-\frac{\zeta}{1+\sigma^2}\right) - \frac{1}{\ln(2)}. \tag{17}$$

Eq. (14) shows that the asymptotic average capacity is a strictly decreasing function of the number of users K. The number of users is upper bounded by the spreading factor, if zero–forcing equalization is addressed. This means, that the minimal asymptotic average capacity is achieved for $K = L$ and given by

$$\min_{\zeta\leq 1}\lim_{K=\zeta L\to\infty}\overline{C}_{\mathrm{ZFDF1}} = \left(1+\sigma^2\right)\log_2\left(1+\frac{1}{\sigma^2}\right)-\frac{1}{\ln(2)}. \tag{18}$$

Surprisingly, the asymptotic loss in capacity, i.e. the gap between orthogonal sequences and randomly chosen ones, is less than *one nat* (about 1.44 bits)

$$\min_{\zeta<1}\lim_{K=\zeta L\to\infty}\log_2\left(1+\frac{1}{\sigma^2}\right)-\overline{C}_{\mathrm{ZFDF1}} = \frac{1}{\ln(2)}-\sigma^2\log_2\left(1+\frac{1}{\sigma^2}\right). \tag{19}$$

Obviously, it has to vanishes for $\sigma^2 \to \infty$ and achieves the maximal value of one nat for $\sigma^2 \to 0$.

Comparing this with the results found for the optimum joint demodulator, see [3], we find that decorrelating decision–feedback equalization is asymptotically optimal if the number of users equals the spreading factor and the powers of the users are identical, but their rates differ. Moreover, it is the counterpart in multiuser equalization to a result found by Price [15, 16] for equalization of intersymbol interference where zero–forcing decision–feedback equalization is also asymptotically optimum as $\sigma^2 \to 0$.

Considering spectral efficiency instead of average capacity, a low number of users cannot be optimum any longer. With the help of (5) and (6), spectral efficiency is plotted as a function of power efficiency in Fig. 4 for $K \to \infty$. Note that the loss in spectral efficiency is larger than one nat for unit load. This is, as spectral efficiency is plotted versus the energy per *bit* which depends itself on the average capacity. In contrast, (19) is based on the energy per symbol. Nevertheless, (19) can be used to calculate the asymptotic loss in power efficiency. As the optimum load converges to 1 for large spectral efficiencies, cf. Figs. 4 or 5, the asymptotic power loss is bounded by

$$V_{\mathrm{ZFDF1}}(\Gamma) = \frac{\min\limits_{\zeta}\left(\dfrac{E_\mathrm{b}}{N_0}\right)_{\mathrm{ZFDF1}}}{\dfrac{2^\Gamma-1}{\Gamma}} < \frac{\dfrac{2^{\Gamma+1/\ln 2}-1}{\Gamma+1/\ln 2}}{\dfrac{2^\Gamma-1}{\Gamma}} = \mathrm{e}+\mathcal{O}\left(\Gamma^{-1}\right). \tag{20}$$

As Ineq. (20) which becomes equality for $\Gamma \to \infty$ shows, the loss of only one nat in spectral efficiency — which obviously corresponds to a factor e in power efficiency — is achieved at least asymptotically for $\Gamma \to \infty$, i.e. $\frac{E_\mathrm{b}}{N_0} \to \infty$.

3.2 Equal Rates

In contrast to the equal power case discussed in the previous section, the preassumption of equal rates prohibits an explicit expression giving the average capacity, as the problem of optimal power assignment cannot be solved analytically for finite spreading gain, as (3) cannot be resolved with respect to C. Nevertheless, the average capacity can be bounded with the help of Proposition 1

$$\log_2\left(1+\frac{L-k}{L}\cdot\frac{E_k}{N_0}\right) < \overline{C}_{\mathrm{ZFDF2}} < \log_2\left(1+\frac{L-k+1}{L}\cdot\frac{E_k}{N_0}\right), \quad \forall 1\leq k\leq K \tag{21}$$

