Spectral Efficiency of CDMA Systems with Linear MMSE Interference Suppression

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***Abstract*— Code-division multiple-access is a promising technique for communication systems. Recently, many demodulation schemes have been proposed which take account of inherent multiple-access interference. In this paper, linear interference suppression as proposed in, e.g., [1], is compared with conventional demodulation based on power-bandwidth plane. This approach is much more general than the comparison of error rates for specific scenarios as used to be done before. In case of interference suppression, spectral efficiency is calculated by a combination of analysis and simulation.**

The coding and modulation scheme used is characterized within power-bandwidth plane for a single-cell scenario as well as for a cellular system. In the latter case, some statements which are valid for the single-cell scenario have to be revised.

***Index Terms*— Cellular systems, code-division multiple access, linear interference suppression, minimum mean-squared error, multiuser communication, spectral efficiency.**

I. Introduction

In CODE-DIVISION multiple access (CDMA) channels, many users share a common channel using spread spectrum (SS) signals with different signature (spreading) sequences. In general, the signature sequences, and therefore the users' waveforms, are not orthogonal, and multiple-access interference (MAI) occurs. In conventional receivers, this interference is modeled as white noise, and demodulation is performed by matched filters. Since the near–far problem has an impact on the uplink of cellular radio applications, stringent power control is inevitable.

The performance of CDMA systems can be improved by receivers which take account of the statistical properties of MAI. The output samples of a bank of matched filters for the waveforms of all users form a set of sufficient statistics [2]. Therefore, joint detectors are able to make use of all information on MAI. The optimum maximum-likelihood detector [2] is able to achieve almost the same performance as a single-user transmission scheme (without MAI). An alternative with low complexity is the linear equalizer which has been derived according to the zero-forcing (ZF) [3] and the minimum mean-squared error (MMSE) [4], [5] criterion. An exchange between the good performance of the optimum detector and the low complexity of the linear receiver is possible using decision-feedback or reduced-state multiuser detection as addressed in [6]–[8], respectively. A survey of recent literature about these and other interference suppression techniques has been given in [9].

The disadvantage of all these joint-detectors is the necessity of matched filters and synchronization units for the waveforms of all users. Although the signals from the other users may not be of interest, they have to be taken into account in the receiver. Another approach to suppression of MAI in CDMA systems is a receiver which does not exploit explicit information about the signature sequences of the other users. This demodulator introduced in [1], [10], and [11] consists of a band-limiting receiver input filter and a discrete-time adaptive filter for interference suppression. In this context, such a receiver will be called scalar linear interference suppression equalizer (SLISE). Like in [1], [10], and [11] the equalizer is assumed to be adjusted according to the MMSE criterion. It should be mentioned that for transmission over dispersive channels, this scheme can be generalized to a decision-feedback equalizer (DFE), cf. [12], [13]. But we do not address intersymbol interference channels further in this paper.

There are various ways to implement SLISEs. Obviously, in systems with band-limited waveforms, the implementation of the discrete-time interference suppression filter of infinite length with oversampling leads to the same performance as the linear equalizer based on a matched filter bank as in [3] and [5], because sampling theorem is fulfilled. A less complex implementation is achieved if the continuous-time receiver input filter is chosen as a filter matched to the chip waveform (chip-matched filter) and the sampling instants are multiples of the chip interval. This results in a receiver which has a superior performance compared to the conventional matched filter but, in general, little worse than the matched filter bank demodulator. In the papers addressing SLISEs [1], [10], [11], [14]–[16], this latter form of implementation is discussed while assuming that the length of the discrete-time filter is equal to the spreading factor. Implications of the selection of the filter length are considered in more detail in Section III of this paper.

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Bandwidth is a resource limited by nature. Therefore, communications engineering has to take care to not waste bandwidth unnecessarily. From this point of view the fundamental limit on spectral (bandwidth) efficiency of CDMA systems is an important topic.

It has been a celebrated result of information theory that spectral efficiency can only be obtained by the cost of power efficiency [17]. This tradeoff is a characteristic of digital transmission schemes and is often illustrated by the position in the power-bandwidth plane [18], [19]. Spectral efficiency of CDMA systems based on conventional demodulation is discussed in [20] and [21]. These considerations, which took place for single-cell systems, are generalized to cellular systems in [21] and [22].

For CDMA systems with interference suppression, spectral efficiency has not been addressed yet. Therefore, in this paper we calculate spectral efficiency, which represents the most important criterion for the selection of transmission schemes. Here, we focus on SLISEs, which have the advantage of low complexity. It should be noted that this receiver provides suppression of other-cell interference without modification. In contrast, joint detectors based on matched filter bank demodulation could defeat other-cell interference only if complexity is increased.

The considerations in this paper are restricted to transmission over nondispersive additive white Gaussian noise (AWGN) channels. The signals of the users are assumed to be asynchronous. We discuss and compare various linear modulation schemes like M-ary phase-shift keying (PSK) or M-ary quadrature amplitude modulation (QAM). In order to derive the information theoretical limits, transmission with Gaussian distributed amplitude coefficients is considered, too.

The organization of the paper is as follows. The communication model is introduced in Section II. In Section III, the implications of chip-synchronous and chip-asynchronous transmission and the impact of filter length on system performance are discussed. Spectral efficiency of single-cell and cellular CDMA systems is calculated in Sections IV and V, respectively. The paper is finished with some concluding remarks in Section VI.

II. Communications Model

Asynchronous CDMA communication over the nondispersive AWGN channel is considered. K users transmit SS signals over a common channel. Without loss of generality, we consider the detection of the signal of user 1. The signals of user 2 up to K are treated as interference. The signal at the receiver site of user 1 is

$$r'(t) = \sum_{i=1}^{K} A_i e^{j\theta_i} \sum_{\nu=-\infty}^{+\infty} a_i[\nu] s_i(t - \nu T_s - \tau_i) + n(t).$$

$a_i[\nu]$ is the (generally complex-valued) amplitude coefficient of user $i \in \{1, \cdots, K\}$ in the modulation interval $\nu \in \mathbb{Z}$. We assume that the $a_i[\nu]$ are uncorrelated and identically distributed random variables with $\mathcal{E}\{|a_i[\nu]|^2\} = 1$ and $\mathcal{E}\{a_i[\nu]\} = 0$. It should be noted that the assumption of uncorrelatedness of the amplitude coefficients is fulfilled by usual channel codes.

