On Channel Estimators for Iterative CDMA Multiuser Receiver in Flat Rayleigh Fading

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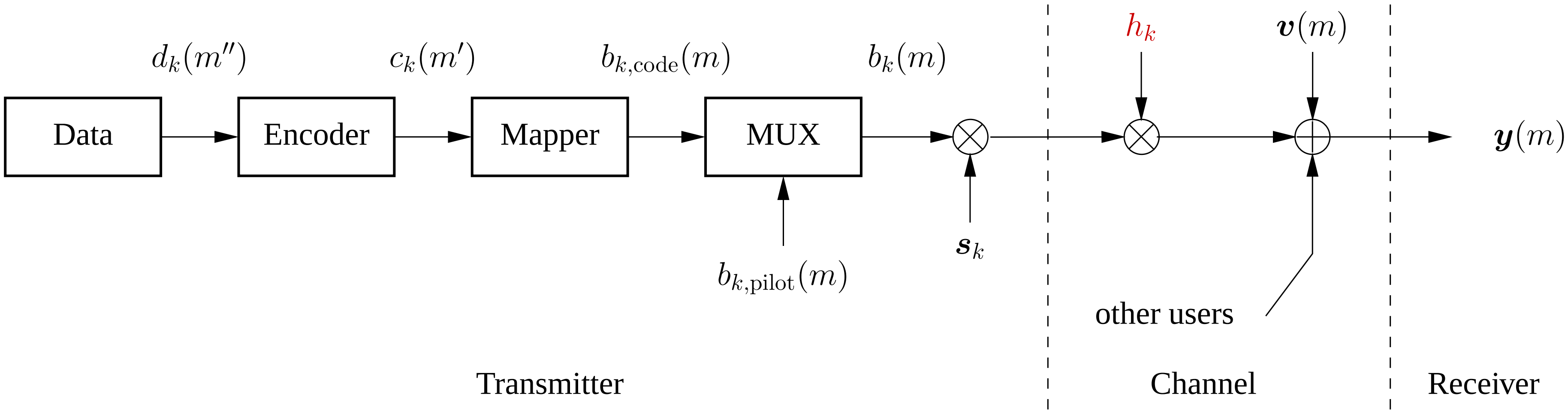
Overview

