ASYMPTOTIC ANALYSIS OF ITERATIVE CHANNEL ESTIMATION AND MULTIUSER DETECTION WITH SOFT FEEDBACK IN MULTIPATH CHANNELS

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ABSTRACT

Large system analysis of a randomly spread direct-sequence code-division multiple-access system in a frequency-selective channel is considered. The receiver uses iterative channel estimation and multiuser detection (MUD) with soft feedback from single-user decoders to refine both the decisions of the linear minimum mean square error (LMMSE) MUD, and the initial pilot-based estimates of the LMMSE channel estimator. Replica method, a tool from statistical physics, and density evolution with Gaussian approximation are used to obtain the asymptotic performance of the LMMSE MUD with soft parallel interference cancellation. The results indicate that as the channel coherence time grows, the performance of corresponding single-user system having perfect channel state information can be approached even in an overloaded multiuser system using channel estimation and vanishing pilot overhead.

1. INTRODUCTION

In this paper, we consider large system analysis of a direct-sequence code-division multiple-access system (DS-CDMA) operating over a multipath fading channel. To mitigate the multiple-access interference (MAI) causing severely degraded performance with linear receivers [1], we take the factor-graph approach proposed in [2,3] and combine the linear data estimation with soft parallel interference cancellation (PIC). Instead of assuming perfect channel state information (CSI), the impact of CSI mismatch on the data estimation is also considered. To reduce the pilot overhead and improve the convergence behavior of the multiuser detector (MUD), we utilize the feedback from the single-user decoders iteratively in refining the initial pilot-based channel estimates. In contrast to hard decision based schemes analyzed by Li *et al.* [4], we consider the soft feedback framework proposed in [5, 6]. This avoids error propagation and accurate initialization usually required in hard decision based schemes. We also remark that in [4] the authors consider least-squares channel estimator and a single-user matched filter with hard PIC. It should be noted that our analysis differs from [7] since we consider also the problem of updating the channel estimates iteratively, and our treatment of CSI mismatch differs from assumptions made in [7, Sec. 2.2].

To assess the performance of iterative data and channel estimation, we combine density evolution with Gaussian approximation (DE-GA) [2, 8] and the replica method, a standard tool from statistical physics. Although the assumptions made in replica method are heuristic and their rigorous justification is still an open problem in mathematical physics, it has been applied recently with great success to a multitude of problems in telecommunications (see, e.g., [9–14]). The replica method allows for the analysis of generalized posterior mean estimators (GPME) [10, 12, 14] that include as special cases both the linear and non-linear minimum mean square error (MMSE) estimators. Due to the space constraints and for the interest of practical implementation, however, this contribution considers only linear MMSE (LMMSE) data and channel estimators.

THIS WORK WAS PARTIALLY SUPPORTED BY THE RESEARCH COUNCIL OF NORWAY (GRANT 171133/V30).

We specialize our numerical results for a system operating over a channel with three equal power paths and channel coherence time of 100 symbols. For a finite size system this corresponds approximately to a data rate of 120 kbps, spreading factor $L = 32$, and mobile speed 120 km/h in a Universal Mobile Telecommunications System (UMTS) network. For the average signal-to-noise ratio (SNR) per information bit $\bar{\gamma}_\mathrm{b} = 6$ dB, fixed load $\alpha = 1.2$, half-rate $(753,561)_8$ convolutional code and Gray encoded quarternary phase shift keying (QPSK), we find that the iterative system achieves at large system limit a multiuser efficiency (ME) $\eta \approx 0.95$ at pilot overhead of 1%. For varying load and fixed code rate, the spectral efficiency, taking into account the pilot overhead, at target bit error rate (BER) $\leq 10^{-5}$ shows a maximum degradation of 1 dB for the iterative receiver compared to perfect CSI.

Throughout this paper, we write $\mathbf{x}_0 \sim \mathbb{P}$ and $\mathbf{x} \sim \mathbb{Q}$ for a random vector (RV) drawn according to the true and postulated probability measure (or distribution), respectively. If $\mathbf{x}$ is a zero-mean proper complex Gaussian RV with covariance matrix $\mathbf{R}$, we write $\mathbf{x} \sim \mathsf{CN}(\mathbf{0},\mathbf{R})$. Calligraphic symbols denote for sets and boldface lowercase and uppercase symbols for (column) vectors and matrices, respectively. For a $M \times N$ matrix $\mathbf{A} = \begin{bmatrix}\mathbf{a}_1 \ \mathbf{a}_2 \ \cdots \ \mathbf{a}_N\end{bmatrix}$, $\mathrm{vec}(\mathbf{A}) = \begin{bmatrix}\mathbf{a}_1^\mathrm{T} \ \mathbf{a}_2^\mathrm{T} \ \cdots \ \mathbf{a}_N^\mathrm{T}\end{bmatrix}^\mathrm{T}$ and operator $\otimes$ is the Kronecker product. For positive definite matrix $\mathbf{A}$ we abbreviate $\mathbf{A} > 0$, $\mathbf{e}_M = [1,1,\ldots,1]^\mathrm{T} \in \mathbb{R}^M$, and $\vec{\mathbf{x}}$ is a $1 \times N$ row vector. $\mathsf{E}\{\mathbf{x}\}$ denotes for statistical expectation and $\mathrm{Cov}\{\mathbf{x}\} = \mathsf{E}\big\{\big(\mathbf{x} - \mathsf{E}\{\mathbf{x}\}\big)\big(\mathbf{x} - \mathsf{E}\{\mathbf{x}\}\big)^\mathrm{H}\big\}$.

2. SYSTEM MODEL AND ASSUMPTIONS

Consider a synchronous uplink DS-CDMA system operating over M-path block fading channel [15]. Following the framework of [1], we assume that the delay spread of the channel is small compared to the symbol time and neglect the effects of intersymbol interference.

Assume the transmission takes place over fading blocks $c = 1,2,\ldots,C$, each having a coherence time of T_c symbols. The users transmit information independently using binary code $\mathscr{C}$. The code words are interleaved and modulated by using Gray mapping onto the standard QPSK signal set $\mathscr{M} = \{\pm \frac{1}{\sqrt{2}} \pm \frac{j}{\sqrt{2}}\}$. We denote the QPSK modulated code word for the user $k = 1,\ldots,K$ by $\mathbf{x}_{0k} = \mathrm{vec}\big(\big[\mathbf{x}_{0k}[1] \cdots \mathbf{x}_{0k}[C]\big]\big), \in \mathscr{M}^N$, where $\mathbf{x}_{0k}[c] \in \mathscr{M}^{\tau_\mathrm{d}}$, and assume that $\mathsf{E}\{\mathbf{x}_{0k}\} = 0$ and $\mathrm{Cov}\{\mathbf{x}_{0k}\} = \mathbf{I}$. In addition, $\tau_\mathrm{p} = T_\mathrm{c} - \tau_\mathrm{d}$ independent identically distributed (IID) pilot symbols $\mathbf{p}_k[c] \in \mathscr{M}^{\tau_\mathrm{p}}$, are drawn uniformly from $\mathscr{M}$ and placed in each block for initial channel estimation. In the limit $N = \tau_\mathrm{d} C \to \infty$, with τ_d fixed, due to random bit-interleaving (cf. Remark 1) the channel can thus be considered to be ergodic over the entire code word.