The symbol waveform of user $i \in \{1, \cdots, K\}$ is denoted by $s_i(t)$. Its energy is $E_s = \int_{-\infty}^{+\infty} |s_i(t)|^2 \, dt$. τ_i and θ_i are the delay and the carrier phase of the signal transmitted by user $i \in \{1, \cdots, K\}$. Without loss of generality, we assume $\tau_1 = \theta_1 = 0$. The $\tau_i \in [0; T_s)$ and $\theta_i \in [0; 2\pi)$, $i \in \{2, \cdots, K\}$ are assumed as independent and uniformly distributed random variables. The modulation interval is denoted by T_s. The $A_i \in \mathbb{R}$ are the amplitudes (square-root of average power) of the received signal from user $i \in \{1, \cdots, K\}$. Without loss of generality, we define $A_1 = 1$. All signals are given in their equivalent complex baseband representation (normalization for equal energy in passband and baseband representation applied, cf. [19]). Then, $n(t)$ is complex-valued white Gaussian noise with *double-sided* power spectral density N_0. It corresponds to real-valued white Gaussian noise with *single-sided* power spectral density N_0.

The symbol waveform of user i is given by

$$s_i(t) = \sum_{\kappa=0}^{N-1} b_i[\kappa] \psi(t - \kappa T_c)$$

where N, $b_i[\kappa]$, and $\psi(t)$ are the spreading factor, the signature sequence of user i, and the chip waveform, respectively. The chips of the signature sequences are assumed to be independent and identically distributed random variables with $\mathcal{E}\{|b_i[\kappa]|^2\} = 1/N$ and $\mathcal{E}\{b_i[\kappa]\} = 0$. Since the chip sequence is normalized to energy 1 per modulation interval, we have $\int_{-\infty}^{+\infty} |\psi(t)|^2 = E_s$. The chip interval, which is denoted by $T_c = T_s/N$, is real-valued and square-root Nyquist (e.g., square-root cosine). In practical applications, band-limited waveforms are used.

Demodulation is performed by a continuous-time chip matched filter with impulse response[1] $\psi^*(-t)/E_s$ and a subsequent discrete-time adaptive linear interference suppression filter. The chip-matched filter output signal is sampled at instants $t = \kappa T_c$. As already stated in Section I, in the general case of chip-asynchronous transmission, some information is lost, so that this kind of demodulation is a heuristic and suboptimum approach. The signal at the output of the chip-matched filter sampled at instants $t = \kappa T_c$ is given by

$$\begin{aligned} r[\kappa] &= r'(t) * \frac{1}{E_s} \psi^*(-t) \Big|_{t=\kappa T_c} \\ &= \sum_{i=1}^{K} A_i e^{j\theta_i} \sum_{\nu=-\infty}^{+\infty} a_i[\nu] \\ &\quad \cdot \sum_{\lambda=0}^{N-1} b_i[\lambda] \varphi'_{\psi}((\kappa - \lambda - \nu N)T_c - \tau_i) + n[\kappa] \quad (1) \end{aligned}$$

where $\varphi'_{\psi}(t) = \psi(t) * \psi^*(-t)/E_s$ denotes the normalized autocorrelation function of the chip waveform. $n[\kappa]$ is discrete-time, complex-valued, white Gaussian noise with zero mean and variance $\sigma_n^2 = N_0/E_s$.

[1] The impulse response of the chip matched filter is normalized to obtain an amplification factor for the information-bearing signal equal to one.

We define the signal part at chip interval κ which is caused by the signal of user i transmitted in modulation interval ν as

$$p_{i,\nu}[\kappa] = A_i e^{j\theta_i} \sum_{\lambda=0}^{N-1} b_i[\lambda]\varphi'_{\psi}((\kappa - \lambda - \nu N)T_c - \tau_i). \quad (2)$$

Therefore, (1) is given by

$$r[\kappa] = \sum_{i=1}^{K} \sum_{\nu=-\infty}^{+\infty} a_i[\nu] p_{i,\nu}[\kappa] + n[\kappa]. \quad (3)$$

The signal $r[\kappa]$ is the input sequence of the interference suppression filter with impulse response $h^*[\kappa]$. For convenience, we define the vectors of length L

$$\mathbf{r}^{\mathsf{T}}[\mu] \triangleq [r[\mu N + w], \cdots, r[\mu N], \cdots, r[\mu N + v]] \quad (4)$$

$$\mathbf{n}^{\mathsf{T}}[\mu] \triangleq [n[\mu N + w], \cdots, n[\mu N], \cdots, n[\mu N + v]] \quad (5)$$

$$\mathbf{p}_{i,\nu}^{\mathsf{T}}[\mu] \triangleq [p_{i,\nu}[\mu N + w], \cdots, p_{i,\nu}[\mu N], \cdots, p_{i,\nu}[\mu N + v]] \quad (6)$$

$$\mathbf{h}^{\mathsf{H}} \triangleq [h^*[-w], \cdots, h^*[0], \cdots, h^*[-v]]$$

with

$$w \triangleq \left\lceil \frac{N+L}{2} - 1 \right\rceil, \quad v \triangleq \left\lceil \frac{N-L}{2} \right\rceil$$

where we assume that the signature sequence $b_1[\lambda]$ of the user of interest is centered within the filter vector. The time index μ in (4) to (6) is according to the symbol interval, i.e., time instants $t = \mu T_s$ are represented. $\lceil \cdot \rceil$ denotes the operator for rounding the argument to the nearest integer toward infinity, and $(\cdot)^{\mathsf{H}}$ is the conjugate transpose operator. The position of the window of the vector representation is chosen to simplify the further computations. Since signature sequences do not change in time, we have

$$\mathbf{p}_{i,\nu}[\mu] = \mathbf{p}_{i,\nu-\mu}[0].$$

Then, (3) leads to

$$\mathbf{r}[\mu] = \sum_{i=1}^{K} \sum_{\nu=-\infty}^{+\infty} a_i[\mu + \nu] \mathbf{p}_{i,\nu}[0] + \mathbf{n}[\mu].$$

$\mathbf{p}_{1,0}[0]$ is the desired signal vector, and $\mathbf{p}_{i,\nu}[0]$, $(i,\nu) \neq (1,0)$ are the *interference vectors*. Since the chip waveform is a square-root Nyquist pulse, the number of interference vectors is equal to $K-1$ if transmission of the signals of all K users is symbol-synchronous.