Multiuser Uplink

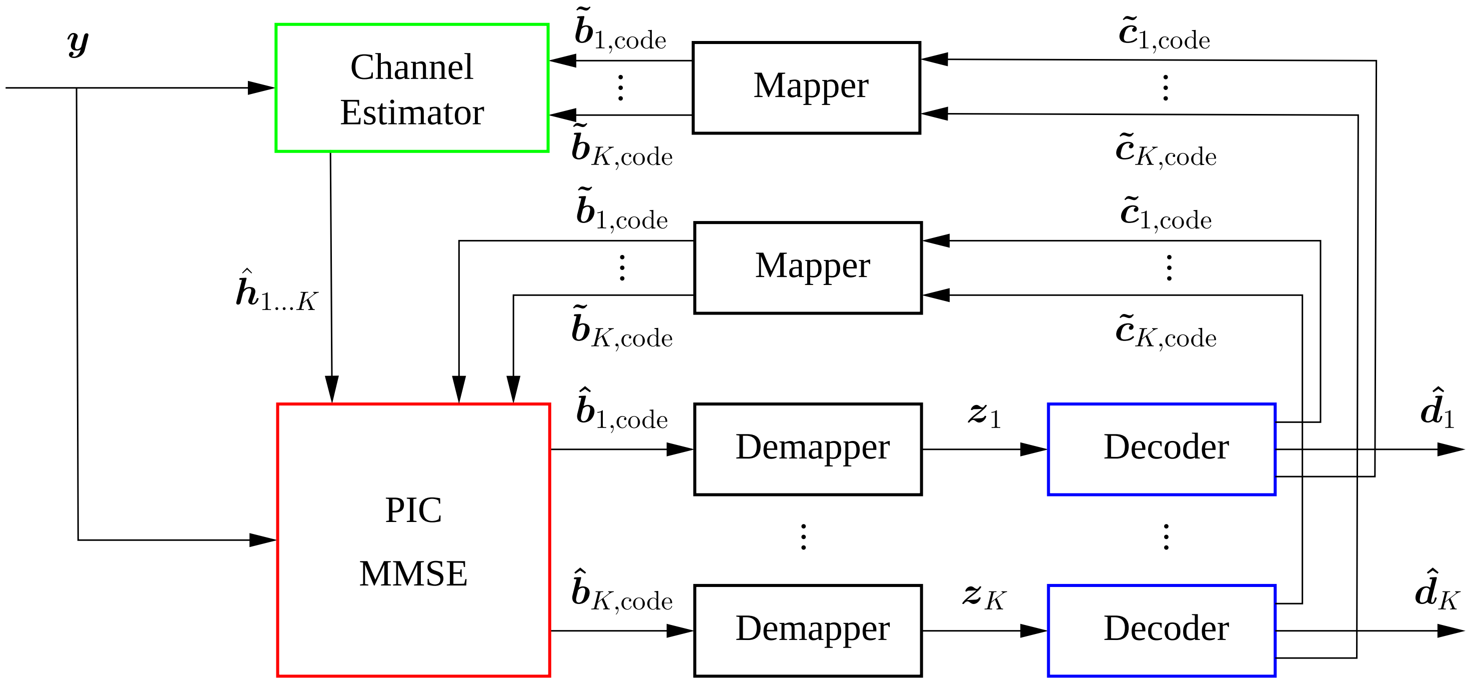


- Synch. CDMA system with **short** random spreading
- Convolutional encoder $\mathcal{C}(5,7)_8$
- QPSK Gray mapping
- J pilots are placed in a preamble

J pilots	$M - J$ code symbols

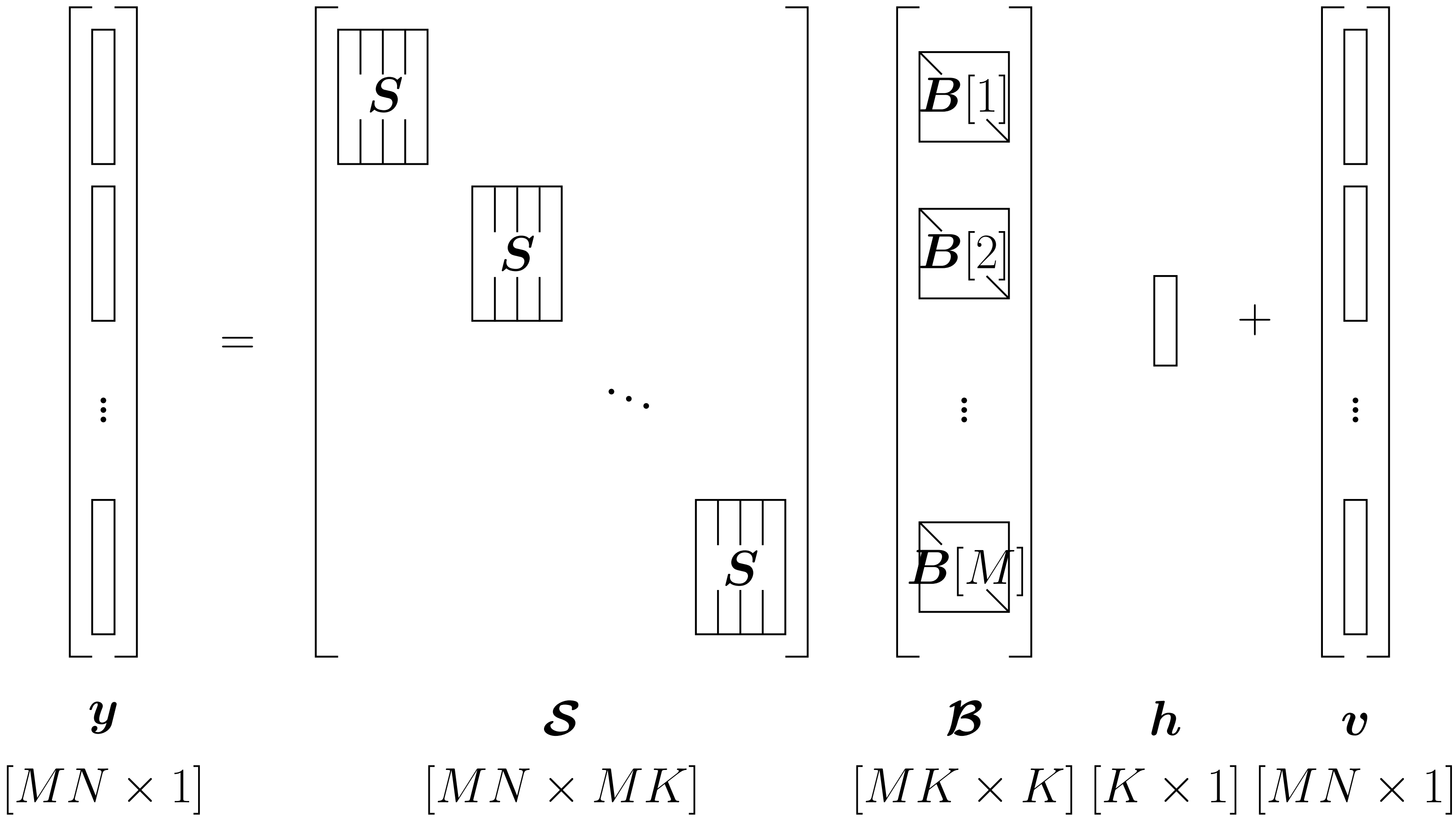
- Single Rayleigh fading tap - constant over M symbols

