Multiuser Interference Mitigation with Multistage Detectors: Design and Analysis for Unequal Powers

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Abstract

Asymptotic multistage detectors reach a good compromise between performance and complexity for large size systems. However, design criteria and performance analysis are available only for additive Gaussian noise channels and equal received power users. In this work we provide equations to precisely calculate the asymptotic weighting of multistage detectors for random CDMA satisfying the individual and joint LMMSE criteria in the projection subspace for scenarios with unequal powers. Additionally, a general expression of the SINR achievable at the filter output as system size grows large is derived. Such equation can be applied to any multistage detector. We specialize this result to both the individual and joint LMMSE multistage detector with asymptotic weighting. We show that the individual LMMSE detector outperforms the joint LMMSE detector in the case of unequal received powers while the detectors are equivalent in the case of equal received powers. Performance degradation of the asymptotic multistage detectors, when used in finite large systems, is negligible compared to the correspondent optimum detectors for finite systems.

1 Introduction

Allowing for mutual interference among multiple transmissions, multiple access communication systems can achieve a significant increase in spectral efficiency. However, dealing with crosstalk considerably increases the complexity of receivers, sometimes even far above what is feasible in practice. Therefore, there is a strong demand for the discovery of algorithms that simplify the signal processing required for theoretically optimum communication. The Linear Minimum Mean Square Error (LMMSE) detector has been proposed with the goal of finding an acceptable compromise between performance and complexity. It is also capable of providing soft information to a subsequent channel decoder. It applies a linear mapping to the output of the matched filter bank, so as to reduce the Multiple Access Interference (MAI). The LMMSE detector yields substantial improvements in performance, while maintaining a lower complexity than the optimum detector investigated in [1]. However computing this mapping in real-time is difficult since the MAI is time-varying. Namely, the LMMSE detector requires the inversion of matrices that are at least of size $\min(K,N) \times \min(K,N)$, where K is the number of active users and N the spreading factor. Obviously its complexity is prohibitive for real-time applications when the system size is large. The multistage detectors aim to efficiently implement the LMMSE detector approximating the desired matrix inversion by a matrix polynomial expansion [2, 3, 4, 5]. As shown in [3, 4], for this class of detectors, a low number of stages is sufficient for near-LMMSE performance, even as system size grows large. However, the polynomial approximations of the LMMSE detector are helpful in practice only if the polynomial coefficients can be calculated more easily than performing matrix inversion. As the coefficients depend on the eigenvalues of the correlation matrix, they are not easy to calculate either. The practical usefulness of this class of detectors derives from the drastic reduction in complexity that can be achieved at the cost of a slight degradation in performance [6] under the assumption of random spreading sequences by approximating optimum weighting by asymptotic weighting [5]. Several different criteria for the weight optimization of multistage detectors have been proposed and their asymptotic performance has been analyzed in the case of equal received powers. However, asymptotic weighting is unknown for unequal received powers and performance has been analyzed only for the optimality criterion proposed in [4]. In this work we focus our attention on multistage detectors in case of unequal received powers providing equations to precisely calculate asymptotic weighting. Namely we design two asymptotic detectors that satisfy the optimality criteria of individual and joint LMMSE in the projection subspace $\chi_M(\mathbf{SA})$ and we refer to them respectively as the asymptotic individual LMMSE detector in $\chi_M(\mathbf{SA})$ and asymptotic joint LMMSE detector in $\chi_M(\mathbf{SA})$. For both the considered detectors the asymptotic weighting is independent of the channel and spreading sequence realizations and depends only on a small set of parameters. This fact substantially reduces the complexity of the weight computation. Additionally, we provide general equations for the performance of any multistage detector in terms of MSE and SINR. By calculating the asymptotic weights for both the individual and the joint LMMSE detector we specialize the general equation for the asymptotic SINR to them. Thus the performance of the two detectors can be easily compared. While their asymptotic performance coincides in the case of equal powers it differs for unequal powers and the individual LMMSE detector in $\chi_M(\mathbf{SA})$ outperforms its counterpart.

Simulations show that the difference in performance between the two LMMSE detectors for finite systems and their respective approximations using asymptotic weighting is negligible.