The received signal within the cth fading block reads [1]

$$\begin{bmatrix}\mathbf{y}_l^\mathrm{p}[c] \\ \mathbf{y}_l^\mathrm{d}[c]\end{bmatrix} = \frac{1}{\sqrt{L}} \sum_{k=1}^{K} \begin{bmatrix}\mathbf{p}_k[c] \\ \mathbf{x}_{0k}[c]\end{bmatrix} \vec{\mathbf{s}}_{kl} \mathbf{h}_{0k}[c] + \sigma_0 \mathbf{w}_l[c] \in \mathbb{C}^{T_\mathrm{c}}, \quad (1)$$

where $l = 1,\ldots,L$ is the chip index, $\mathbf{y}_l^\mathrm{p}[c] \in \mathbb{C}^{\tau_\mathrm{p}}$, $\mathbf{y}_l^\mathrm{d}[c] \in \mathbb{C}^{\tau_\mathrm{d}}$, are the received signals during the pilot and data transmission phases, respectively, and $\mathbf{w}_l[c] \sim \mathsf{CN}(\mathbf{0},\mathbf{I})$ denotes for the additive white Gaussian noise. We write $\mathscr{Y} = \{\mathbf{y}_l^{\mathrm{p}}[c], \mathbf{y}_l^{\mathrm{d}}[c] \,|\, \forall l, c\}$, and let the set of all channel coefficients, pilot symbols, data symbols and spreading sequences be denoted by $\mathscr{H}_0 = \{\mathbf{h}_{0k}[c] \,|\, \forall k, c\}$, $\mathscr{X}_{\mathrm{p}} = \{\mathbf{p}_k[c] \,|\, \forall k, c\}$, $\mathscr{X}_0 = \{\mathbf{x}_{0k}[c] \,|\, \forall k, c\}$ and $\mathscr{S} = \{\vec{\mathbf{s}}_{kl} \,|\, \forall k, l\}$, respectively, where $\vec{\mathbf{s}}_{kl} = [s_{kl}^1 \cdots s_{kl}^M] \sim \mathsf{CN}(\mathbf{0},\mathbf{I}_M)$ are IID (cf. [1, Theorems 3 and 4]) RVs for all $k = 1,\ldots,K, l = 1,\ldots,L$. In the following we assume that for $c = 1,\ldots,C$ and $k = 1,\ldots,K$, the channel vectors $\mathbf{h}_{0k}[c] = [h_{0k}^1[c] \cdots h_{0k}^M[c]]^{\mathrm{T}} \sim \mathbb{P}(\mathbf{h}_{0k}[c]) = \mathsf{CN}(\mathbf{0}, \frac{\overline{p}}{M}\mathbf{I}_M)$, are IID and the average received signal-to-noise ratio is $\overline{\gamma} = \overline{p}/\sigma_0^2$ for all users k and fading blocks c.

Remark 1. *In the following, it is implicitly assumed that the system fulfills the following assumptions (cf. [2, Ch. IV-A]):*

- *Bit-interleaved coded modulation [16] with trellis termination is used for error correction coding. Each user has the same basic code $\mathscr{C}$, but the random uniform bit and symbol-level interleavers are chosen independently for each user.*
- *Phase randomization is introduced by the transmitters, letting us to concentrate on all-ones code word in the DE-GA analysis.*

We omitted the phase randomization in (1) *for convenience.*

2.1 Iterative Estimation and Detection

Consider the general framework of [5, 6], and let $\mathscr{I} = \{\mathscr{Y}, \mathscr{X}_{\mathrm{p}}, \mathscr{S}\}$. At iteration $i = 0$, channel estimation based on $\mathscr{I}$ is first performed $\langle\langle h_k^m[c]\rangle\rangle_{(0)} = \int h_k^m[c]\mathbb{Q}(\mathrm{d}\mathscr{H}^{(0)}|\mathscr{I})$, where we denoted $\mathscr{H}^{(0)} = \{h_k^m[c] \,|\, \forall k, c, m\}$. The information at MUD is $\mathscr{I}$ and distribution $\mathbb{Q}(\mathscr{H}^{(0)} \,|\, \mathscr{I})$, provided by the channel estimator. In our case, the MUD performs then LMMSE data estimation and PIC based on *extrinsic information* obtained by the single-user decoders [2, 3]. The soft estimates of the transmitted symbols based on the *a posteriori* probabilities from the decoders are $\tilde{\mathscr{X}}^{(0)} = \{\tilde{\mathbf{x}}_k^{(0)}[c] \,|\, \forall k, c\}$ where $\tilde{\mathbf{x}}_k^{(0)}[c] = [\tilde{x}_{k\tau_{\mathrm{p}}+1}^{(0)}[c] \cdots \tilde{x}_{kT_{\mathrm{c}}}^{(0)}[c]]^{\mathrm{T}}$, $\tilde{x}_{kn}^{(0)}[c] = \int x_{kn}^{(0)}[c]\mathbb{Q}_{\mathsf{APP}}(\mathrm{d}\mathscr{X}_k^{(0)}|\mathscr{I})$, and $\mathscr{X}_k^{(0)} = \{x_{kn}^{(0)}[c] \,|\, n = \tau_{\mathrm{p}} + 1,\ldots,T_{\mathrm{c}}, c = 1,\ldots,C\}$. The feedback from MUD to channel estimator for $i = 1$ is then $\mathbb{Q}_{\mathsf{APP}}(\tilde{\mathscr{X}}^{(0)}|\mathscr{I})$, that is used to postulate a new measure $\mathbb{Q}(\mathscr{H}^{(1)}|\mathscr{I})$.

2.2 LMMSE Channel Estimation With Soft Information

Let us drop the block index and concentrate on one fading block. We postulate a channel model at ith iteration for $l = 1,\ldots,L$

$$\begin{bmatrix}\mathbf{y}_l^{\mathrm{p}} \\ \mathbf{y}_l^{\mathrm{d}}\end{bmatrix} = \frac{1}{\sqrt{L}}\sum_{k=1}^{K}\begin{bmatrix}\mathbf{p}_k \\ \tilde{\mathbf{x}}_k^{(i-1)}\end{bmatrix}\vec{\mathbf{s}}_{kl}\mathbf{h}_k + \sum_{m=1}^{M} s_{kl}^m \begin{bmatrix}\mathbf{0} \\ h_k^m \Delta\tilde{\mathbf{x}}_k^{(i-1)}\end{bmatrix} + \sigma\mathbf{w}_l,$$

where $\Delta\tilde{\mathbf{x}}_k^{(i-1)} = \mathbf{x}_{0k} - \tilde{\mathbf{x}}_k^{(i-1)} \in \mathbb{C}^{\tau_{\mathrm{d}}}$, is the error in data estimation, and $\mathbf{h}_k = [h_k^1 \cdots h_k^M]^{\mathrm{T}} \sim \mathbb{Q}(\mathbf{h}_k) = \mathsf{CN}(\mathbf{0}, \frac{\overline{p}}{M}\mathbf{I})$. Since the channel was assumed to be ergodic over the code word, we treat $\mathbf{h}_k$ and $\Delta\tilde{\mathbf{x}}_k^{(i-1)}$ as being statistically independent, which simplifies the analysis considerably. Unfortunately, due to the multiplicative error term $\Delta\tilde{\mathbf{x}}_k^{(i-1)}$, letting $\sigma^2 = \sigma_0^2$ does not yield an LMMSE estimator.