Receiver



- **Detector** - parallel interference canceller (PIC), LMMSE filter
- **Channel estimator** - use soft feedback values
- **Decoder** - symbol by symbol MAP decoder (BCJR)

Signal Model for a Single Block



M block length, N spreading seq. length, K # users

Multiuser Detector

1 - Parallel Interference Cancellation

$$\tilde{\boldsymbol{y}}_k^{(i)}(m) = \boldsymbol{y}(m) - \tilde{\boldsymbol{S}}^{(i)}\tilde{\boldsymbol{b}}^{(i)}(m) + \tilde{\boldsymbol{s}}_k^{(i)}\tilde{b}_k^{(i)}(m)$$

with $\tilde{\boldsymbol{S}}^{(i)} = \boldsymbol{S}\,\mathrm{diag}\left(\hat{\boldsymbol{h}}^{(i)}\right)$.

2 - LMMSE Filtering

$$\hat{b}_k^{(i+1)}(m) = \left(\boldsymbol{w}_k^{(i)}\right)^{\mathrm{H}} \tilde{\boldsymbol{y}}_k^{(i)}(m)$$

The filter is the solution to

$$\boldsymbol{w}_k^{(i)}(m) = \underset{\boldsymbol{w}}{\mathrm{argmin}}\, \underset{\boldsymbol{v}}{\mathbb{E}} \left\{ \left| b_k(m) - \boldsymbol{w}^{\mathrm{H}}\tilde{\boldsymbol{y}}_k^{(i)}(m) \right|^2 \right\}.$$

Decoder

The code symbol **a-posteriori probability** is defined as

$$\mathrm{APP}_k(m') \triangleq \Pr\left\{c_k(m') = +1 | \boldsymbol{z}_k\right\}$$

and can be computed via the BCJR algorithm.

For a **Gaussian channel** the extrinsic probability and the a-posteriori probability satisfy the following relation

$$\mathrm{APP}_k(m') \sim \mathrm{EXT}_k(m') \; \frac{1}{\sqrt{2\pi\nu_k^2}} \exp\left(\frac{|z_k(m') - \mu_k|^2}{2\nu_k^2}\right).$$

Soft Symbol QPSK Mapping

$$\tilde{b}_{k,\mathrm{code}}(m) = \frac{1}{\sqrt{2}} \{2\mathrm{APP}_k(2m') - 1 + j[2\mathrm{APP}_k(2m' + 1) - 1]\}$$

and similarly for extrinsic information.