2 System Model and Notations

Let us consider a synchronous CDMA communication system with spreading factor N and K physical users. Additionally let us assume unequal powers of the users at the receiver and additive Gaussian noise. Then, the baseband equivalent signals at the chip matched filter output are given by:

$$\mathbf{r} = \mathbf{SAb} + \mathbf{n} \tag{1}$$

where $\mathbf{r}$ is the N-dimensional received vector, $\mathbf{b}$ is the K-dimensional vector of transmitted symbols, $\mathbf{S}$ is the $N \times K$ matrix of signature sequences and $\mathbf{A}$ is the $K \times K$ diagonal matrix of amplitudes. $\mathbf{n}$ is the vector noise with covariance matrix $\sigma^2\mathbf{I}$. The transmitted symbols belong to a finite alphabet in $\mathbb{C}$, they are zero mean and satisfy the relation $E\{\mathbf{bb}^H\} = \mathbf{I}$. The elements of $\mathbf{S}$ satisfy one of the following statements:

A-1 they are i.i.d. with $|s_{11}| \leq \frac{\log N}{\sqrt{N}}$, $\mathrm{E}\{s_{11}\} = 0$, $\mathrm{E}\{|s_{11}|^2\} = \frac{1}{N}$ and finite fourth moment;

A-2 they are i.i.d., Gaussian with $\mathrm{E}\{s_{11}\} = 0$ and $\mathrm{E}\{|s_{11}|^2\} = \frac{1}{N}$.

The additive noise is white complex Gaussian with $\mathrm{E}\{\mathbf{n}^H\} = \mathbf{0}$ and $\mathrm{E}\{\mathbf{nn}^H\} = \sigma^2\mathbf{I}$. The matrix $\mathbf{A}$ is completely known and the sequence of the eigenvalue distribution of $\mathbf{A}^H\mathbf{A}$ converges almost surely, as $K \to \infty$, to a nonrandom distribution function with upper bounded support. Hereinafter we denote by $F_{|\mathbf{A}|^2}(\lambda)$ the asymptotic distribution. Furthermore, we denote with $\chi_M(\mathbf{SA})$ the subspace in the space of the $K \times N$ matrices in $\mathbb{C}$ defined by

$$\chi_M(\mathbf{SA}) = \mathrm{span}\{\mathbf{R}^m\mathbf{A}^H\mathbf{S}^H\}|_{m=0}^{M-1} \tag{2}$$

where $\mathbf{R} = \mathbf{A}^H\mathbf{S}^H\mathbf{SA}$. As shown in [2], the true LMMSE detector is in $\chi_K(\mathbf{SA})$

3 Asymptotic Detector Design

Applying the MMSE criterion jointly to all the users or individually to each user we obtain respectively the joint LMMSE detector in $\chi_M(\mathbf{SA})$ with structure $\mathbf{L} = \sum_{m=0}^{M-1} w_m\mathbf{R}^m\mathbf{A}^H\mathbf{S}^H$ and the individual LMMSE detector in $\chi_M(\mathbf{SA})$ with structure $\mathbf{L} = \sum_{m=0}^{M-1}\mathbf{W}_m\mathbf{R}^m\mathbf{A}^H\mathbf{S}^H$ where $\mathbf{W}_m = \mathrm{diag}(w_{1m}, w_{2m}, \ldots, w_{Km})$. The optimum scalar weights w_m minimizing the MSE $\mathrm{E}\left\{\left\|\sum_{m=0}^{M-1} w_m\mathbf{R}^m\mathbf{A}^H\mathbf{S}^H\mathbf{r} - \mathbf{b}\right\|\right\}$ for the joint LMMSE detector in $\chi_M(\mathbf{SA})$ are given by [2]:

$$\mathbf{w} = \mathbf{\Phi}^{-1}\mathbf{c} \tag{4}$$

where the M-dimensional vector $\mathbf{c}$ and the elements of the $M \times M$ matrix $\mathbf{\Phi}$ can be expressed in terms of the traces of the powers of $\mathbf{R}$ as $\Phi_{ij} = \mathrm{trace}(\mathbf{R}^{i+j}) + \sigma^2\mathrm{trace}(\mathbf{R}^{i+j-1})$ and $c_i = \mathrm{trace}(\mathbf{R}^i)$.