Our solution is to assume that $\mathbb{Q}_{\mathsf{APP}}(\mathscr{X}^{(i-1)}|\mathscr{I})$ is a product of K distributions whose mean and covariance are given by DE-GA [8], and postulate that vector $\Delta\mathbf{w}_{km}^{(i-1)} = \mathrm{vec}\{[\mathbf{0}, h_k^m \Delta\tilde{\mathbf{x}}_k^{(i-1)}]\} \in \mathbb{C}^{T_{\mathrm{c}}}$ conditioned on $\mathscr{I}$ is a zero-mean complex Gaussian RV[1]. Denoting $\Delta\mathscr{W}^{(i-1)} = \{\Delta\mathbf{w}_{km}^{(i-1)} \,|\, \forall k, m\}$, the GPME for user k and path m is then given by

$$\langle\langle h_k^m\rangle\rangle_{(i)} = \frac{\mathsf{E}_{\mathscr{H},\Delta\mathscr{W}^{(i-1)}}\{h_k^m \mathbb{Q}(\mathscr{Y} \,|\, \mathscr{I}, \mathscr{H}, \tilde{\mathscr{X}}^{(i-1)}, \Delta\mathscr{W}^{(i-1)})\}}{\mathsf{E}_{\mathscr{H},\Delta\mathscr{W}^{(i-1)}}\{\mathbb{Q}(\mathscr{Y} \,|\, \mathscr{I}, \mathscr{H}, \tilde{\mathscr{X}}^{(i-1)}, \Delta\mathscr{W}^{(i-1)})\}}. \tag{2}$$

[1]We interpret $\mathbf{x} \sim \mathsf{CN}(\mathbf{0},\mathbf{0}) \iff \mathbf{x} = \mathbf{0}$.

It can be shown, that for $\sigma^2 = \sigma_0^2$, $\langle\langle h_k^m\rangle\rangle_{(i)}$ corresponds to the LMMSE estimator proposed, e.g., in [17].

3. PERFORMANCE ANALYSIS

Let $\mathbb{Q}(\mathscr{H}^{(i)} \,|\, \mathscr{I})$, where $\mathbf{h}_k^{(i)} \,|\, \mathscr{I} \sim \prod_{m=1}^{M} \mathsf{CN}(\langle\langle h_k^m[c]\rangle\rangle_{(i)}, \xi_c^{(i)})$ are IID[2] and $\xi_c^{(i)} = \xi_{km}^{(i)}[c], \forall k, m$ are the per-path mean squared errors (MSEs) [1], be the information provided by the channel estimator at ith iteration, and consider estimation of data symbols $\mathbf{x}_{01}[c]$.

Proposition 1. *Let first $N = \tau_{\mathrm{d}}C \to \infty$ and then $K = \alpha L \to \infty$ with α and τ_{d} finite and fixed. The multiuser efficiency of the first user for the LMMSE MUD with soft-PIC converges then almost surely to*

$$\eta_1^{(i)}[c] = \sum_{m=1}^{M} |\langle\langle h_1^m[c]\rangle\rangle_{(i)}|^2 \left[\overline{\gamma}\left(\xi_c^{(i)} + 1/\beta_{\mathrm{d}}^{(i)}\right)\right]^{-1},$$

at ith iteration, where $\beta_{\mathrm{d}}^{(i)}$ is the unique solution to the fixed point equation given in (3) *at the top of the next page, in which $\overline{\gamma}_{\max}$ is the upper bound for the truncated SNR distribution, μ is a function*

$$\mu(\overline{\gamma}\eta^{(i-1)}[c]) = \left[Q^{-1}\left(\varepsilon(\overline{\gamma}\eta^{(i-1)}[c])\right)\right]^2,$$

where $Q(x) = \int_x^\infty \mathrm{D}z$, $\mathrm{D}z = (2\pi)^{-1/2}\exp(-z^2/2)\mathrm{d}z$, and $\varepsilon(\cdot)$ is the bit error probability of the extrinsic bits of a channel code in ergodic Rayleigh fading channel with average SNR $\overline{\gamma}\eta^{(i-1)}[c]$.

Proposition 1 provides the ME of the MUD, given the MSEs $\{\xi_c^{(i)}\}$. This is used in turn in DE-GA analysis to derive the statistics for the channel estimator. The next proposition obtains $\{\xi_c^{(i)}\}$.

Definition 1. *Consider the system model* (1) *and let us define a corresponding flat fading single-user channel*

$$\mathbf{z}_{km} = \mathbf{x}_{0k}^{\mathrm{p,d}} h_{0k}^m + \tilde{\mathbf{w}}_{0k}^m, \quad \mathbf{x}_{0k}^{\mathrm{p,d}} = \mathrm{vec}([\mathbf{p}_k \; \mathbf{x}_{0k}]),$$

where $\tilde{\mathbf{w}}_{0k}^m \sim \mathsf{CN}(\mathbf{0}, \mathbf{\Sigma}_0^{(i)})$, for all $m = 1,2,\ldots,M$. Let us also define a postulated measure $\mathbb{Q}$ based on the channel

$$\mathbf{z}_{km} = \mathbf{x}_k^{(i-1)} h_k^m + \Delta\mathbf{w}_{km}^{(i-1)} + \tilde{\mathbf{w}}_{km}^{(i)}, \quad \mathbf{x}_k^{(i-1)} = \mathrm{vec}([\mathbf{p}_k \; \tilde{\mathbf{x}}_k^{(i-1)}]),$$

where $\Delta\mathscr{W}_k^{(i-1)} = \{\Delta\mathbf{w}_{km}^{(i-1)} \,|\, \forall m\}$, and $\tilde{\mathbf{w}}_{km}^{(i)} \sim \mathsf{CN}(\mathbf{0}, \mathbf{\Sigma}^{(i)}), \forall k = 1,\ldots,K, m = 1,\ldots,M$. The corresponding single-user GPME reads

$$\langle h_k^m\rangle_{(i)} = \frac{\mathsf{E}_{\mathbf{h}_k,\Delta\mathscr{W}_k^{(i-1)}}\{h_k^m \prod_{m=1}^{M}\mathbb{Q}(\mathbf{z}_{km} \,|\, h_k^m, \mathbf{p}_k, \tilde{\mathbf{x}}_k^{(i-1)}, \Delta\mathbf{w}_{km}^{(i-1)})\}}{\mathsf{E}_{\mathbf{h}_k,\Delta\mathscr{W}_k^{(i-1)}}\{\prod_{m=1}^{M}\mathbb{Q}(\mathbf{z}_{km} \,|\, h_k^m, \mathbf{p}_k, \tilde{\mathbf{x}}_k^{(i-1)}, \Delta\mathbf{w}_{km}^{(i-1)})\}}, \tag{4}$$