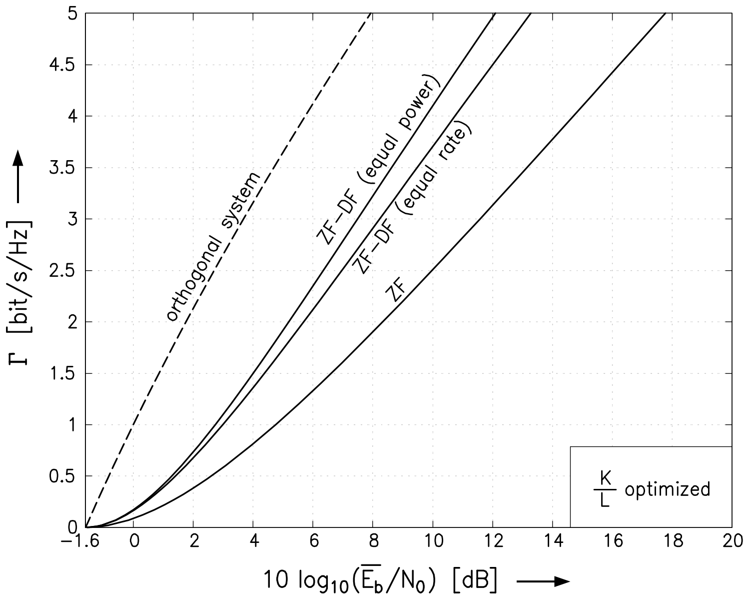

Figure 4: Power–bandwidth–plane for zero–forcing CDMA equalization with and without decision–feedback for optimized load. For comparison the dashed line shows an orthogonal system.

yielding bounds for the users' individual signal–to–noise ratios

$$\frac{2^{\overline{C}_{\mathrm{ZFDF2}}}-1}{1-(k-1)/L} < \frac{E_k}{N_0} < \frac{2^{\overline{C}_{\mathrm{ZFDF2}}}-1}{1-k/L}. \tag{22}$$

The energies per symbol and per bit averaged over all users are then bounded by

$$\frac{2^{\overline{C}_{\mathrm{ZFDF2}}}-1}{K}\sum_{k=1}^{K}\frac{L}{L-k+1} < \frac{\overline{E_\mathrm{s}}}{N_0} < \frac{2^{\overline{C}_{\mathrm{ZFDF2}}}-1}{K}\sum_{k=1}^{K}\frac{L}{L-k} \tag{23}$$

and

$$\frac{2^{\Gamma L/K}-1}{\Gamma}\ln\left(\frac{L+1}{L-K+1}\right) < \left(\frac{\overline{E_\mathrm{b}}}{N_0}\right)_{\mathrm{ZFDF2}} < \frac{2^{\Gamma L/K}-1}{\Gamma}\ln\left(\frac{L-1}{L-K-1}\right), \tag{24}$$

respectively. In the asymptotic case both bounds coincide. Thus, we have

$$\lim_{K=\zeta L\to\infty}\left(\frac{\overline{E_\mathrm{b}}}{N_0}\right)_{\mathrm{ZFDF2}} = -\ln(1-\zeta)\,\frac{2^{\Gamma/\zeta}-1}{\Gamma}. \tag{25}$$

This relationship is illustrated for optimized load ζ in Fig. 4. The gap to the orthogonal bound is smaller than without decision–feedback, but slightly larger than in the equal power case. The asymptotic loss is bounded by

$$V_{\mathrm{ZFDF2}}(\Gamma) = \frac{\min\limits_{\zeta}\left(\frac{\overline{E_\mathrm{b}}}{N_0}\right)_{\mathrm{ZFDF2}}}{\frac{2^\Gamma-1}{\Gamma}} = \frac{\inf\limits_{c}\lim\limits_{\zeta\to\frac{\Gamma}{\Gamma+c}}\left(\frac{\overline{E_\mathrm{b}}}{N_0}\right)_{\mathrm{ZFDF2}}}{\frac{2^\Gamma-1}{\Gamma}} \tag{26}$$

for some positive constant c. With (25), the asymptotic loss can be shown by Taylor expansions to be

$$V_{\mathrm{ZFDF2}}(\Gamma) = \min_{c>0} 2^c \ln\left(1+\frac{\Gamma}{c}\right)\left(1+\mathcal{O}\left(2^{-\Gamma}\right)\right). \tag{27}$$

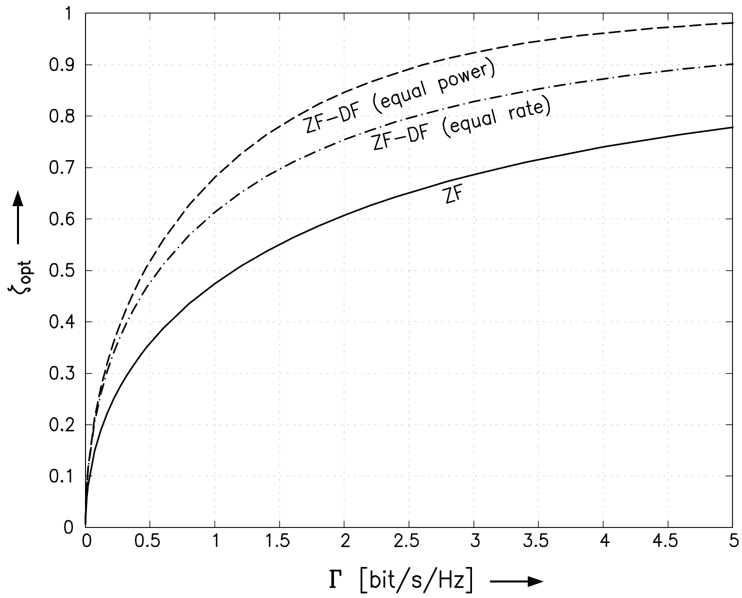

Figure 5: Ratios of users to spreading factor ζ_{opt} that maximize spectral efficiency Γ_{ZF} (solid line), Γ_{ZFDF1} (dashed line) and Γ_{ZFDF2} (dash–dotted line), respectively.