A recursive structure of the individual LMMSE detector in $\chi_L(\mathbf{SA})$ is provided in [4] [1] and it is equivalent to the following:

$$\mathbf{w}_k = \mathbf{\Phi}_k^{-1}\mathbf{c}_k \tag{5}$$

where[2] $(\mathbf{w}_k)_l = (\mathbf{W}_l)_{kk} = w_{kl}$, $(\mathbf{c}_k)_l = (\mathbf{R}^{l+1})_{kk}$ and $(\mathbf{\Phi}_k)_{lm} = (\mathbf{R}^{l+m})_{kk} + \sigma^2(\mathbf{R}^{l+m-1})_{kk}$. The same individually optimum filter could be obtained maximizing the sum of the SINR_k.

The asymptotic multistage detectors are based on the idea of approximating the weights of detectors in $\chi_M(\mathbf{SA})$ for large systems with the weights of the corresponding detectors as $K, N \to \infty$, keeping β constant.

The asymptotic weights of the joint LMMSE detector in $\chi_L(\mathbf{SA})$ satisfy

$$\mathbf{w}^\infty = (\mathbf{\Phi}^\infty)^{-1}\mathbf{c}^\infty \tag{6}$$

where $c_i^\infty = \mathrm{E}(\lambda^i)$, $\Phi_{ij}^\infty = \mathrm{E}(\lambda^{i+j}) + \sigma^2\mathrm{E}(\lambda^{i+j-1})$ and λ are the eigenvalues of $\mathbf{R}$.

Assuming the existence of the limits

$$\lim_{\substack{N,K\to\infty \\ \frac{K}{N}=\beta}} (\mathbf{R}^l)_{kk} = R_{kk,\infty}^l \qquad \forall k \in \mathbb{N}, \qquad 1 \leq l \leq M^2.$$

we can also consider the asymptotic individual MMSE detector in $\chi_M(\mathbf{SA})$ with weights

$$\mathbf{w}_k^\infty = (\mathbf{\Phi}_k^\infty)^{-1}\mathbf{c}_k^\infty \tag{7}$$

where $\mathbf{\Phi}_k^\infty$ is a matrix with elements $(\mathbf{\Phi}_k^\infty)_{lm} = R_{kk,\infty}^{l+m} + \sigma^2 R_{kk,\infty}^{l+m-1}$ and $\mathbf{c}_k^\infty$ is a vector such that $(\mathbf{c}_k^\infty)_j = R_{kk,\infty}^j$. The convergence of $(\mathbf{R}^l)_{kk}$ to a deterministic limit as $N, K \to \infty$ with $\frac{K}{N}$ constant is proven in the following theorem and the asymptotic value $R_{kk,\infty}^l$ is also provided.

[1] In [4] the individual LMMSE detector in $\chi_L(\mathbf{SA})$ is referred to as MultiStage Wiener Filter.

[2] $(.)_x$ denotes the element x of the argument.

$$\varphi(i_0,i_1,\dots i_{l-1}) = \begin{cases} 1 & \text{for } i_0=1,\ i_0+\sum_{j=1}^{l-1} j i_j = l \text{ and } \sum_{j=1}^{l-1}(j+1)i_j \le l \\ \frac{\left(\sum_{j=1}^{l-1} i_j\right)!}{\prod_{j=1}^{l-1} i_j!}\left(\sum_{r=1}^{\alpha-1} g_{\alpha-r}[\gamma-1]r + 2\sum_{r=1}^{\alpha} g_{\alpha+1-r}[\gamma-2]r + \sum_{r=1}^{\alpha+1} g_{\alpha+2-r}[\gamma-3]r\right) & \\ & \text{for } i_0>1,\ i_0+\sum_{j=1}^{l-1} j i_j = l \text{ and } \sum_{j=1}^{l-1}(j+1)i_j \le l \\ 0 & \text{else} \end{cases} \quad (3)$$

Theorem 1 *Let $\mathbf{A}$ be a $K\times K$ diagonal matrix in $\mathbb{C}$ with bounded elements and such that the sequence of the eigenvalue distribution of $\mathbf{A}^H\mathbf{A}$ converges almost surely, as $K\to\infty$, to a nonrandom distribution function with upper bounded support. Let $\mathbf{S}$ be an $N\times K$ matrix in $\mathbb{C}$ satisfying hypothesis A-1. If a_{kk} is the k-th element of $\mathbf{A}$, then, for any $k,l\in\mathbb{N}$, $(\mathbf{R}^l)_{kk}$ converges almost surely, as $N,K\to\infty$, with $\beta=\frac{K}{N}$ constant, to the deterministic quantity $R^l_{kk,\infty}$ depending on $|a_{kk}|^2$*