that can be shown to be equivalent to the LMMSE estimator in [17]. The true and postulated effective noise covariance matrices are given by the solutions to the coupled fixed point equations

$$\mathbf{\Sigma}_0^{(i)} = \sigma_0^2\mathbf{I}_{T_{\mathrm{c}}} + \alpha \lim_{K\to\infty}\frac{1}{K}\sum_{k=1}^{K}\sum_{m=1}^{M} E_{km}(\mathbf{\Sigma}_0^{(i)}, \mathbf{\Sigma}^{(i)}), \tag{5}$$

$$\mathbf{\Sigma}^{(i)} = \sigma^2\mathbf{I}_{T_{\mathrm{c}}} + \alpha \lim_{K\to\infty}\frac{1}{K}\sum_{k=1}^{K}\sum_{m=1}^{M} V_{km}(\mathbf{\Sigma}_0^{(i)}, \mathbf{\Sigma}^{(i)}), \tag{6}$$

respectively. Denote also $\mathbf{v}_{km}^{(i-1)} = \mathbf{x}_k^{(i-1)} h_k^m + \Delta\mathbf{w}_{km}^{(i-1)}$, so that

$$E_{km}(\mathbf{\Sigma}_0^{(i)}, \mathbf{\Sigma}^{(i)}) = \mathrm{Cov}\left\{\mathbf{x}_{0k}h_{0k}^m - \langle\mathbf{v}_{km}^{(i-1)}\rangle \,\middle|\, \mathscr{X}_0, \mathscr{X}_{\mathrm{p}}, \tilde{\mathscr{X}}^{(i-1)}\right\},$$

$$V_{km}(\mathbf{\Sigma}_0^{(i)}, \mathbf{\Sigma}^{(i)}) = \mathrm{Cov}\left\{\mathbf{v}_{km}^{(i-1)} - \langle\mathbf{v}_{km}^{(i-1)}\rangle \,\middle|\, \mathscr{X}_0, \mathscr{X}_{\mathrm{p}}, \tilde{\mathscr{X}}^{(i-1)}\right\},$$

are the error covariance and covariance matrices of the estimator $\langle\mathbf{v}_{km}^{(i-1)}\rangle$, respectively.

[2]This is not true in general even though the channel coefficients are IID for all k. We make this assumption to keep the problem tractable.

$$
\begin{aligned}
\frac{1}{\beta_{\mathrm{d}}} = \sigma_0^2 + \alpha(M-1)\frac{\xi_c^{(i)}}{1+\xi_c^{(i)}\beta_{\mathrm{d}}} + \alpha\int_0^{\overline{\gamma}_{\max}} \mathrm{d}p \frac{p^{M-1}}{\int_0^{\overline{\gamma}_{\max}} p^{M-1}\exp\left(-p/\left(\overline{p}/M-\xi_c^{(i)}\right)\right)\mathrm{d}p}\exp\left(-\frac{p}{\overline{p}/M-\xi_c^{(i)}}\right) \\
\times\int_{\mathbb{R}^2}\frac{\frac{p}{2}\left[2-\tanh^2\left(z_1\sqrt{\mu\left(\overline{\gamma}\eta^{(i-1)}[c]\right)}+\mu\left(\overline{\gamma}\eta^{(i-1)}[c]\right)\right)-\tanh^2\left(z_2\sqrt{\mu\left(\overline{\gamma}\eta^{(i-1)}[c]\right)}+\mu\left(\overline{\gamma}\eta^{(i-1)}[c]\right)\right)\right]+\xi_c^{(i)}}{1+\beta_{\mathrm{d}}\left\{\frac{p}{2}\left[2-\tanh^2\left(z_1\sqrt{\mu\left(\overline{\gamma}\eta^{(i-1)}[c]\right)}+\mu\left(\overline{\gamma}\eta^{(i-1)}[c]\right)\right)-\tanh^2\left(z_2\sqrt{\mu\left(\overline{\gamma}\eta^{(i-1)}[c]\right)}+\mu\left(\overline{\gamma}\eta^{(i-1)}[c]\right)\right)\right]+\xi_c^{(i)}\right\}}\mathrm{D}z_1\mathrm{D}z_2
\end{aligned} \tag{3}
$$

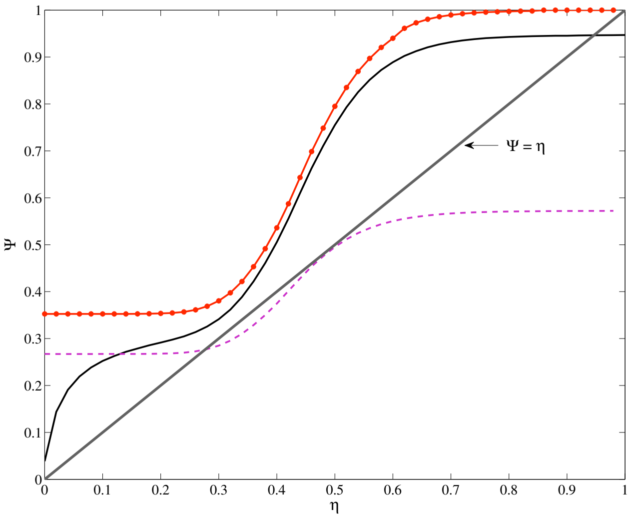

Figure 1: Mapping function Ψ for the LMMSE MUD with PIC. Three equal power paths, coherence time of $T_{\mathrm{c}} = 100$ symbols, $\overline{\gamma}_{\mathrm{b}} = 6$ dB, system load $\alpha = 1.2$, rate-1/2 convolutional code with generators $(753,561)_8$ and Gray encoded QPSK. Dotted line for perfect CSI, solid line for iterative channel estimation ($\tau_{\mathrm{p}} = 1$) and dashed line for non-iterative channel estimation ($\tau_{\mathrm{p}} = 10$).

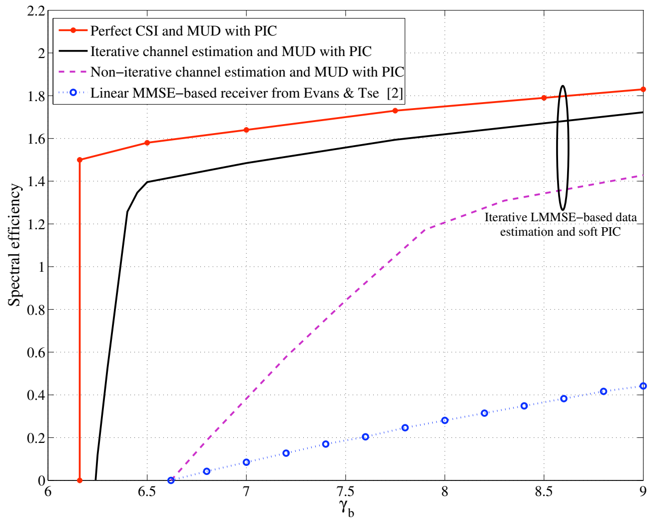

Figure 2: Spectral efficiency vs. $\overline{\gamma}_{\mathrm{b}}$ at target BER $\leq 10^{-5}$. Three equal power paths, coherence time of $T_{\mathrm{c}} = 100$ symbols, rate-1/2 convolutional code with generators $(753,561)_8$ and Gray encoded QPSK. For iterative MUD and channel estimation $\tau_{\mathrm{p}} = 1$. To guarantee early convergence and positive throughput also with non-iterative channel estimators, $\tau_{\mathrm{p}} = 15$ pilots were used for them.