The discrete-time interference suppression filter, which is used to generate the decision variable (demodulator output signal)

$$d[\mu] = \mathbf{h}^{\mathsf{H}} \mathbf{r}[\mu]$$

is adjusted according to the MMSE criterion by

$$\mathbf{h} = \left(\mathbf{P} + \mathbf{p}_{1,0}[0]\mathbf{p}_{1,0}^{\mathsf{H}}[0]\right)^{-1} \mathbf{p}_{1,0}[0]$$

with $\mathbf{P}$ denoting the covariance matrix of total noise at the output of the sampling unit. Following [1], it is given by

$$\mathbf{P} = \sum_{i=1}^{K} \sum_{\nu=-\infty}^{+\infty} \mathbf{p}_{i,\nu}[0]\mathbf{p}_{i,\nu}^{\mathsf{H}}[0] - \mathbf{p}_{1,0}[0]\mathbf{p}_{1,0}^{\mathsf{H}}[0] + \mathcal{E}\{\mathbf{n}[\mu]\mathbf{n}^{\mathsf{H}}[\mu]\}.$$

The maximum signal-to-interference ratio (MSIR) for an optimally adjusted equalizer filter has been calculated in, e.g., [1], yielding

$$\mathrm{MSIR} = \mathbf{p}_{1,0}^{\mathsf{H}}[0]\mathbf{P}^{-1}\mathbf{p}_{1,0}[0]. \quad (7)$$

The decision variable $d[\cdot]$ is directly fed into the channel decoder. Thus, no reliability information is lost by detection. Moreover, the transmission system of each user from the SS modulator input to the SLISE output can be well modeled as an equivalent AWGN channel with signal-to-noise ratio equal to MSIR [23]. Then, the maximum signal-to-interference ratio can be denoted by $\mathrm{MSIR} = E_s/I_0$ with I_0 as the (constant) spectral density of total interference. With this assumption, the result in (7) can be used for the analysis of systems with channel coding, too. It is the basis for the numerical evaluations in Sections IV and V.

III. Impact of Chip-Asynchronism

The SLISE introduced in [1] and [10] is a heuristic approach to suppression of MAI in asynchronous CDMA systems. In the general case of chip-asynchronous transmission, this receiver degrades compared to the matched filter bank demodulator with succeeding linear equalization [5], because the SLISE does not exploit any information about time shifts which are fractions of the chip interval.

Since the chip waveform is square-root Nyquist, each transmitted chip pulse causes a signal at exclusively *one* filter tap if the signals of all users are chip-synchronous. In the general case of chip-asynchronous signals, each chip sent by user $i \neq 1$ may cause interference at all taps of the discrete-time equalizer filter. If this interference is generated by a chip located outside of the processing window of length L (equal to the filter length), its statistical properties cannot be exploited efficiently to increase signal-to-interference ratio at demodulator output. It represents *unprocessed* MAI.

In literature, the effects of chip-asynchronism on the system performance have not been addressed if the SLISE is used without oversampling. In this section, the problem is discussed in order to show that chip-asynchronism has only minor impact on the performance of SLISEs.

Inserting (2) into (6) leads to the following representation of the interference vector due to the symbol with index ν sent by user i and received at time 0:

$$\mathbf{p}_{i,\nu}[0] = A_i e^{j\theta_i} \sum_{\lambda=0}^{N-1} b_i[\lambda] \begin{bmatrix} \varphi'_{\psi}\big((w - \lambda - \nu N)T_c - \tau_i\big) \\ \vdots \\ \varphi'_{\psi}\big((-\lambda - \nu N)T_c - \tau_i\big) \\ \vdots \\ \varphi'_{\psi}\big((v - \lambda - \nu N)T_c - \tau_i\big) \end{bmatrix}.$$

The interference energy *at the input of the discrete-time equalizer filter* caused by the symbol sent in modulation interval ν by user i for a given signature sequence is $||\mathbf{p}_{i,\nu}[0]||^2/L$. Since we are interested in the average system performance, we model the chips in the signature sequences as independent random variables as stated in Section II. Then, the average interference energy due to *one* symbol is

$$\frac{1}{L}\mathcal{E}\{\|\mathbf{p}_{i,\nu}[0]\|^2\} = \frac{A_i^2}{LN}\sum_{\kappa=v}^{w}\sum_{\lambda=0}^{N-1}\varphi_\psi'^2((\kappa-\lambda-\nu N)T_c-\tau_i) \tag{8}$$

where we used $\mathcal{E}\{|b_i[\kappa]|^2\}=1$ and assumed that the chip waveform is real-valued.

We define

$$\tau_i \triangleq \ell_i T_c + \tilde{\tau}_i$$

where $\ell_i \in \{0,\cdots,N-1\}$ and $\tilde{\tau}_i \in [0;T_c)$ are the delay of user i in multiples of the chip interval, and the portion of delay which is smaller than the chip interval, respectively. Both random variables are assumed to be uniformly distributed. Thus, (8) yields

$$\frac{1}{L}\mathcal{E}\{\|\mathbf{p}_{i,\nu}[0]\|^2\} = \frac{A_i^2}{LN}\sum_{\kappa=v}^{w}\sum_{\lambda=0}^{N-1}\varphi_\psi'^2((\kappa-\lambda-\nu N-\ell_i)T_c-\tilde{\tau}_i). \tag{9}$$

In the following, we focus on the impact of chip-asynchronism on performance. At first glance, one may expect that the interference from symbols outside the finite impulse response (FIR) filter window causes a significant degradation. This interference, which vanishes in the chip-synchronous case ($\tilde{\tau}_i=0$), consists of terms as given in (9) if the condition

$$\kappa-\lambda-\nu N-\ell_i \neq 0;\quad \kappa\in\{v,\cdots,w\};\quad \lambda,\ell_i\in\{0,\cdots,N-1\} \tag{10}$$

is fulfilled.

In order to simplify the following considerations, we assume that the length of the discrete-time filter is an odd multiple of the spreading factor and define

$$\Lambda \triangleq \frac{L}{N} \in \{1,3,5,7,\cdots\}. \tag{11}$$

The restriction to odd numbers is introduced to avoid the discussion of two cases. Furthermore, the other case would lead to a more complex notation and thus reduce readability. By inserting (11), (10) can be given equivalently by

$$\nu \in \{-\infty,\cdots,-(\Lambda+3)/2\}\cup\{(\Lambda+3)/2,\cdots,+\infty\}.$$

The interference energy appearing for chip-asynchronous transmission and vanishing in the chip-synchronous case

$$E_{\text{cas}} = \underbrace{\sum_{i=2}^{K}\sum_{\nu=-\infty}^{-(\Lambda+3)/2}\frac{1}{L}\mathcal{E}\{\|\mathbf{p}_{i,\nu}[0]\|^2\}}_{E_{\text{pre}}} + \underbrace{\sum_{i=2}^{K}\sum_{\nu=(\Lambda+3)/2}^{+\infty}\frac{1}{L}\mathcal{E}\{\|\mathbf{p}_{i,\nu}[0]\|^2\}}_{E_{\text{fol}}}$$

