Multiuser Detection for Continuous Phase CDMA

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Abstract — **This paper studies linear multiuser detection for code-division multiple-access (CDMA) with continuous-time constant envelope and continuous phase. The restriction to continuous phase is found to cause negligible degradation of multiuser efficiency when detected by means of linear multiuser detectors. This holds for both classical linear multiuser detection as well as for multistage detection based on universal weight design.**

1 INTRODUCTION

Constant envelope (CE) modulation offers significant advantages compared to amplitude modulation with respect to the design and linearity demands of the radio-frequency (RF) hardware, in general, and of the power amplifier, in particular.

The combination of continuous phase (CP) modulation and CDMA was first addressed in [1] in a very general setting. In the sequel, various ways to combine CP modulation with CDMA were given further attention [2, 3, 4, 5, 6, 7]. All these methods suffer from one or more of the following drawbacks: The signal has spectral sidelobes, the signal is colored, linear multiuser detection suffers from performance degradation. In [8], the author proposed to use continuous phase random chips to generate time-limited constant envelope spreading sequences. By means of an iterative process spreading waveforms could be found whose spectral sidelobes are suppressed by more than 60dB while keeping the spectrum approximately white in the passband. Reference [8] also claimed that linear multiuser detection could be implemented easily without substantial performance degradation, but neither proof nor empirical evidence was given due to space limitations. A quantitative analysis of linear multiuser detection for the method proposed in [8] is the focus of this work.

2 SYSTEM MODEL

Consider a conventional CDMA system, see e.g. [9], with time-limited spreading waveforms. The spreading waveforms are optimized iteratively by repeated spectral shaping and renormalization to constant envelope until they achieve suitable spectral properties, i.e. high stopband attenuation and approximately constant passband spectrum. In order to avoid phase jumps at the symbol transitions, multiple spreading waveforms are assigned to each other. All spreading waveforms of a user are very similar and significantly differ only at the first and/or last chips. The choice of the spreading waveform is symbol dependent and made such that phase jumps are avoided. An algorithm to create such a set of waveforms is given in [8].

Let $s_{k,\mu}(t)$ and $b_{k,\mu}$ respectively denote the spreading waveform and data symbol of user k at symbol clock cycle μ. Let $h_k(t)$ denote the channel impulse response of user k. Define the virtual spreading waveform

$$c_{k,\mu}(t) = h(t) * s_{k,\mu}(t) \tag{1}$$

as the convolution of the channel impulse response and the actual spreading waveform. Then, the received signal is given by

$$y(t) = \sum_{k=1}^{K} \sum_{\mu} c_{k,\mu}(t-\mu T) b_{k,\mu} + n(t) \tag{2}$$

with K, T, and $n(t)$ denoting the number of users, the symbol interval, and additive white Gaussian noise, respectively.

3 MULTIUSER DETECTION

Decision-directed multiuser detection is invariant to whether linear CDMA or the CE-CDMA proposed in [8] is considered. Therefore, we restrict the following considerations to linear multiuser detection.

Consider now the case of synchronized users on a non-dispersive channel, i.e. $h(t) = \delta(t)$. Let $\boldsymbol{s}_{k,\ell}$ denote the ℓ^{th} spreading waveform of user k represented as a vector of time samples. Furthermore, assume that the data symbols of user k are chosen with equal probability from a regular M-ary phase-shift keying constellation $\mathcal{B} = \{z \in \mathbb{C} : z^M = 1\}$. Then, with standard arguments of linear algebra, see e.g. [10, Sec. III.A], the linear minimum mean-squared error (LMMSE) detector for user k is given by

$$\overline{\boldsymbol{s}}_k^\dagger \left(\sigma^2 \mathbf{I} + \frac{1}{M^2} \sum_{k' \neq k} \sum_{i=1}^{M^2} \boldsymbol{s}_{k',i} \boldsymbol{s}_{k',i}^\dagger \right)^{-1} \boldsymbol{y} \tag{3}$$