Proposition 2. *Assuming the distribution of the feedback is given by the DE-GA, conditioned on $\{\mathscr{X}_0, \mathscr{X}_{\mathrm{p}}, \tilde{\mathscr{X}}^{(i-1)}\}$ the distribution of $\langle h_k^m\rangle_{(i)}$ is identical to that of $\langle\langle h_k^m\rangle\rangle_{(i)}$ in the large system limit.*

Corollary 1. *Let $\zeta_0^{(i)}$ be the solution to the fixed point equation $\zeta_0^{(i)}\mathbf{I} = \sigma_0^2 + \alpha M\mathsf{E}\{E_{k,m}(\zeta_0^{(i)}\mathbf{I}, \zeta_0^{(i)}\mathbf{I})\}$ in the large system limit. The MSE $\xi_{km}^{(i)}$ for user k and path m of the iterative LMMSE channel estimator* (2) *satisfies then $\xi_{km}^{(i)}\mathbf{I} = \mathsf{E}\{E_{k,m}(\zeta_0^{(i)}\mathbf{I}, \zeta_0^{(i)}\mathbf{I})\}$.*

4. NUMERICAL EXAMPLES

Let us consider a half-rate convolutional code with generator polynomials $(753,561)_8$. With QPSK the average SNR per information bit is then $\overline{\gamma}_{\mathrm{b}} = \overline{\gamma}$. A coherence time of 100 symbols is assumed, corresponding roughly to a mobile user moving at the speed of 120 km/h and with bit-rate 120 kbps and $L = 32$ in the UMTS network. We remark that the numerical examples given here are based on the asymptotic results $N = \tau_{\mathrm{d}}C \to \infty, K = \alpha L \to \infty$ obtained in Section 3 and for finite systems they are only approximations. In general, however, the large system analysis yield good predictions already for small system sizes [1, 12] and, thus, we feel the insight gained from this analysis is valuable also in practice.

Figure 1 shows the mapping function [2, Eq. (55)]

$$
\eta^{(i)} = \Psi(\eta^{(i-1)}, \alpha, \overline{\gamma}, \xi^{(i)}, M)
$$

for the receiver with iterative channel estimation and MUD with PIC. Note that the MSEs $\xi^{(i)}$ implicitly depend on the coherence time, number of pilots, average received SNR and the reliability of the feedback. Curves for perfect CSI and non-iterative channel estimator with $\tau_{\mathrm{p}} = 10$ pilots are reported for comparison. At load $\alpha = 1.2$ and $\overline{\gamma}_{\mathrm{b}} = 6$ dB, the iterative system converges to $\eta \approx 0.95$ using only a single pilot symbol. Note that the iterative channel estimation changes the shape of the DE curve so that the MUD converges to high ME even with a very low starting point at $\eta \approx 0.04$. This behavior comes from the use of *a posteriori* probabilities in the feedback, which helps improving the MSEs even in regions where the PIC based MUD has not yet started converging. This is in contrast to the results in [4] where a good initialization was found to be critical. With *extrinsic* information based channel estimation (curve not shown) eight pilots are needed in order to converge similar performance, and with non-iterative channel estimation a maximum ME of $\eta \approx 0.27$ is achieved.

Spectral efficiency, taking into account the loss in information rate due to adding pilot symbols, versus $\overline{\gamma}_{\mathrm{b}}$ at target BER $\leq 10^{-5}$ is shown in Figure 2. With perfect CSI, the threshold SNR for positive spectral efficiency is 6.16 dB and for iterative channel estimation with $T_{\mathrm{c}} = 100$ and $\tau_{\mathrm{p}} = 1$, $\overline{\gamma}_{\mathrm{b}} = 6.24$ dB. With channel estimation, however, there is no "jump", and the slope of the curve is finite everywhere. We note that by using soft feedback based iterative channel estimator the threshold value for the positive rate is shifted to left and the shape of the curve changed at low $\overline{\gamma}_{\mathrm{b}}$ compared to the estimator in [1]. At high SNR, the slope of the spectral efficiency curve seems to be the same with that of perfect CSI, and the loss in spectral efficiency is about 1 dB for the iterative and 3 dB for the non-iterative channel estimation.

5. SKETCH OF PROOF OF PROPOSITION 2

Let us consider ith iteration for the LMMSE channel estimator (2), The denominator in (2) is denoted by $Z(\cdot)$ in the following. Henceforth we omit the iteration index for clarity.

Assumption 1 (Self-averaging property and replica continuity)**.** *The free energy at thermodynamic equilibrium can be written as*

$$F = \lim_{u\to 0}\frac{\partial}{\partial u}\lim_{K=\alpha L\to\infty}\frac{1}{K}\log \Xi_K^{(u)},$$

where $\Xi_K^{(u)} = \mathsf{E}\{Z^u(\mathscr{Y},\mathscr{H},\mathscr{S},\mathscr{X}_{\mathrm{p}},\tilde{\mathscr{X}}^{(i-1)},\Delta\mathscr{W}) \mid \mathscr{X}_0,\mathscr{X}_{\mathrm{p}},\tilde{\mathscr{X}}^{(i-1)}\}$ *is evaluated for positive integers u and analytic continuity in the vicinity of* 0 *is assumed to hold. In the following* $\mathsf{E}_{\mathrm{c}}\{\cdot\} = \mathsf{E}\{\cdot \mid \mathscr{X}_0,\mathscr{X}_{\mathrm{p}},\tilde{\mathscr{X}}^{(i-1)}\}$.