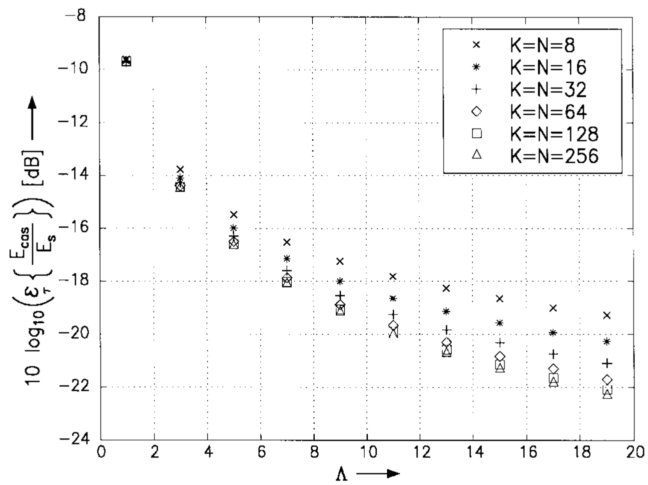

Fig. 1. Upper bound on additional interference energy due to chip-asynchronism (averaged over the users' signal delays) versus filter length normalized to spreading factor.

is the sum of the interference energy from previous and following symbols E_{pre} and E_{fol}, respectively. With (9), the interference power caused by following symbols reads

$$E_{\text{fol}} = \frac{1}{LN}\sum_{i=2}^{K}A_i^2 \cdot \sum_{\nu=(\Lambda+3)/2}^{+\infty}\sum_{\kappa=v}^{w-1}\sum_{\lambda=0}^{N-1}\varphi_\psi'^2((\kappa-\lambda-\nu N)T_c-\tau_i).$$

Several substitutions of the summation indexes lead to

$$E_{\text{fol}} = \frac{1}{LN}\sum_{i=2}^{K}A_i^2\sum_{\kappa=1}^{L+N}\sum_{\lambda=L+2N+1}^{+\infty}\varphi_\psi'^2((\kappa-\lambda)T_c-\tau_i).$$

Now, we assume that $|\varphi_\psi'(t)|$ is upper bounded by $T_c/|\pi t|$. This is valid for, e.g., square-root cosine pulses with zero roll-off. Furthermore, $\sum_{\lambda=\lambda_1}^{\lambda_2} f(\lambda) \le \int_{\lambda_1-1}^{\lambda_2} f(\alpha)\,d\alpha$ holding obviously for all decreasing functions leads to an upper bound

$$\begin{aligned}E_{\text{fol}} &\le \frac{T_c^2}{LN\pi^2}\sum_{i=2}^{K}A_i^2\sum_{\kappa=1}^{L+N}\int_{L+2N}^{+\infty}\frac{-d\alpha}{((\kappa-\alpha)T_c-\tau_i)^2}\\ &= \frac{T_c}{LN\pi^2}\sum_{i=2}^{K}A_i^2\sum_{\kappa=1}^{L+N}\frac{-1}{(\kappa-L-2N)T_c-\tau_i}.\end{aligned}$$

Applying $\sum_{\kappa=\kappa_1}^{\kappa_2} f(\kappa) \le \int_{\kappa_1}^{\kappa_2+1} f(\alpha)\,d\alpha$, which is obviously valid for all increasing functions, yields

$$\begin{aligned}E_{\text{fol}} &\le \frac{T_c}{LN\pi^2}\sum_{i=2}^{K}A_i^2\int_{1}^{L+N+1}\frac{-d\alpha}{(\alpha-L-2N)T_c-\tau_i}\\ &= \frac{1}{LN\pi^2}\sum_{i=2}^{K}A_i^2\ln\frac{(L+2N-1)T_c+\tau_i}{(N-1)T_c+\tau_i} \overset{L\to\infty}{\longrightarrow} 0.\end{aligned}$$

Equivalently, it can be shown that $E_{\text{past}} \overset{L\to\infty}{\longrightarrow} 0$ yielding

$$\lim_{L\to\infty} E_{\text{cas}} = 0.$$

TABLE I
REQUIRED SIGNAL-TO-INTERFERENCE RATIO $(E_s/I_0)_r$ IN ORDER TO ACHIEVE BIT ERROR RATE $\mathrm{BER}_r = 10^{-3}$ FOR 4PSK WITH CONVOLUTIONAL CODING AND SEVERAL TRELLIS-CODED MODULATION SCHEMES (EACH WITH CONSTRAINT LENGTH SEVEN)

	R_c	R [bit/sym.]	$(E_b/I_0)_r$	$(E_s/I_0)_r$
4PSK	1/3	2/3	2.30 dB	−2.47 dB
4PSK	2/3	4/3	3.20 dB	1.44 dB
16QAM	3/4	3	6.31 dB	11.08 dB
32QAM	4/5	4	8.70 dB	14.72 dB
64QAM	5/6	5	10.71 dB	17.70 dB

This means that for a sufficiently long equalizer filter, chip-asynchronism does not lead to a significant degradation compared to chip-synchronous transmission. Fig. 1 illustrates that a filter three to seven times as long as the spreading factor is sufficient for the modulation schemes which are discussed later, cf. Table I. In the limit of infinite filter length, the degradation vanishes. This fact is the reason why in the following exclusively chip-synchronous transmission is considered.

IV. SPECTRAL EFFICIENCY OF SINGLE-CELL SYSTEMS

The position within the power-bandwidth plane is one of the most important characteristics of transmission schemes. This kind of representation is well known for single-user communication. For multiuser systems based on CDMA with conventional demodulation, spectral efficiency is computed in [20] and [21]. In the following, we present the calculation of spectral efficiency of CDMA systems with demodulation by SLISEs and consider single-cell systems first in this section. For convenience, it is assumed that the signals of all users at demodulator input have the same power. In the uplink, this assumption is equivalent to perfect power control. In order to simplify the simulation procedure, *chip-synchronous* transmission is assumed. Since we have shown in Section III that the degradation by chip-asynchronism is low if sufficiently long equalizer filters are applied, this is a reasonable approach.

In Sections II and III, no restrictions have been introduced with respect to the chip waveform. In the following, we assume that its spectrum is constant within the interval $[-1/(2T_c), 1/(2T_c)]$ and zero outside, which is equivalent to the assumption of a square-root cosine pulse with zero roll-off. In [24] and [25], it is shown that this choice of chip waveform leads to maximum spectral efficiency of CDMA systems based on conventional demodulation.

The discrete-time channel from modulator input to demodulator output is well modeled as an equivalent AWGN channel with a signal-to-interference ratio which depends on the channel situation and the demodulation scheme.