$$R^l_{kk,\infty} = \sum_{\substack{(i_0,i_1,\dots i_{l-1}):\\ i_0+\sum_{j=1}^{l-1} j i_j = l\\ \sum_{j=1}^{l-1}(j+1)i_j\le l}} \varphi(i_0,i_1,\dots i_{l-1})|a_{kk}|^{2i_0}\prod_{n=1}^{l-1}(\beta m^n_{\mathbf{R}})^{i_n}$$

where $m^k_{\mathbf{R}} = \mathrm{E}\left\{\frac{1}{K}\mathrm{tr}\left(\mathbf{R}^k\right)\right\}$ and $(i_0,i_1,\dots i_{l-1})$ is an l-tuple of nonnegative integers. $\varphi(i_0,i_1,\dots i_{l-1})$ are nonnegative integer coefficients given in (3). α and γ in (3) are defined by $\alpha=\sum_{j=1}^{l-1} i_j$ and $\gamma = l-\sum_{j=1}^{l-1}(j+1)i_j$ respectively. $\forall r,s>0$ $g_r[s]$ in (3) satisfies the recursive equation

$$g_r[s] = g_{r-1}[s] + g_r[s-1] \quad (8)$$

with initializing values

$$g_1[0]=1, \qquad g_r[0]=0, \qquad g_1[s]=1. \quad (9)$$

Furthermore, by convention, it is stated $\sum_{\substack{r=1\\t\ge1}}^{t} g_{t+1-r}[-1]r = 1$ and $\sum_{\substack{r=1\\t\ge1}}^{t} g_{t+1-r}[s]r = 0$ for $s<-1$. □

The proof of this theorem is in [7]. The moments $m^k_{\mathbf{R}}$ can be derived recursively according to the following theorem:

Theorem 2 *Let $\mathbf{A}$ and $\mathbf{S}$ be as in Theorem 1 and let $m^l_{|\mathbf{A}|^2}$ be the eigenvalue moments of the diagonal matrix $\mathbf{A}\mathbf{A}^H$, then the asymptotic eigenvalue moments of $\mathbf{R}$ are given by*

$$m^l_{\mathbf{R}} = \sum_{\substack{(i_0,i_1,\dots i_{l-1}):\\ i_0+\sum_{j=1}^{l-1} j i_j = l\\ \sum_{j=1}^{l-1}(j+1)i_j\le l}} \varphi(i_0,i_1,\dots i_{l-1}) m^{i_0}_{|\mathbf{A}|^2}\prod_{k=1}^{l-1}\left(\beta m^k_{\mathbf{R}}\right)^{i_k}$$

where the l-tuple $(i_0,i_1,\dots i_{l-1})$ and the coefficients $\varphi(i_0,i_1,\dots i_{l-1})$ are defined as in Theorem 1. For the initializing moment it results $m^1_{\mathbf{R}} = m^1_{|\mathbf{A}|^2}$. □

Theorem 2 is proven in [7].

Noting that for signals having the same power, i.e. $\mathbf{A}\mathbf{A}^H = \mathcal{P}\mathbf{I}$, $(\mathbf{R}^l)_{kk}$ converges almost surely to a value that does not depend on the index k, we obtain the following corollary:

Corollary 1 *Let $\mathbf{S}$ be as in Theorem 1 and let the $K\times K$ diagonal matrix $\mathbf{A}$ be such that $\mathbf{A}\mathbf{A}^H=\mathcal{P}\mathbf{I}$, then, for any $l,k\in\mathbb{N}$, $(\mathbf{R}^l)_{kk}$ converges almost surely, as $N,K\to\infty$ with $\frac{K}{N}$ constant to the non-random quantity*

$$(\mathbf{R}^l)_{kk} \xrightarrow{a.s.} m^l_{\mathbf{R}}.$$

□

For $\mathbf{A}\mathbf{A}^H=\mathcal{P}\mathbf{I}$ a closed-form expression of the eigenvalue moments is in [8].