Due to random spreading, the L channels are IID, and thus,

$$\Xi_K^{(u)} = \mathsf{E}_{\mathrm{c}}\left\{\left[\mathsf{E}_{\mathscr{S}}\left\{\prod_{a=0}^{u}\frac{1}{(\pi\sigma_a^2)^{T_{\mathrm{c}}}}\exp\left(-\frac{1}{\sigma_a^2}\left\|\mathbf{y}-\sqrt{\alpha}\mathbf{v}_a\right\|^2\right)\right.\right.\right.$$
$$\left.\left.\left.\Bigg|\{\mathscr{H}_a\}_{a=0}^u,\{\mathscr{X}_a\}_{a=0}^u\right\}\mathrm{d}\mathbf{y}\right]^L\right\},$$

where the elements of the set $\tilde{\mathscr{S}} = \{s_k^m \mid k=1,\ldots,K, m=1,\ldots,M\}$ are IID and have the same distribution as $\{s_{kl}^m\}$. We also denoted $\mathscr{X}_a = \{\mathbf{x}_{ak}\}_{k=1}^K$, $\mathscr{H}_a = \{h_{ak}^m \mid k=1,\ldots,K, m=1,\ldots,M\}$ and $\Delta\mathscr{W}_a = \{\Delta\mathbf{w}_{ak}^m \mid k=1,\ldots,K, m=1,\ldots,M\}$ for all $a=0,1,\ldots,u$, and wrote $\sigma^2=\sigma_a^2$ for replica indices $a=1,2,\ldots,u$. The random vectors $\{\mathbf{v}_a\}_{a=0}^u$ are given by

$$\mathbf{v}_0 = \frac{1}{\sqrt{K}}\sum_{k=1}^{K}\sum_{m=1}^{M}s_k^m h_{0k}^m\mathbf{x}_{0k}^{\mathrm{p,d}},$$
$$\mathbf{v}_a = \frac{1}{\sqrt{K}}\sum_{k=1}^{K}\sum_{m=1}^{M}s_k^m\left(h_{ak}^m\mathbf{x}_{ak}+\Delta\mathbf{w}_{ak}^m\right),$$

where we denoted $\mathbf{x}_{0k}^{\mathrm{p,d}} = \mathrm{vec}([\mathbf{p}_k,\mathbf{x}_{0k}])$, $\mathbf{x}_{ak} = \mathrm{vec}([\mathbf{p}_k,\tilde{\mathbf{x}}_k])$, $a=1,\ldots,u$. By the central limit theorem, conditional on $\{\{\mathscr{H}_a\}_{a=0}^u,\{\mathscr{X}_a\}_{a=0}^u,\{\Delta\mathscr{W}_a\}_{a=0}^u\}$, as $K\to\infty$ the vector $\mathbf{v} = \mathrm{vec}(\mathbf{v}_0,\mathbf{v}_1,\ldots,\mathbf{v}_u)$ converges to zero-mean Gaussian random vector with conditional covariance matrix

$$\mathbf{Q} = \lim_{K\to\infty}\overline{\mathbf{Q}}_K = \lim_{K\to\infty}\frac{1}{K}\sum_{k=1}^{K}\sum_{m=1}^{M}\mathrm{vec}(\mathbf{\Omega}_{km})\mathrm{vec}(\mathbf{\Omega}_{km})^{\mathrm{H}}$$

where $\mathbf{\Omega}_{km} = \left[\mathbf{x}_{0k}^{\mathrm{p,d}}h_{0k}^m+\Delta\mathbf{w}_{0k}^m,\ldots,\mathbf{x}_{uk}h_{uk}^m+\Delta\mathbf{w}_{uk}^m\right]\in\mathbb{C}^{T_{\mathrm{c}}\times(u+1)}$. Following [10, Appendix II] it can be shown that

$$\Xi_K^{(u)} = \mathsf{E}_{\mathrm{c}}\left\{\exp\left[K\alpha^{-1}\left(G_u(\overline{\mathbf{Q}}_K)+O(K^{-1})\right)\right]\right\},$$

where $G_u(\overline{\mathbf{Q}}_K)$ is given in (7) at the top of the next page. Applying a vector form complex Gaussian integral

$$\int e^{-\mathbf{y}^{\mathrm{H}}\mathbf{A}\mathbf{y}+2\Re\{\mathbf{b}^{\mathrm{H}}\mathbf{y}\}}\mathrm{d}\mathbf{y} = \frac{\pi^{T_{\mathrm{c}}}}{\det(\mathbf{A})}e^{\mathbf{b}^{\mathrm{H}}\mathbf{A}^{-1}\mathbf{b}},\quad \mathbf{A}>0,$$

first for the integral with respect to $\mathbf{y}$ in (7), rearranging terms, and using it again for the expectation over $\mathbf{v}\sim\mathsf{CN}(\mathbf{0},\overline{\mathbf{Q}}_K)$ gives

$$G_u(\overline{\mathbf{Q}}_K) = -uT_{\mathrm{c}}\log(\pi\sigma^2) - T_{\mathrm{c}}\log\left(1+u\frac{\sigma_0^2}{\sigma^2}\right)$$
$$-\log\det\left(\mathbf{I}_{(u+1)T_{\mathrm{c}}}+\mathbf{A}\overline{\mathbf{Q}}_K\right),$$

where

$$\mathbf{A} = \frac{\alpha}{\sigma^2+u\sigma_0^2}\left[\begin{array}{c|c} u & -\mathbf{e}_u^{\mathrm{T}} \\ \hline -\mathbf{e}_u & (1+u\frac{\sigma_0^2}{\sigma^2})\mathbf{I}_u - \frac{\sigma_0^2}{\sigma^2}\mathbf{e}_u\mathbf{e}_u^{\mathrm{T}}\end{array}\right]\otimes\mathbf{I}_{T_{\mathrm{c}}}.$$

We define next a new probability measure

$$\mu_{\overline{\mathbf{Q}}_K}(J) = \mathsf{E}_{\mathrm{c}}\left\{1_J\left[\overline{\mathbf{Q}}_K\left(\{\mathscr{H}_a\}_{a=0}^u,\{\mathscr{X}_a\}_{a=0}^u\right)\right]\right\} \tag{8}$$

where J is a convex set in V, a space of $(u+1)T_{\mathrm{c}}\times(u+1)T_{\mathrm{c}}$ positive definite matrices, so that

$$\Xi_K^{(u)} = \int\exp\left[K\beta^{-1}G_u(\overline{\mathbf{Q}}_K)\right]\mu_{\overline{\mathbf{Q}}_K}(\mathrm{d}\mathbf{Q})+O(K^{-1}). \tag{9}$$

Applying Gärtner-Ellis theorem and Varadhan's lemma to (9) and neglecting the vanishing terms yields

$$F = \lim_{u\to 0}\frac{\partial}{\partial u}\sup_{\mathbf{Q}\in V}\left\{\alpha^{-1}G_u(\mathbf{Q})-I_u(\mathbf{Q})\right\}, \tag{10}$$

where the rate function I_u of (8) reads

$$I_u(\mathbf{Q}) = \sup_{\tilde{\mathbf{Q}}\in V}\left\{\mathrm{tr}(\tilde{\mathbf{Q}}\mathbf{Q}) - \lim_{K\to\infty}\frac{1}{K}\log\sum_{k=1}^{K}M_k^{(u)}(\tilde{\mathbf{Q}})\right\}, \tag{11}$$

and the corresponding kth moment generating function is

$$M_k^{(u)}(\tilde{\mathbf{Q}}) = \mathsf{E}_{\mathrm{c}}\left\{\exp\left[\sum_{m=1}^{M}\mathrm{vec}(\mathbf{\Omega}_{km})^{\mathrm{H}}\tilde{\mathbf{Q}}\,\mathrm{vec}(\mathbf{\Omega}_{km})\right]\right\}.$$