Approximated Concepts

Linear signal model

$$\boldsymbol{y} = \boldsymbol{\mathcal{S}}\boldsymbol{\mathcal{B}}\boldsymbol{h} + \boldsymbol{v}$$

with $\boldsymbol{h} \sim \mathcal{CN}(\mathbf{0}, \boldsymbol{I}_K)$ and $\boldsymbol{v} \sim \mathcal{CN}(\mathbf{0}, \sigma_v^2 \boldsymbol{I}_N)$.

Approximation Soft feedback data is treated as deterministic

$$\tilde{\boldsymbol{\mathcal{B}}} = \left[\boldsymbol{\mathcal{B}}_{\text{Pilots}};\ \ \tilde{\boldsymbol{\mathcal{B}}}_{\text{Feedback}}\right]$$

1 - Approx. Least-Squares Estimator (ALS)

$$\hat{\boldsymbol{h}}_{\text{ALS}} = \left(\tilde{\boldsymbol{\mathcal{B}}}^{\text{H}} \boldsymbol{\mathcal{S}}^{\text{H}} \boldsymbol{\mathcal{S}} \tilde{\boldsymbol{\mathcal{B}}}\right)^{-1} \tilde{\boldsymbol{\mathcal{B}}}^{\text{H}} \boldsymbol{\mathcal{S}}^{\text{H}} \boldsymbol{y}$$

2 - Approx. linear MMSE Estimator (ALMMSE)

$$\hat{\boldsymbol{h}}_{\text{ALMMSE}} = \left(\tilde{\boldsymbol{\mathcal{B}}}^{\text{H}} \boldsymbol{\mathcal{S}}^{\text{H}} \boldsymbol{\mathcal{S}} \tilde{\boldsymbol{\mathcal{B}}} + \sigma_v^2 \boldsymbol{I}_K\right)^{-1} \tilde{\boldsymbol{\mathcal{B}}}^{\text{H}} \boldsymbol{\mathcal{S}}^{\text{H}} \boldsymbol{y}$$

Linear MMSE (LMMSE) Concept

3 - Improved Estimator Considering Symbol Reliability

$$\hat{\boldsymbol{h}}_{\text{LMMSE}} = \boldsymbol{C}_{hy}\boldsymbol{C}_{yy}^{-1}\boldsymbol{y}$$

$$\boldsymbol{C}_{yh} = \mathop{\mathbb{E}}_{\boldsymbol{\mathcal{B}},\boldsymbol{h},\boldsymbol{v}}\left\{\boldsymbol{y}\boldsymbol{h}^{\text{H}}\right\} = \boldsymbol{\mathcal{S}}\tilde{\boldsymbol{\mathcal{B}}}\boldsymbol{T}$$

$$\boldsymbol{C}_{yy} = \mathop{\mathbb{E}}_{\boldsymbol{\mathcal{B}},\boldsymbol{h},\boldsymbol{v}}\left\{\boldsymbol{y}\boldsymbol{y}^{\text{H}}\right\} = \boldsymbol{\mathcal{S}}\mathop{\mathbb{E}}_{\boldsymbol{\mathcal{B}}}\left\{\boldsymbol{\mathcal{B}}\boldsymbol{T}\boldsymbol{\mathcal{B}}^{\text{H}}\right\}\boldsymbol{\mathcal{S}}^{\text{H}} + \sigma_v^2\boldsymbol{I}_{KM}.$$

Statistical independence among code symbols leads to

$$\mathbb{E}\left\{b_p(m)b_q^*(n)\right\} = \begin{cases} \tilde{b}_p(m)\tilde{b}_q^*(n) & \text{for} \quad p \neq q, m \neq n \\ 1 & \text{for} \quad p = q, m = n. \end{cases}$$

$\boldsymbol{T}$ is the correlation matrix of the channel taps

$$\boldsymbol{T} = \operatorname{diag}\left(\sigma_{\text{h},1}^2, \sigma_{\text{h},2}^2, \ldots, \sigma_{\text{h},K}^2\right).$$