The asymptotic loss of the decorrelator is proportional to spectral efficiency, see (13), while it is constant for decorrelating decision–feedback with equal powers, cf. (20). For equal rates, it is shown in (27) to be proportional to the logarithm of spectral efficiency. This is a significant improvement compared to decorrelation without decision–feedback and only a small drawback to the equal power case.

4 Summary and Conclusions

Decorrelating multiuser detectors with and without decision–feedback have been analyzed. The performance variability of the decorrelating detector has been enabled to be derived analytically and the average capacity has been given for finite number of users. Moreover, the spectral efficiency has been shown to decrease slightly with the spreading factor, if the system load is fixed. Thus, it is *not* necessary to provide large spreading factors in order to neglect the degradation caused by performance variability. Decorrelating decision–feedback detectors have been found to achieve higher capacity when they are operated with equal power users rather than with equal rate users. This suggests the application of a time–sharing or rate–splitting [17] protocol to exploit all theoretical resources.

Acknowledgement

The author would like to thank Johannes Huber, Sergio Verdú, Peter Schramm and Alexander Lampe for stimulating hints and valuable discussions.

References

[1] Sergio Verdú. Minimum probability of error for asynchronous Gaussian multiple–access channels. *IEEE Transactions on Information Theory*, IT–32(1):85–96, January 1986.

[2] Sergio Verdú. *Multiuser Detection.* Cambridge University Press, New York, 1998.

[3] Sergio Verdú and Shlomo Shamai (Shitz). Spectral efficiency of CDMA with random spreading. *IEEE Transactions on Information Theory*, 45(2):622–640, March 1999.

[4] C. E. Shannon. A mathematical theory of communications. *The Bell System Technical Journal*, 27:623–656, October 1948.

[5] Thomas M. Cover and Joy A. Thomas. *Elements of Information Theory.* John Wiley & Sons, New York, 1991.

[6] Andrew J. Viterbi. *CDMA.* Addison–Wesley, 1995.

[7] A.G. Burr. Performance of linear separation of CDMA signals with FEC coding. In *Proc. of IEEE International Symposium on Information Theory (ISIT)*, page 354, Ulm, Germany, June/July 1997.

[8] Branimir R. Vojcic. Information theoretic aspects of multiuser detection. In *Proc. of Symposium on Interference Rejection and Signal Separation for Wireless Communications (IRSS'97)*, Washington D.C., March 1997.

[9] David Tse and Stephen Hanly. Linear multiuser receivers: Effective interference, effective bandwidth and user capacity. *IEEE Transactions on Information Theory*, 45(2):641–657, March 1999.

[10] Upamanyu Madhow and Michael L. Honig. On the average near–far resistance for MMSE detection of direct sequence CDMA signals with random spreading. *To appear in IEEE Transactions on Information Theory*, 45, September 1999.

[11] Ralf R. Müller, Peter Schramm, and Johannes B. Huber. Spectral efficiency of CDMA systems with linear interference suppression (in German). In *Proc. of IEEE Workshop Kommunikationstechnik*, pages 93–97, Ulm, Germany, January 1997. ITUU–TR–1997/01. English version available via http://www-nt.e-technik.uni-erlangen.de/~dcg.

[12] Ralf R. Müller. Multiuser receivers for randomly spread signals: Fundamental limits with and without decision–feedback. *Submitted to IEEE Transactions on Information Theory*, June 1998.

[13] Ralf Müller. *Power and Bandwidth Efficiency of Multiuser Systems with Random Spreading.* Shaker–Verlag, Aachen, 1999.

[14] Ralf R. Müller and Johannes B. Huber. Capacity of cellular CDMA systems applying interference cancellation and channel coding. In *Proc. of Communication Theory Mini Conference (CTMC) at IEEE Globecom*, pages 179–184, Phoenix, AZ, November 1997.

[15] R. Price. Nonlinear feedback equalized PAM versus capacity for noisy filter channels. In *Proc. of IEEE International Conference on Communications (ICC)*, pages 22.12–22.17, Philadelphia, PA, June 1972.

[16] M. Vedat Eyuboğlu. Detection of coded modulation signals on linear severely distorted channels using decision–feedback noise prediction with interleaving. *IEEE Transactions on Communications*, 36(4):401–409, April 1988.

[17] Bixio Rimoldi and Rüdiger Urbanke. A rate–splitting approach to the Gaussian multiple–access channel. *IEEE Transactions on Information Theory*, 42(2):364–375, March 1996.