From (10) and (11) we get

$$\mathbf{Q}^* = \lim_{K\to\infty}\frac{1}{K}\sum_{k=1}^{K}\frac{1}{M_k^{(u)}(\tilde{\mathbf{Q}}^*)}\mathsf{E}_{\mathrm{c}}\left\{\sum_{m=1}^{M}\mathrm{vec}(\mathbf{\Omega}_{km})\mathrm{vec}(\mathbf{\Omega}_{km})^{\mathrm{H}}\right.$$
$$\left.\times\exp\left[\sum_{m=1}^{M}\mathrm{vec}(\mathbf{\Omega}_{km})^{\mathrm{H}}\tilde{\mathbf{Q}}^*\mathrm{vec}(\mathbf{\Omega}_{km}^{\mathrm{H}})\right]\right\} \tag{12}$$

$$\tilde{\mathbf{Q}}^* = -\alpha^{-1}\left(\mathbf{I}_{(u+1)T_{\mathrm{c}}}+\mathbf{A}\mathbf{Q}^*\right)^{-1}\mathbf{A}. \tag{13}$$

Assumption 2 (Replica symmetry)**.** *The saddle-point solution to* (12) – (13) *is invariant to the permutations of replica indices,*

$$\mathbf{Q}^* = \left[\begin{array}{c|c}\mathbf{R} & -\mathbf{e}_u^{\mathrm{T}}\otimes\mathbf{M} \\ \hline -\mathbf{e}_u\otimes\mathbf{M}^{\mathrm{H}} & \mathbf{I}_u\otimes(\mathbf{P}-\mathbf{U})+\mathbf{e}_u\mathbf{e}_u^{\mathrm{T}}\otimes\mathbf{U}\end{array}\right]$$
$$\tilde{\mathbf{Q}}^* = \left[\begin{array}{c|c}\mathbf{C} & -\mathbf{e}_u^{\mathrm{T}}\otimes\mathbf{D} \\ \hline -\mathbf{e}_u\otimes\mathbf{D}^{\mathrm{H}} & \mathbf{I}_u\otimes(\mathbf{G}-\mathbf{F})+\mathbf{e}_u\mathbf{e}_u^{\mathrm{T}}\otimes\mathbf{F}\end{array}\right]$$

where $\{\mathbf{U},\mathbf{P},\mathbf{G},\mathbf{F},\mathbf{R},\mathbf{C}\}$ *are* $T_{\mathrm{c}}\times T_{\mathrm{c}}$ *Hermitian matrices.*

From (13) we get with Assumption 2,

$$\mathbf{C}=\mathbf{0},\quad \mathbf{G}=\mathbf{F}-\mathbf{D},\quad \mathbf{D}=\mathbf{\Sigma}^{-1},\quad \mathbf{F}=\mathbf{\Sigma}^{-1}\mathbf{\Sigma}_0\mathbf{\Sigma}^{-1},$$

where we denoted

$$\mathbf{\Sigma}_0 = \sigma_0^2\mathbf{I}_{T_{\mathrm{c}}}+\alpha\left(\mathbf{R}-(\mathbf{M}+\mathbf{M}^{\mathrm{H}})+\mathbf{U}\right), \tag{14}$$
$$\mathbf{\Sigma} = \sigma^2\mathbf{I}_{T_{\mathrm{c}}}+\alpha(\mathbf{P}-\mathbf{U}). \tag{15}$$

The moment generating function under Assumption 2 is given in (16) at the top of the next page. Since $\mathbf{\Sigma}_0$ is positive definite, we

$$\exp\left(G_u(\overline{\mathbf{Q}}_K)\right) = \mathsf{E}_{\mathbf{v}}\left\{(\pi\sigma_0^2)^{-T_c}\int \exp\left[-\left(\frac{1}{\sigma_0^2}+\frac{u}{\sigma^2}\right)\|\mathbf{y}\|^2 + 2\Re\left\{\sqrt{\alpha}\left(\frac{1}{\sigma_0^2}\mathbf{v}_0+\frac{1}{\sigma^2}\sum_{a=1}^{u}\mathbf{v}_a\right)^{\mathrm{H}}\mathbf{y}\right\}\right]\mathrm{d}\mathbf{y}\right.$$
$$\left.\times(\pi\sigma^2)^{-uT_c}\exp\left[-\frac{\alpha}{\sigma_0^2}\|\mathbf{v}_0\|^2-\frac{\alpha}{\sigma^2}\sum_{a=1}^{u}\|\mathbf{v}_a\|^2\right]\middle|\,\overline{\mathbf{Q}}_K\right\},\qquad \mathbf{v}\sim\mathsf{CN}(\mathbf{0},\overline{\mathbf{Q}}_K) \tag{7}$$

$$M_k^{(u)}(\tilde{\mathbf{Q}}) = \mathsf{E}_{\mathrm{c}}\left\{\prod_{m=1}^{M}\exp\left[\sum_{a=1}^{u}\left(\mathbf{x}_{ak}h_{ak}^m+\Delta\mathbf{w}_{ak}^m\right)^{\mathrm{H}}\mathbf{G}\left(\mathbf{x}_{ak}h_{ak}^m+\Delta\mathbf{w}_{ak}^m\right)+2\Re\left\{\left(\mathbf{x}_{0k}^{\mathrm{p,d}}h_{0k}^m\right)^{\mathrm{H}}\mathbf{D}\left(\mathbf{x}_{ak}h_{ak}^m+\Delta\mathbf{w}_{ak}^m\right)\right\}\right.\right.$$
$$\left.\left.+\sum_{b=1,b\neq a}^{u}\left(\mathbf{x}_{ak}h_{ak}^m+\Delta\mathbf{w}_{ak}^m\right)^{\mathrm{H}}\mathbf{F}\left(\mathbf{x}_{bk}h_{bk}^m+\Delta\mathbf{w}_{bk}^m\right)\right]\right\} \tag{16}$$

can write $\mathbf{F}=\sqrt{\mathbf{F}}\sqrt{\mathbf{F}}^{\mathrm{H}}$, where $\sqrt{\mathbf{F}}=\mathbf{\Sigma}^{-1}\sqrt{\mathbf{\Sigma}_0}$. Re-introducing the iteration index and denoting $\mathbf{x}_k^{(i-1)}=\mathrm{vec}\left(\left[\mathbf{p}_k,\tilde{\mathbf{x}}_k^{(i-1)}\right]\right)$, an application of the Hubbard-Stratonovich transform[3] gives

$$M_k^{(u)}(\tilde{\mathbf{Q}}) = \mathsf{E}_{\mathrm{c}}\left\{\left(\int\prod_{m=1}^{M}\mathrm{d}\mathbf{z}_{km}\right)\prod_{m=1}^{M}f\left(\mathbf{z}_{km}\middle|\,\mathbf{x}_{0k}^{\mathrm{p,d}}h_{0k}^m;\mathbf{\Sigma}_0\right)\right.$$
$$\left.\times\left(\frac{\mathsf{E}_{\mathbf{h}^k,\Delta\mathscr{W}_k^{(i-1)}}\left\{\prod_{m=1}^{M}f\left(\mathbf{z}_{km}\middle|\,\mathbf{x}_k^{(i-1)}h_k^m+\Delta\mathbf{w}_{km}^{(i-1)};\mathbf{\Sigma}\right)\right\}}{\prod_{m=1}^{M}f\left(\mathbf{z}_{km}\middle|\,\mathbf{0};\mathbf{\Sigma}\right)}\right)^{u}\right\},$$

where we used the fact that replicated RVs are IID and denoted $\Delta\mathscr{W}_k^{(i-1)}=\{\Delta\mathbf{w}_{km}^{(i-1)}\,|\,m=1,\ldots,M\}$, and

$$f(\mathbf{z}\,|\mathbf{y};\mathbf{R})=\left[\pi^{T_c}\det(\mathbf{R})\right]^{-1}\exp\left(-(\mathbf{z}-\mathbf{y})^{\mathrm{H}}\mathbf{R}^{-1}(\mathbf{z}-\mathbf{y})\right).$$