Each user applies a transmission scheme which is characterized by the transmission rate R [bit/symbol]. Depending on the selected transmission scheme, different signal-to-interference ratios $(E_s/I_0)_r$ are required to achieve a given bit error rate BER_r. For the numerical evaluations in this paper, we assume $\mathrm{BER}_r = 10^{-3}$.

We will discuss the spectral efficiency of various transmission schemes. The considered modulation schemes are 4PSK with Gray mapping, either without channel coding or with convolutional codes with various rates R_c, and $(M \geq 8)$-ary Ungerboeck coded modulation [26] based on QAM or PSK. In each case, the used convolutional code has constraint length seven. In Table I, the signal-to-interference ratios at the bit error rate of 10^{-3}, which were determined by means of simulation of a single-user link, are given. Furthermore, the equivalent ratios of energy per binary information symbol-to-interference power density, denoted by $(E_b/I_0)_r$, are shown. Here, we use

$$E_s = RE_b. \tag{12}$$

Additionally, we are interested in the performance of CDMA communication with SLISE if transmission at the capacity of the equivalent AWGN channel is assumed. This hypothetical system is called transmission scheme with *optimum channel coding* in the following. Channel capacity depends on the used signal set. The maximum transmission rate (with respect to *complex-valued* symbols)

$$R = \log_2\left(1 + \frac{E_s}{I_0}\right) \quad \left[\frac{\text{bit}}{\text{symbol}}\right]$$

is attainable for Gaussian distributed amplitude coefficients while for the signal sets of M-ary PSK and QAM, channel capacity can be calculated by numerical integration, e.g., [27]. In all cases, the maximum transmission rate is given as a function of signal-to-interference ratio

$$R = g(E_s/I_0). \tag{13}$$

The inverse function is denoted by $g^{-1}(\cdot)$.

In order to calculate spectral efficiency of the CDMA system, first we have to determine the signal-to-interference ratio as a function of signal-to-noise ratio E_s/N_0

$$\frac{E_s}{I_0} = f_{K,N}\left(\frac{E_s}{N_0}\right). \tag{14}$$

The parameters are the number of users K and the spreading factor N. For each combination of K and N, a function results whose inverse is denoted by $f_{K,N}^{-1}(\cdot)$.

In the case of conventional demodulation, it can be given analytically by

$$\frac{E_s}{I_0} = f_{K,N}\left(\frac{E_s}{N_0}\right) = \frac{1}{\dfrac{K-1}{N} + \dfrac{N_0}{E_s}} \tag{15}$$

e.g., [21] and [28]. This equation is valid for chip-synchronous transmission if the chip waveform is square-root Nyquist, and for chip-asynchronous transmission with the chip waveform assumed here.

In contrast to the decorrelating detector analyzed in [29], the *average* signal-to-interference ratio of SLISE is estimated by means of simulation as in [7, Fig. 6]. For each measurement, 4096 scenarios of sequences and time offsets are selected randomly as stated in Section II. The signature sequences are generated by pseudorandom binary chips. It should be noted that the distribution of the chips has negligible effect on performance. For signal sets with complex amplitude coefficients [e.g. $(M \geq 4)$-ary QAM], complex-valued chips lead to similar results. For each scenario, signal-to-interference ratio is computed applying (7). The average signal-to-interference ratios calculated as the harmonic means of these signal-to-interference ratios and are used as lookup tables representing the functions $f_{K,N}(\cdot)$ defined in (14). Here, we neglect the variance of signal-to-interference ratio, which tends toward zero for high spreading factors N [29], [30].

The definitions of $(E_s/I_0)_r$, R, $g(\cdot)$, and $f_{K,N}(\cdot)$ as given previously provide computation of the spectral efficiency of CDMA systems.

For single-user systems, spectral efficiency is defined as $1/B = 1/(B_s T_b)$, the inverse of the bandwidth normalized to the equivalent period T_b of binary information symbols. B_s is the physical bandwidth. Since we assume chip waveforms with roll-off zero, $B_s = 1/T_c = N/T_s = N/(RT_b)$ and the spectral efficiency is $1/B = R/N$. N/R is often termed as *bandwidth expansion* or *processing gain*.

In the case of multiuser communication, where several users share a common frequency band, the definition of spectral efficiency is generalized to

$$\gamma = \frac{K}{B} = \frac{K}{N}R \quad \left[\frac{\text{bit}}{\text{s}\cdot\text{Hz}}\right] \tag{16}$$

cf. [20], where spectral efficiency is called *total transmission rate efficiency*. The number of users K which are possible at BER_r is a function of E_s/N_0 and E_b/N_0, respectively. Therefore, spectral efficiency $\gamma = \gamma(E_b/N_0)$ is a function of E_b/N_0, too. It characterizes the performance of the transmission scheme used and is derived in the following.

For CDMA communication based on conventional demodulation, spectral efficiency can be given analytically using (12), (15), and (16) by

$$\gamma = \left(\frac{E_b}{I_0}\right)_r^{-1} - \left(\frac{E_b}{N_0}\right)^{-1} + \frac{R}{N} \tag{17}$$

where we again assume that $(E_b/I_0)_r$ is required to achieve the specified bit error rate BER_r.

We interpret (17) for the case of large numbers of users $K \gg 1$ or large spreading factors $N \gg 1$, respectively. Then, the last summand diminishes, hence yielding

$$\lim_{E_b/N_0 \to \infty} \gamma = \left(\frac{E_b}{I_0}\right)_r^{-1} \tag{18}$$

which has already been stated in [20]. I.e., spectral efficiency is limited by $(E_b/I_0)_r$. For single-user systems, error-free transmission is possible if $E_b/I_0 > \ln(2) = 1/\log_2(e)$ [18]. Therefore, in the case of optimum channel coding, the maximum spectral efficiency is given by $\log_2(e) \approx 1.44$ bit/(s·Hz) and will be achieved at transmission rate $R \to 0$.

For systems with SLISEs, spectral efficiency is calculated as follows. We assume the spreading factor N to be fixed. It should be noted that the performance of CDMA systems both with conventional demodulation and SLISE is determined approximately by the ratio K/N only, see also [14]. Since spectral efficiency is a function of K/N, cf. (16), it changes only slightly if N is varied. This fact has been proved by numerical evaluations of spectral efficiency for various values of N.

For the calculation of spectral efficiency, we have to distinguish between the use of standard channel coding with fixed transmission rate and optimum channel coding where transmission rate is a parameter which can be optimized. First, the previous case is considered. We assume that BER_r is specified, and the respective signal-to-interference ratio $(E_s/I_0)_r$ is given in Table I. In the case of conventional demodulation, spectral efficiency is calculated by (17). If demodulation is performed by SLISE, spectral efficiency is determined as follows. The number of users is varied as $K = 1, 2, 3, \cdots$. For each K, spectral efficiency $\gamma = RK/N$ and $E_b/N_0 = f_{K,N}^{-1}((E_s/I_0)_r)/R$ are calculated. Each pair represents one point in the power-bandwidth plane.