Theorems 1 and 2 and Corollary 1 can be extended to the case of Gaussian distribution for the elements of $\mathbf{S}$ and can be rewritten with hypothesis A.2 instead of hypothesis A.1 using the following lemma:

Lemma 1 *Let $\mathbf{S}$ be an $N\times K$ matrix with statistically independent and identically Gaussian distributed elements with zero mean and variance $\mathrm{E}\left\{|s_{11}|^2\right\}=\frac{1}{N}$. Then, as $N,K\to\infty$ with $\frac{K}{N}$ constant,*

$$\Pr\left\{\bigvee_{i=1}^{N}\bigvee_{j=1}^{K}\left(|S_{ij}|>\frac{\log N}{\sqrt{N}}\right)\right\}\to 0 \quad (10)$$

i.e. almost surely all the elements of $\mathbf{S}$ satisfy the condition A-1 asymptotically. □

4 Asymptotic Performance Analysis

The MSE and the SINR of user k for any multistage detector with weight vector $\overline{\mathbf{w}}_k$ are given respectively by

$$\mathrm{MSE}_k = 1-2\,\mathrm{Re}\left(\mathbf{c}_k^T\overline{\mathbf{w}}_k\right)+\overline{\mathbf{w}}_k^H\mathbf{\Phi}_k\overline{\mathbf{w}}_k. \quad (11)$$

and

$$\mathrm{SINR}_k = \frac{\overline{\mathbf{w}}_k^H\mathbf{c}_k\mathbf{c}_k^T\overline{\mathbf{w}}_k}{\overline{\mathbf{w}}_k^H\left(\mathbf{\Phi}_k-\mathbf{c}_k\mathbf{c}_k^T\right)\overline{\mathbf{w}}_k}. \quad (12)$$

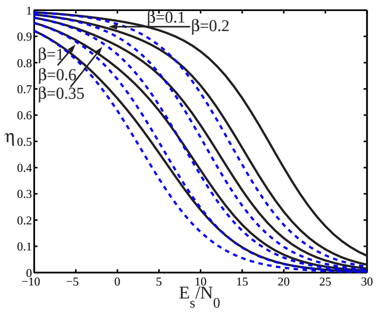

Figure 1: Multiuser efficiency η versus SNR for the asymptotic individual LMMSE detector in $\chi_M(\mathbf{SA})$ with $M=4$ (solid line) and $M=2$ (dashed line)

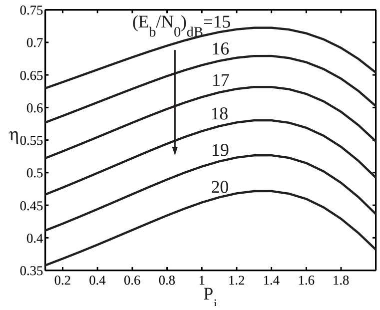

Figure 2: Multiuser efficiency versus power of the user of interest for the asymptotic joint LMMSE detector. Parameter setting: $M=4$, $\beta=\frac{1}{2}$

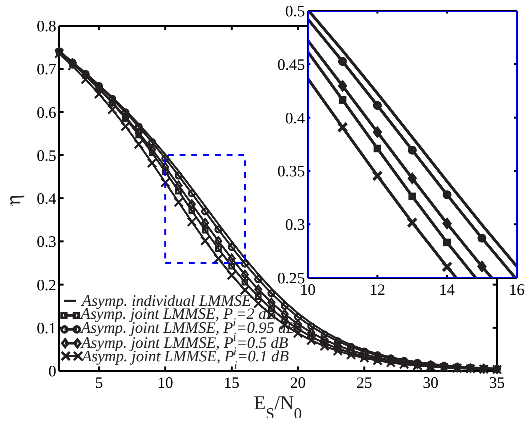

Figure 3: η_k versus $\frac{E_s}{N_0}$ for $\beta=0.5$ and varying P_k. Comparison of the individual LMMSE detector to the joint LMMSE detector

where Re(.) is the real part operator. Specializing the previous equations to the case of the individual LMMSE detector in $\chi_M(\mathbf{SA})$ we obtain:

$$\mathrm{MSE}_{ind,k} = 1-\mathbf{c}_k^T\mathbf{\Phi}_k^{-1}\mathbf{c}_k \tag{13}$$

$$\mathrm{SINR}_{ind,k} = \frac{\mathbf{c}_k^T\mathbf{\Phi}_k^{-1}\mathbf{c}_k}{1-\mathbf{c}_k^T\mathbf{\Phi}_k^{-1}\mathbf{c}_k} = \frac{1}{\mathrm{MSE}_{ind,k}}-1 \tag{14}$$