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where

$$\overline{\boldsymbol{s}}_k = \frac{1}{M^2}\sum_{\ell=1}^{M^2} \boldsymbol{s}_{k,\ell}, \tag{4}$$

$\boldsymbol{y}$ is the received signal in chip space and σ^2 is the variance of the appropriately band-limited and sampled noise. The signal-to-interference-and-noise ratio (SINR) of user k is given by [10, Sec. III.A]

$$\gamma_k = \overline{\boldsymbol{s}}_k^\dagger \left(\sigma^2\mathbf{I} + \frac{1}{M^2}\sum_{k'\neq k}\sum_{i=1}^{M^2} \boldsymbol{s}_{k',i}\boldsymbol{s}_{k',i}^\dagger\right)^{-1} \overline{\boldsymbol{s}}_k. \tag{5}$$

With help of the matrix inversion lemma it can also be expressed as

$$\gamma_k = \frac{1}{M^2}\mathbf{1}^\dagger\left[\mathbf{I} - \boldsymbol{A}\right]^{-1}\boldsymbol{A}\mathbf{1} \tag{6}$$

with

$$\boldsymbol{A} = \boldsymbol{S}_k^\dagger\left(\sigma^2\mathbf{I} + \boldsymbol{S}\boldsymbol{S}^\dagger\right)^{-1}\boldsymbol{S}_k \tag{7}$$

where $\boldsymbol{S}_k = [\boldsymbol{s}_{k,1}, \boldsymbol{s}_{k,2}, \ldots, \boldsymbol{s}_{k,M^2-1}, \boldsymbol{s}_{k,M^2}]/M$, $\boldsymbol{S} = [\boldsymbol{S}_1, \boldsymbol{S}_2, \ldots, \boldsymbol{S}_{K-1}, \boldsymbol{S}_K]$ and $\mathbf{1}$ denoting the all one vector.

For asynchronous users and dispersive channels, a generalization of (3) to (6) can be derived straightforwardly by means of block matrices. To avoid to interrupt the flow of this paper with awkward notation, we leave this exercise to the interested reader.

Due to its linearity, a linear multiuser detector cannot track which of the signature waveforms of a user is used for communicating a certain data symbol. By this lack of knowledge, the detector (3) needs to cope with the expected signature waveform (4). Thus, the signature waveforms of a user should be very similar for two reasons: 1) Differences in the signature waveforms reduce the average of the signature waveforms in (4) and thus reduce the received signal energy. 2) The more the signature waveforms of an interfering user differ among each other the larger the subspace spanned by the interference from this user.

The chips which differ within the set of signature sequences of a user are called *guard chips*. In the proposed CE-CDMA, *guard chips* are deployed to guarantee for smooth phases at symbol transitions. The number of required guard chips hardly depends on the spreading factor. Thus, the relative overhead due to guard chips decreases with growing spreading factor.

4 NUMERICAL RESULTS

We consider the performance of the LMMSE detector for random spreading and various numbers of users and spreading factor. We aim to compare the performance of CDMA with continuous phase against the performance of pure random CDMA with root-raised cosine chip pulses with roll-off factor $\alpha = 0.22$ for various spreading factors. We expect CE signature waveforms to perform the better the larger the spreading factor N, since the guard chips account for a smaller portion of the signature sequence.