Second, the case of optimum channel coding is considered. Here, the number of users K and the transmission rate $R \in [0;\infty)$ are free parameters. For each pair of K and R, the spectral efficiency $\gamma = RK/N$ and the corresponding E_b/N_0 are calculated. The latter is obtained by use of (13) and (14), yielding

$$E_b/N_0 = f_{K,N}^{-1}\big(g^{-1}(R)\big)/R.$$

For each K, the point with minimum E_b/N_0 is selected. It represents the transmission scheme with the transmission rate which is optimized to attain the highest power efficiency for the spectral efficiency given by each choice of parameter K. The optimum transmission rate R_{opt} is of interest and is considered, too.

In the following, we present some numerical examples for the spectral efficiency of CDMA systems with SLISE and conventional demodulation. The spreading factor is given by $N = 64$. Spectral efficiency is computed for $\mathrm{BER}_r = 10^{-3}$. In the case of optimum channel coding, transmission takes place at channel capacity without any errors. Furthermore, we assume perfect power control. The results are shown in Figs. 2 to 5.

First, we consider symbol-synchronous transmission. The length of the equalizer filter is equal to the spreading factor. Then, the number of interference vectors is equal to $K - 1$. Conventional demodulation leads to the best performance if a low-rate code is used. Increasing code rate causes decreasing spectral efficiency for all values of E_b/N_0 [20], [21]. This fact is true for signal sets with $M > 4$, too. Therefore, exclusively convolutional coding with rate 1/3 is used if conventional demodulation is considered. As shown in (18), spectral efficiency saturates at the inverse of the required signal-to-interference ratio.

In comparison to conventional demodulation, SLISEs provide a significant gain. The optimum choice of the transmission scheme depends on the signal-to-noise ratio. If E_b/N_0 is high, the transmission rate should be high as well.

An important difference between SLISE and conventional demodulation is that the level of saturation is not exclusively determined by $(E_b/I_0)_r$ but also by the rate R of the single-user transmission system. This is the motivation for considering larger signal sets in the following.

In Fig. 2, spectral efficiency of M-ary QAM with $M = 4$, 16, 32, 64 is shown. Different transmission schemes with different rates R lead to maximum spectral efficiency depending

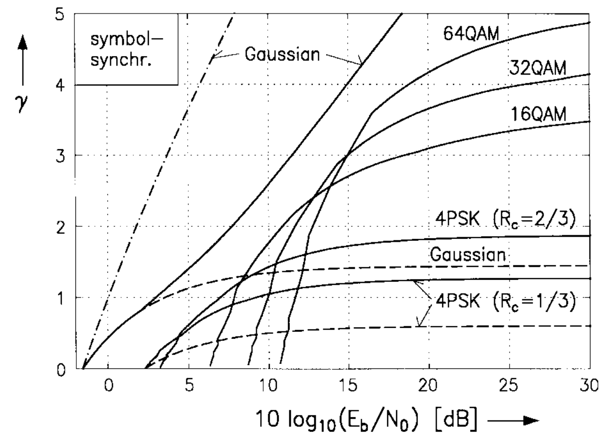

Fig. 2. Spectral efficiency of symbol-synchronous CDMA systems (single-cell) with SLISE (—) and conventional demodulation (– –); M-ary PSK/QAM with convolutional code (constraint length seven), and Gaussian distributed amplitude coefficients with optimum channel coding, spreading factor $N = 64$, symbol-synchronous transmission; for comparison: spectral efficiency for orthogonal waveforms and Gaussian distributed amplitude coefficients (– · –).

on the signal-to-noise ratio. If the number of levels is not restricted, spectral efficiency can be increased arbitrarily at cost of power efficiency. This fact stands in contrast to conventional CDMA and is similar to single-user communication.

Also in Fig. 2, the absolute limit of spectral efficiency of both CDMA with SLISE and conventional demodulation is shown for optimum channel coding and Gaussian distributed amplitude coefficients. Furthermore, the information theoretical limit (optimum channel coding, Gaussian distributed amplitude coefficients) for single-user transmission is presented. This is achieved by multiuser systems if the waveforms of the users are orthogonal [27]. As already discussed, spectral efficiency of conventional CDMA saturates at approximately 1.44 bit/(s · Hz). In contrast, spectral efficiency of CDMA with SLISE is not bounded. For low E_b/N_0, it is equal to that of conventional CDMA. With increasing spectral efficiency, power efficiency severely decreases compared to the case of orthogonal waveforms. The reason is the suboptimality of linear equalization. By using more efficient multiuser demodulation schemes like DFE, reduced-state sequence estimation or maximum-likelihood sequence estimation, this degradation could be reduced [31].

For demodulation by SLISE, the number of interference vectors is greater than $K-1$ if transmission is asynchronous and filter length is finite, cf. [1]. Then, a degradation is caused compared to the symbol-synchronous case. The number of interference vectors decreases to $K-1$ if filter length grows toward infinity. Noticeably, signal-to-interference ratio is higher than for symbol-synchronous transmission in this case. This fact is pointed out in Table II by exemplary simulation results.

In order to show the improvement by increasing filter length, simulations for $\Lambda = 1$ and $\Lambda = 3$ were performed. The results are presented in Figs. 3 and 4, respectively. Since filter lengths of two or three times the spreading factor seem to be a good compromise between performance and receiver complexity, the case $\Lambda = 3$ is exclusively considered further.

TABLE II
SIGNAL-TO-INTERFERENCE RATIO MEASURED FOR A CDMA SYSTEM WITH SLISE; SPREADING FACTOR $N = 10$, $K = 8$ USERS, AND $10\log_{10}(E_s/N_0) = 7$ dB; SIMULATION OF 10^6 SCENARIOS ("SYNC." MEANS SYMBOL-SYNCHRONOUS)

Λ	1	2	3	4	5 to 8	sync.
E_s/I_0 [dB]	2.28	3.26	3.47	3.52	3.53	3.37

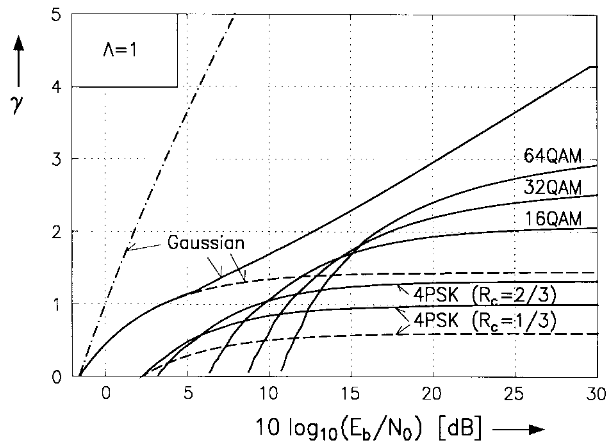

Fig. 3. Spectral efficiency of symbol-asynchronous CDMA systems (single-cell) with SLISE (—) and conventional demodulation (– –); M-ary PSK/QAM with convolutional code (constraint length seven), and Gaussian distributed amplitude coefficients with optimum channel coding, spreading factor $N = 64$, $\Lambda = 1$; for comparison: spectral efficiency for orthogonal waveforms and Gaussian distributed amplitude coefficients (– · –).