Simulation Parameters

- Spreading sequence length $N = 8$
- Number of users $K = 12$, overloaded system $\alpha = 1.5$
- Block length $M = 100$
- Number of pilots $J = 10$

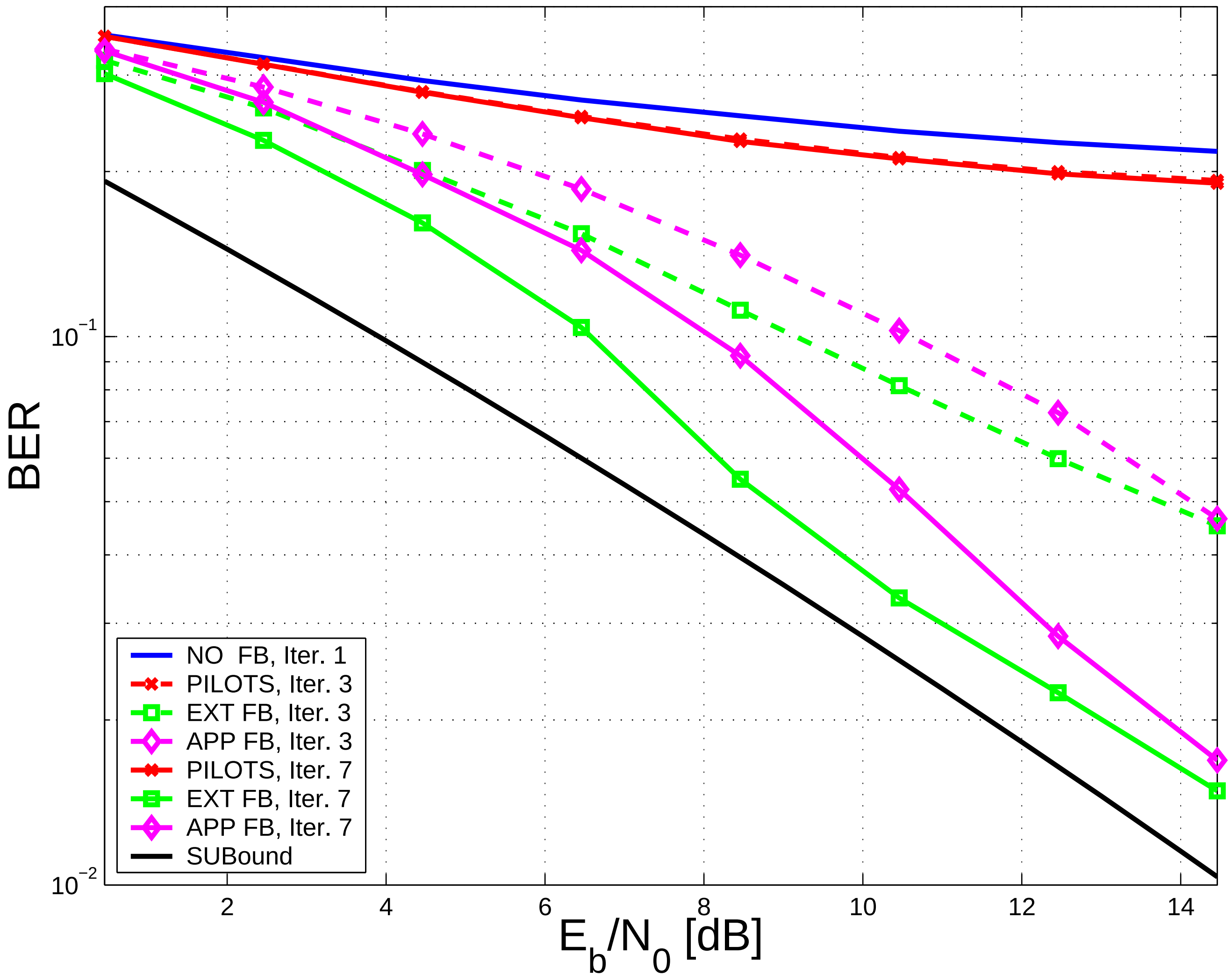
- Energy per information bit to noise level ratio

$$E_b/N_0 \triangleq \frac{1}{\sigma_v^2} \frac{M}{M-J}$$

- Single-User Bound (SUBound)