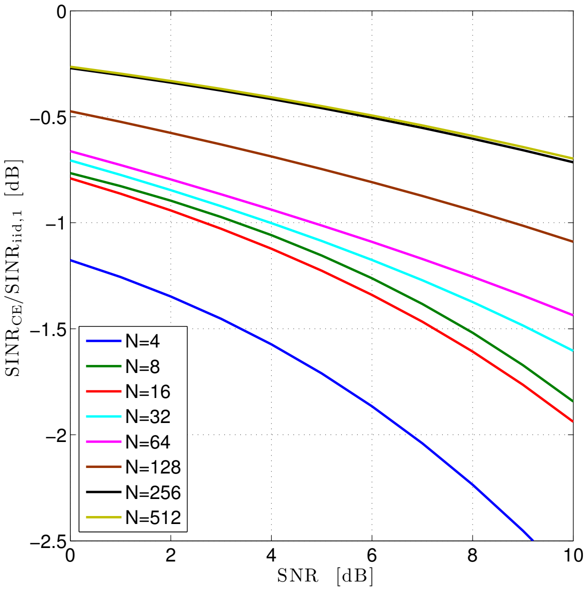

Figure 1: Ratio of SINR of CE random waveform sets for QPSK to SINR of i.i.d. random waveforms for $N = 4, 8, 16, 32, 64, 128, 256, 512$ and $K = \frac{3}{4}N$ shown by the blue, green, red, cyan, magenta, brown, black, and yellow lines, respectively.

In Fig. 1 we show the SINR of a CDMA system with CE signature waveforms relative to the SINR of a CDMA system with root-raised cosine pulses for $M = 4$, i.e. quaternary phase shift keying (QPSK). The basic CE waveforms are obtained by 10^4 primary iterations according to [8, Eqs. (8) and (9)] and the sets of $M^2 = 16$ waveforms per user are found by 10^3 secondary iterations according to [8, Eqs. (11) and (12)][1]. Depending on SNR and spreading factor, the degradation ranges from more than 2.5 dB to less than 0.3 dB. In general, degradation increases with SNR. As expected, larger spreading factors give better results with the surprising exception of $N = 8$ and $N = 16$. The exception results from the fact that going for $N = 8$

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[1] The other parameters for generation of the CE signature waveforms were chosen as in [8], i.e. Markov sequences with state transition probability $\frac{1}{3}$, half-sine-wave pulses as initial choice and filtering with root-raised cosine-pulses with roll-off factor 0.1 and Nyquist frequency $1.1N/T$

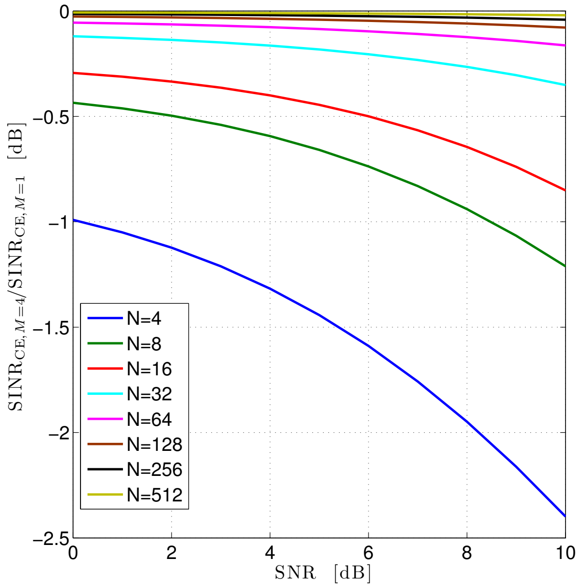

Figure 2: Ratio of the SINR of CE random waveform sets ($M = 4$) to the SINR of CE-CDMA when using a single signature waveform per user for $N = 4, 8, 16, 32, 64, 128, 256, 512$ and $K = \frac{3}{4}N$ shown by the blue, green, red, cyan, magenta, brown, black, and yellow lines, respectively.

to $N = 16$, the properties of i.i.d. sequences improve more than the properties of CE signature waveforms.

The comparison, in Fig. 1 is not fair, since both system do not occupy precisely the same bandwidth. The spectra of the CE-CDMA systems are shown in [8, Fig. 3] for various spreading factors. Defining the bandwidth by means of 50 dB stop-band attenuation, CE-CDMA systems with $N \geq 64$ occupy less bandwidth than conventional CDMA systems whose bandwidth[2] stretches from $-(1+\alpha)N/T/2$ to $+1(1+\alpha)N/T/2$. For small spreading factors, however, CE-CDMA systems occupy larger bandwidth due to spectral regrowth. The severe sidelobes for small spreading factors results from the strict time-limitation of the signature waveforms and time-frequency uncertainty.