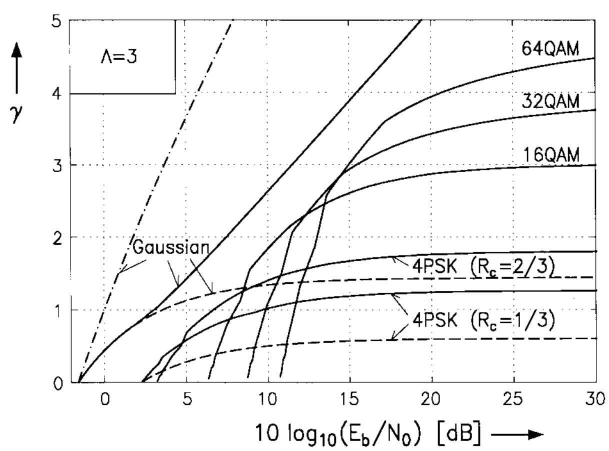

Fig. 4. Spectral efficiency of symbol-asynchronous CDMA systems (single-cell) with SLISE (—) and conventional demodulation (– –); M-ary PSK/QAM with convolutional code (constraint length seven), and Gaussian distributed amplitude coefficients with optimum channel coding, spreading factor $N = 64$, $\Lambda = 3$.

Previously, we already discussed performance of CDMA systems with optimum channel coding and Gaussian distributed amplitude coefficients. In this context, signal sets like M-PSK and M-QAM are also of interest. Spectral efficiency of several transmission schemes with optimum channel coding is shown in Fig. 5. Like in the case of single-user transmission, large signal sets lead to superior spectral efficiency for all E_b/N_0. The reason is that transmission rate is a free parameter. For low E_b/N_0, the gain by using more modulation levels is small. Remarkably, spectral efficiency of 4PSK modulation with optimum channel coding may be greater than 2

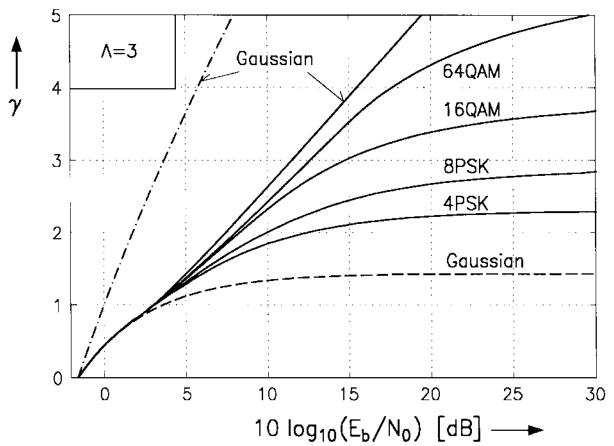

Fig. 5. Spectral efficiency of symbol-asynchronous CDMA systems (single-cell) with SLISE (—) and conventional demodulation (−−); M-ary PSK/QAM and Gaussian distributed amplitude coefficients with optimum channel coding, spreading factor $N = 64$, $\Lambda = 3$; for comparison: spectral efficiency for orthogonal waveforms and Gaussian distributed amplitude coefficients (− · −).

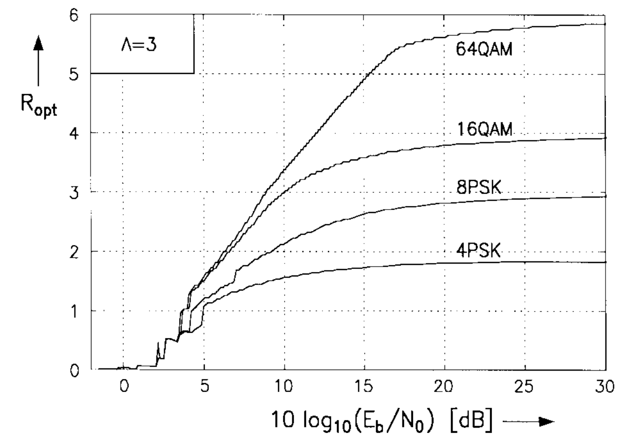

Fig. 6. Optimum transmission rate for symbol-asynchronous CDMA systems (single-cell) with SLISE; M-ary PSK/QAM with optimum channel coding, spreading factor $N = 64$, $\Lambda = 3$.

bit/(s · Hz), which is the maximum spectral efficiency of single-user transmission systems. This is not a contradiction, but a well-known result of multiuser information theory, e.g., [27].

With growing signal-to-noise ratio, signals sets comprising discrete points with equal probability have an increasing loss in power efficiency against signal sets with Gaussian distributed amplitude coefficients. This difference is the *shaping gain* [32]. Its maximum value is equal to $10\log_{10}(\pi e/6) = 1.53$ dB, and will be achieved for an infinite number of signal points.

It is important to note that for $10\log_{10}(E_b/N_0) \lesssim 2$ dB both conventional demodulation and SLISE lead to the same spectral efficiency. In this context, the optimum transmission rate R_{opt}, corresponding to the results in Fig. 5, is of interest and shown in Fig. 6. The optimum rate is zero for $10\log_{10}(E_b/N_0) \lesssim 2$ dB and increases with growing signal-to-noise ratio but does not reach the maximum rate of the signal set. This means that uncoded transmission is not optimum even for very high signal-to-noise ratios. In the case of conventional CDMA, the optimum rate is zero for all signal-to-noise ratios. It is remarkable that for the signal-to-noise ratios which correspond to an optimum rate of zero, SLISE and conventional demodulation lead to the same spectral efficiency. The value of E_b/N_0 where R_{opt} becomes greater than zero first, seems to be the break-even point of interference suppression.