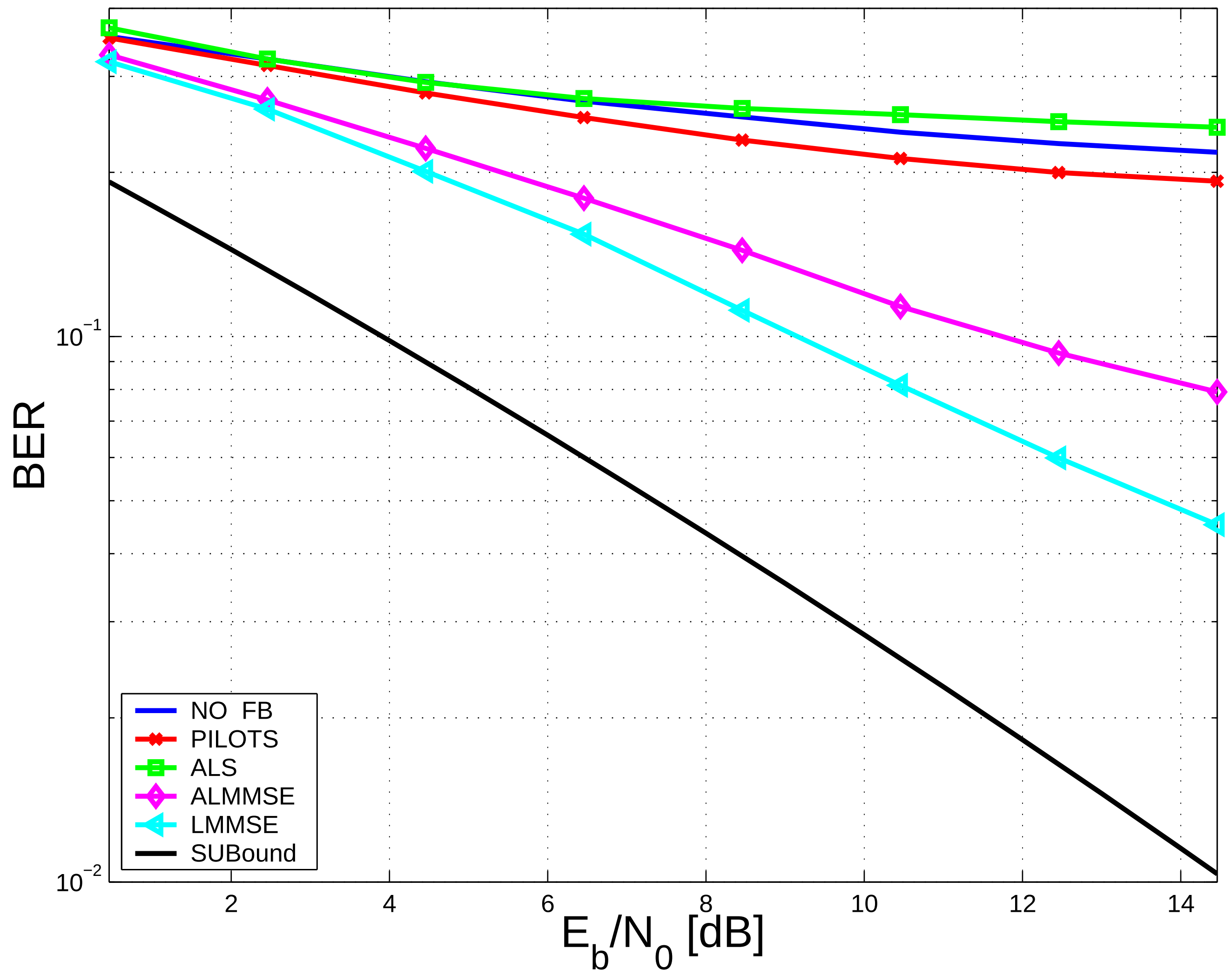
Performance of one user with perfect channel knowledge.