For the joint LMMSE detector in $\chi_M(\mathbf{SA})$ the performance is given by:

$$\mathrm{MSE}_{jnt,k} = 1-2\mathbf{c}_k^T\mathbf{\Phi}^{-1}\mathbf{c}+\mathbf{c}^T\mathbf{\Phi}^{-1}\mathbf{\Phi}_k\mathbf{\Phi}^{-1}\mathbf{c} \tag{15}$$

$$\mathrm{SINR}_{jnt,k} = \frac{\left(\mathbf{c}_k^T\mathbf{\Phi}^{-1}\mathbf{c}\right)^2}{\mathrm{MSE}_{jnt,k}-\left(\mathbf{c}_k^T\mathbf{\Phi}^{-1}\mathbf{c}-1\right)^2} \tag{16}$$

and in general

$$\mathrm{SINR}_{jnt,k} = \begin{cases} <\mathrm{MSNIR}_k & \text{for} \quad M<K \\ =\mathrm{MSNIR}_k & \text{for} \quad M\geq K \end{cases}. \tag{17}$$

For $M\geq K$ the equality is due to the fact that the joint LMMSE detector coincides with the true LMMSE [2] and the latter maximizes jointly the SINR_k for all the users. In asymptotic conditions, the performance of both the individual and the joint LMMSE detectors in $\chi_M(\mathbf{SA})$ is obtained from (13)-(14) and (15)-(16) respectively substituting $\mathbf{c}_k$ for $\mathbf{c}_k^\infty$, $\mathbf{\Phi}_k$ for $\mathbf{\Phi}_k^\infty$, $\mathbf{c}$ for $\mathbf{c}^\infty$ and $\mathbf{\Phi}$ for $\mathbf{\Phi}^\infty$. Then, in the asymptotic case the performance depends only on the moments of the distribution $F_{|\mathbf{A}|^2}(\lambda)$, $\mathcal{P}_k$, $m^l_{|\mathbf{A}|^2}$, β and σ^2. Equations (11) and (12) allow the performance evaluation of both the asymptotic LMMSE detectors when they are used in real scenarios with finite system size. For the asymptotic joint LMMSE detector in $\chi_M(\mathbf{SA})$ we find:

$$\mathrm{MSE}^\infty_{jt,k} = 1+\left((\mathbf{c}^\infty)^T(\Phi^\infty)^{-1}\mathbf{\Phi}_k-2\mathbf{c}_k^T\right)(\Phi^\infty)^{-1}\mathbf{c}^\infty$$

and

$$\mathrm{SINR}^\infty_{jnt,k} = \frac{1}{\dfrac{(\mathbf{c}^\infty)^T(\Phi^\infty)^{-1}\mathbf{\Phi}_k(\Phi^\infty)^{-1}\mathbf{c}^\infty}{\left((\mathbf{c}^\infty)^T(\Phi^\infty)^{-1}\mathbf{c}_k\right)^2}-1}.$$

Analogous relations can be written for the individual LMMSE detector in $\chi_M(\mathbf{SA})$ and the relative SINR degradation can be derived. However all these equations depends also on the specific realizations of $\mathbf{A}$ and $\mathbf{S}$ and not only on $\mathcal{P}_k$, $m^l_{|\mathbf{A}|^2}$, β and σ^2 as was the case in asymptotic conditions.

5 Numerical Results

Numerical results and simulations presented in this paper were obtained assuming flat Rayleigh block fading channels with unitary variance. Then the eigenvalues of $\mathbf{A}$, $F_{|\mathbf{A}|^2}(\lambda)$, are asymptotically χ^2 distributed with 2 degrees of freedom and characteristic function

$$\Phi_{|\mathbf{A}|^2}(\omega) = \frac{1}{1-i\omega}. \tag{18}$$

Calculating the eigenvalue moments from the relation $i^n m^n_{|\mathbf{A}|^2} = \left.\frac{\mathrm{d}^n\Phi_{|\mathbf{A}|^2}}{\mathrm{d}\omega^n}\right|_{\omega=0}$ and the SINR for the joint and the individual LMMSE detector in $\chi_M(\mathbf{SA})$ in asymptotic conditions we obtain the multiuser efficiency η_k for the two detectors with

$$\eta_k = \frac{\sigma^2}{P_k}\mathrm{SINR}_k. \tag{19}$$

The multiuser efficiency of the asymptotic individual LMMSE detector, η^∞_{ind}, is independent of P_k. In Figure 1 the families of the curves η^∞_{ind} versus $\frac{E_s}{N_0}$ parameterized with respect to the system load β are plotted for $M=2$ in dashed lines and for $M=4$ in solid lines. An increment in the number of stages allows a substantial improvement in scenarios with low load and high $\frac{E_s}{N_0}$ while it is not equally effective in scenarios with low SINR and high load. In contrast to many other detectors analyzed in literature η^∞_{jnt} depends on P_k as shown in Figure 2 and in Figure 3. This dependence is stronger for low system loads and high SNR while it tends to vanish for systems heavily loaded and low SNR.