In order to solely investigate the effect of multiple spreading sequences per user without the complication of incompatible spectrum occupation, we compare CE-CDMA with sets of M^2 spreading sequences per user against CE-CDMA with only a single spreading sequence in Fig. 2. Note that using only a single spreading sequence per user results in phase jumps at symbol transitions and is not of practical use. This comparison is, therefore, only of theoretical interest. It demonstrates that the loss decreases with growing spreading factor and vanishes in the limit $N \to \infty$. Furthermore, the loss is within fractions of decibels for spreading factors as low as $N = 8$.

A fair comparison against conventional CDMA with root-raised cosine pulses is tricky. First, let us agree on a definition of spectral occupation by defining the two-sided bandwidth B as the width of the spectrum where the power density is above a certain threshold, say -30 dB. Second, let the Nyquist frequency f_{N} of the root-raised cosine pulse differ from the chip rate in such a way that the total bandwidths of the two systems match, i.e. $1.22f_{\mathrm{N}} = B/2$. Such a comparison might look fair at first glance. In fact, it is fairer than the one in Fig. 1, but it is still not totally fair. It is biased towards the conventional CDMA system due to the following reasons: The reduced Nyquist frequency of the conventional CDMA system creates inter-chip interference. This does not pose an obstacle for multiuser detection. In fact, it actually aids multiuser detection, as will be argued in the following paragraph. However, it severely increases the peak-to-average power ratio. In practice, conventional CDMA system try to avoid inter-chip interference.

Synchronous CDMA systems cannot utilize excess bandwidth (bandwidth exceeding the limit N/T) [11], i.e. the excess bandwidth is wasted bandwidth. However, when reducing the Nyquist frequency below $N/T/2$, the excess bandwidth becomes partially (eventually totally) useful to span dimensions in signal space and aid multiuser detection. Therefore, the comparison outlined above unfairly favors conventional CDMA. Nevertheless, CE-CDMA achieves considerable performance in this comparison, see Fig. 3. Particularly, for high spreading factors, CE-CDMA hardly falls behind conventional CDMA in terms of SINR.

5 CONCLUSIONS

We have demonstrated how to apply LMMSE multiuser detection to CE-CDMA systems deploying multiple data-dependent signature waveforms per user. We find that CE comes at small to no cost in terms of SINR for moderate to high spreading factors. The generalizations of standard LMMSE detectors are minor and do not pose an obstacle for complexity saving implementations by means of

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[2] In the published manuscript, a factor of 2 is lacking at several places. In this corrected version, the missing factors are marked in red.

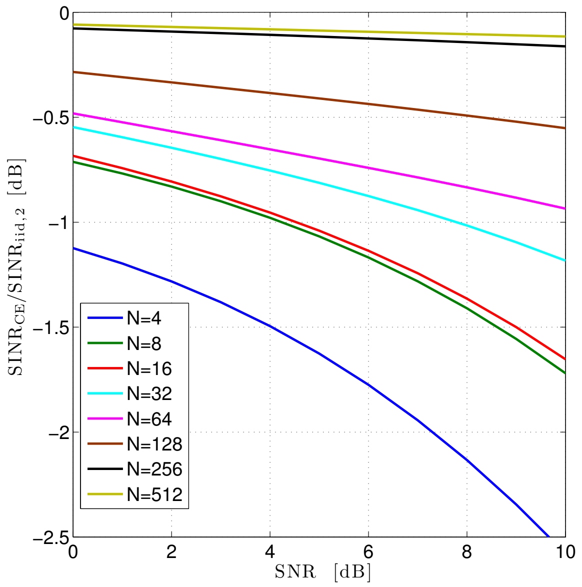

Figure 3: Same ratio as in Fig. 1, but for i.i.d. random waveforms with bandwidths constrained to the respective −30 dB bandwidths of the CE signature waveforms.

multistage detectors[3].

Acknowledgments

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[3] See [12] and [13] for an overview on multistage detectors for synchronous and asynchronous CDMA systems, respectively.