As $u\to 0$, $M_k^{(u)}(\tilde{\mathbf{Q}})\to 1$, and (12) can be written as

$$\mathbf{Q}^* = \lim_{K\to\infty}\frac{1}{K}\sum_{k=1}^{K}\sum_{m=1}^{M}\mathsf{E}_{\mathrm{c}}\left\{\left(\int\prod_{m=1}^{M}\mathrm{d}\mathbf{z}_{km}\right)\right.$$
$$\times\,\mathsf{E}_{\mathbf{h}_{0k},\Delta\mathscr{W}_k^{(i-1)}}\left\{\mathbf{\Omega}_{km}\mathbf{\Omega}_{km}^{\mathrm{H}}\prod_{m=1}^{M}f\left(\mathbf{z}_{km}\middle|\,\mathbf{x}_{0k}^{\mathrm{p,d}}h_{0k}^m;\mathbf{\Sigma}_0\right)\right.$$
$$\left.\left.\times\left(\frac{\prod_{m=1}^{M}f\left(\mathbf{z}_{km}\middle|\,\mathbf{x}_k^{(i-1)}h_k^m+\Delta\mathbf{w}_{km}^{(i-1)};\mathbf{\Sigma}\right)}{\mathsf{E}_{\mathbf{h}_k,\Delta\mathscr{W}_k^{(i-1)}}\left\{\prod_{m=1}^{M}f\left(\mathbf{z}_{km}\middle|\,\mathbf{x}_k^{(i-1)}h_k^m+\Delta\mathbf{w}_{km}^{(i-1)};\mathbf{\Sigma}\right)\right\}}\right)\right\}\right\}. \tag{17}$$

We get from (5) and (17), along with (14) – (15) the proposition. The formal proof of decoupling via convergence of the joint moments [12, Sec. IV-B], [14] is omitted due to space constraints.

REFERENCES

[1] J. Evans and D. N. C. Tse, “Large system performance of linear multiuser receivers in multipath fading channels,” *IEEE Trans. Inform. Theory*, vol. 46, no. 6, pp. 2059–2078, Sep. 2000.

[2] J. Boutros and G. Caire, “Iterative multiuser joint decoding: Unified framework and asymptotic analysis,” *IEEE Trans. Inform. Theory*, vol. 48, no. 7, pp. 1772–1793, Jul. 2002.

[3] G. Caire, R. R. Müller, and T. Tanaka, “Iterative multiuser joint decoding: Optimal power allocation and low-complexity implementation,” *IEEE Trans. Inform. Theory*, vol. 50, no. 9, pp. 1950–1973, Sep. 2004.

[3] $e^{\|\mathbf{b}\|^2}=[\pi^{T_c}\det(\mathbf{A})]^{-1}\int e^{-\mathbf{y}^{\mathrm{H}}\mathbf{A}^{-1}\mathbf{y}+2\Re\{\mathbf{b}^{\mathrm{H}}(\sqrt{\mathbf{A}})^{-1}\mathbf{y}\}}\mathrm{d}\mathbf{y},\quad \mathbf{A}>0$

[4] H. Li, S. M. Betz, and H. V. Poor, “Performance analysis of iterative channel estimation and multiuser detection in multipath DS-CDMA channels,” *IEEE Trans. Signal Processing*, vol. 55, no. 5, pp. 1981–1993, May 2007.

[5] P. D. Alexander and A. J. Grant, “Iterative channel and information sequence estimation in CDMA,” in *Proc. IEEE Int. Symp. Spread Spectrum Techniques and Applications*, vol. 2, New Jersey, USA, Sep. 6–8 2000, pp. 593–597.

[6] G. Caire, A. Tulino, and E. Biglieri, “Iterative multiuser joint decoding and parameter estimation: A factor-graph approach,” in *Proc. IEEE Inform. Theory Workshop*, Cairns, Australia, Sep. 2–7 2001, pp. 36–38.

[7] H. Li and H. V. Poor, “Impact of channel estimation errors on multiuser detection via the replica method,” *EURASIP J. Wirel. Commun. Netw.*, no. 2, pp. 175–186, Apr. 2005.

[8] S. Chung, T. J. Richardson, and R. L. Urbanke, “Analysis of sum-product decoding of low-density parity-check codes using a Gaussian approximation,” *IEEE Trans. Inform. Theory*, vol. 47, no. 2, pp. 657–670, Feb. 2001.

[9] H. Nishimori, *Statistical Physics of Spin Glasses and Information Processing*. New York: Oxford University Press, 2001.

[10] T. Tanaka, “A statistical-mechanics approach to large-system analysis of CDMA multiuser detectors,” *IEEE Trans. Inform. Theory*, vol. 48, no. 11, pp. 2888–2910, Nov. 2002.

[11] R. R. Müller, “Channel capacity and minimum probability of error in large dual antenna array systems with binary modulation,” *IEEE Trans. Signal Processing*, vol. 51, no. 11, pp. 2821–2828, Nov. 2003.

[12] D. Guo and S. Verdú, “Randomly spread CDMA: Asymptotics via statistical physics,” *IEEE Trans. Inform. Theory*, vol. 51, no. 6, pp. 1983–2010, Jun. 2005.

[13] K. Takeda, A. Hatabu, and Y. Kabashima, “Statistical mechanical analysis of the linear vector channel in digital communication,” *J. Phys. A: Math. Theor.*, vol. 40, no. 47, pp. 14 085–14 098, 2007.

[14] K. Takeuchi, T. Tanaka, and T. Yano, “Asymptotic analysis of general multiuser detectors in MIMO DS-CDMA channels,” *IEEE J. Select. Areas Commun.*, vol. 26, no. 3, pp. 486–496, Apr. 2008.

[15] L. H. Ozarow, S. Shamai, and A. D. Wyner, “Information theoretic considerations for cellular mobile radio,” *IEEE Trans. Veh. Technol.*, vol. 43, no. 2, pp. 359–378, May 1994.

[16] G. Caire, G. Taricco, and E. Biglieri, “Bit-interleaved coded modulation,” *IEEE Trans. Inform. Theory*, vol. 44, pp. 927–946, May 1998.

[17] M. Lončar, R. R. Müller, J. Wehinger, C. Mecklenbräuker, and T. Abe, “Iterative channel estimation and data detection in frequency-selective fading MIMO channels,” *European Trans. Telecommun.*, vol. 15, no. 5, pp. 459–470, Sep./Oct. 2004.