V. Spectral Efficiency of Cellular Systems

In cellular environments, SLISE is advantageous compared to joint demodulation because in the latter case, interference from other cells could be coped with only by increased receiver complexity. In contrast, receivers with SLISE need not be changed compared to the single-cell case since the equalizer filter is adjusted to estimated statistical properties of the interference and does not need any information about signature sequences of other users.

For convenience, we assume a simplified scenario of cells with hexagonal shape. The distribution of users within each cell is uniform. For simplified analysis, we neglect the interference caused by cells which are not neighbors of the considered cell. Only the uplink is addressed because it limits user capacity of the total system. In each cell, perfect power control is assumed. The analysis is further simplified by not taking into account slow or fast fading. Exclusively, the attenuation due to power loss is considered. Following [33], it is given by

$$\alpha(r) = \left(\frac{r}{r_0}\right)^{-D} \tag{19}$$

where r, r_0, and D are the distance from the base station, an arbitrary constant for normalization, and the attenuation coefficient, respectively. Here, we assume $D = 4$, which is a typical value for urban environments [33].

As a consequence of the cellular environment, the definition of spectral efficiency has to be generalized again. Now, K is defined as the number of users per cell. Then, (16) can be used to calculate spectral efficiency γ [bit/(s · Hz · cell)], cf. [21, Ch. 6], [22].

For conventional CDMA systems, spectral efficiency is calculated by numerical integration [21, Ch. 6], [22]. By the use of the ratio ζ of other-cell interference (at base station for given cell) to intracell interference introduced in [34], it is given, in equivalence to (17), by

$$\gamma = \frac{1}{1+\zeta}\left[\left(\frac{E_b}{I_0}\right)_r^{-1} - \left(\frac{E_b}{N_0}\right)^{-1}\right] + \frac{R}{N}.$$

Taking into account six neighbored cells and assuming $D = 4$, $\zeta = 0.38$ yields. If an infinite number of cells were assumed, ζ would be equal to 0.44, cf. [21, Ch. 6], [22].

No analytical expression of spectral efficiency has been found for CDMA systems with SLISE. Therefore, it is determined by means of simulations equivalently to Section IV. The simulation has been extended by taking into account users in six neighboring cells, which apply perfect power control by the corresponding base station. The distances of the users from the corresponding base station and from the base station where signal-to-interference ratio is calculated are taken into account by (19). The computation of the signal-to-interference ratio and spectral efficiency is as described in Section IV.

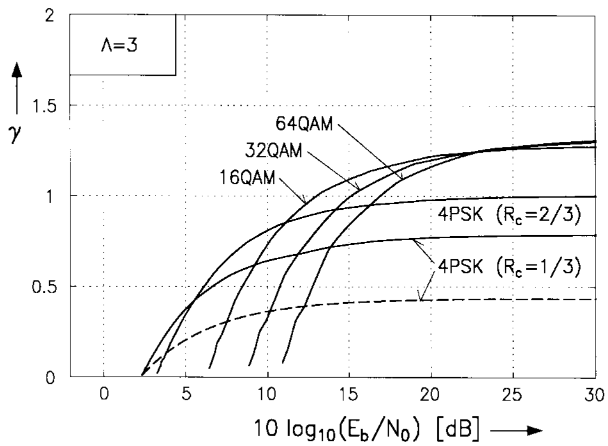

Fig. 7. Spectral efficiency of cellular CDMA systems with SLISE (—) and conventional demodulation (– –); M-ary PSK/QAM with convolutional code (constraint length seven), spreading factor $N = 64$, symbol-asynchronous transmission, $\Lambda = 3$; $D = 4$.

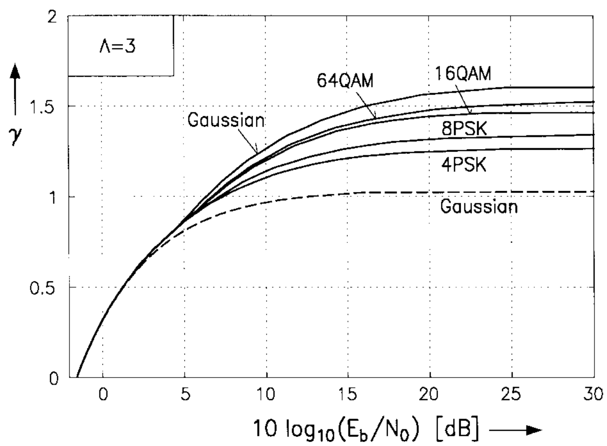

Fig. 8. Spectral efficiency of cellular CDMA systemswith SLISE (—) and conventional demodulation (– –); M-ary PSK/QAM and Gaussian distributed amplitude coefficients with optimum channel coding, spreading factor $N = 64$, symbol-asynchronous transmission, $\Lambda = 3$; $D = 4$.

Numerical examples for spectral efficiency are presented in Figs. 7 and 8 for an equalizer filter with length equal to three times the spreading factor. They correspond to Figs. 4 and 5 for the single-cell case.

It turns out that the increase of MAI by the interferers in neighboring cells leads to a reduction of spectral efficiency, which is more severe if spectral efficiency is high for the single-cell scenario. The values of saturation for conventional CDMA are reduced by a factor of 1.3 and 1.16 for optimum channel coding and for convolutional coding with rate $1/3$, respectively. The degradation of CDMA with SLISE is more severe. As shown in Fig. 7, signal sets with more than 16 points do not lead to a significant increase of spectral efficiency for high signal-to-noise ratios, which is in contrast to the corresponding single-cell scenario.

In Fig. 8, spectral efficiency is depicted for optimum channel coding and various signal sets. Like in Fig. 5, large signal sets lead to the best performance. But the choice $M > 16$ seems to be unreasonable for practical applications because of the little additional gain. Especially, the result for Gaussian distributed amplitude coefficients shows that in cellular environments, spectral efficiency of CDMA systems with SLISE seems to saturate.

VI. Summary

In this paper, CDMA communication with scalar linear MMSE interference suppression is investigated. After the introduction of the communication system, it was first shown that chip-asynchronism has a minor impact on performance if the length of the equalizer filter is equal to several times the spreading factor. Restricted to chip-synchronous transmission, spectral efficiency is determined for CDMA systems with both linear interference suppression and conventional demodulation.

It is shown that linear interference suppression outperforms conventional demodulation significantly. The gain, which is enormous in single-cell systems, is reduced in cellular environments. It is demonstrated that different modulation schemes are optimum depending on signal-to-noise ratio. In cellular environments with standard parameters, signal sets with more than 16 points are not useful.

Acknowledgment

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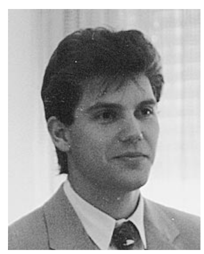

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