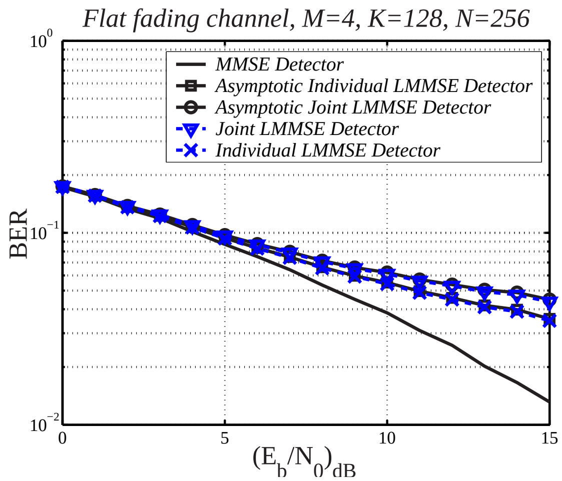

Figure 4: BER versus $\frac{E_b}{N_0}$ for $\beta = 0.5$ and $M = 4$

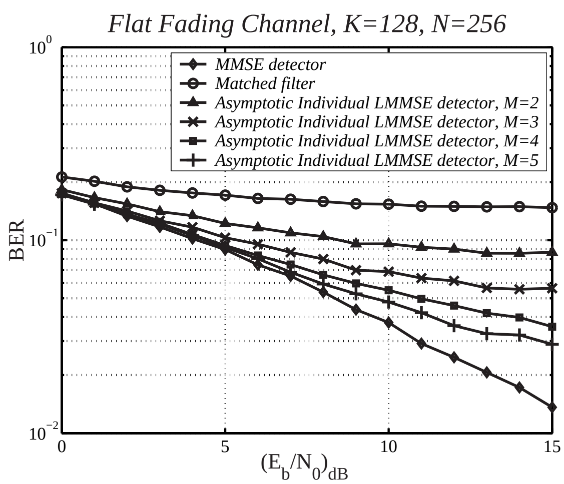

Figure 5: BER versus $\frac{E_b}{N_0}$ for $\beta = 0.5$ and varying number of stages

The performance degradation of both the asymptotic LMMSE detectors in $\chi_M(\mathbf{SA})$ compared to the corresponding LMMSE detectors in $\chi_M(\mathbf{SA})$ and the true LMMSE were assessed by simulations. The simulations were performed using $\frac{\pi}{4}$-QPSK modulation and assuming perfect knowledge of the channel. Figure 4 shows the BER versus $\frac{E_b}{N_0}$ for multistage detectors with $M = 4$ and $\beta = 0.5$. The performance degradation due to the asymptotic approximation of weights is completely negligible and the curves of the asymptotic individual and joint LMMSE detectors almost match the correspondent detectors with exact weighting. Figure 5 shows the performance improvements of the individual LMMSE detector for increasing number of stages.

6 Conclusions

In this paper we derived the asymptotic weighting for the individual and the joint LMMSE detectors in $\chi_M(\mathbf{SA})$ for systems with unequal received powers. Since the asymptotic weights depend on a small set of parameters, namely the noise variance, the system load and the received powers of the users, the complexity of the weight computation decreases substantially compared to the exact solution for finite-size systems, which, on the contrary, depends on the channel realization and on the spreading sequences. The performance assessment has shown that the degradation due to the asymptotic approximation is hardly observable. We also proposed a general equation applicable to any multistage detector in $\chi_M(\mathbf{SA})$ and we specialized it to both the asymptotic LMMSE detectors. The performance analysis showed that the improvement due to an increasing number of stages is lower for low SNR and high load and it is more sensible for high SNR and low system loads.

7 Acknowledgments

The authors would like to thank M. Umlauft and M. Lončar for the useful suggestions aiming to improve the quality of this paper.

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