APP vs. EXT in the Feedback

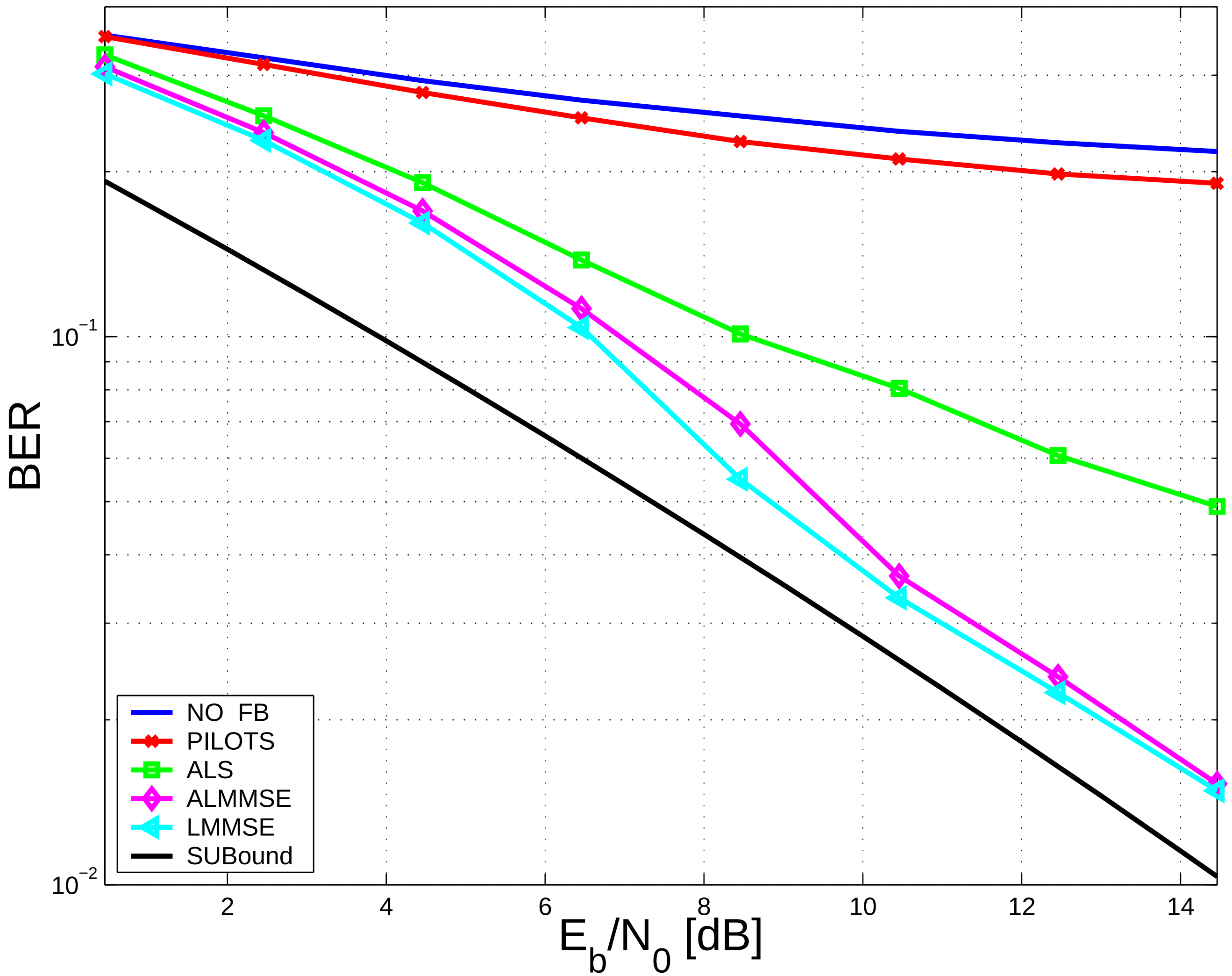


Extrinsic FB better than APP FB for channel estimation.

BER after 3rd Iteration



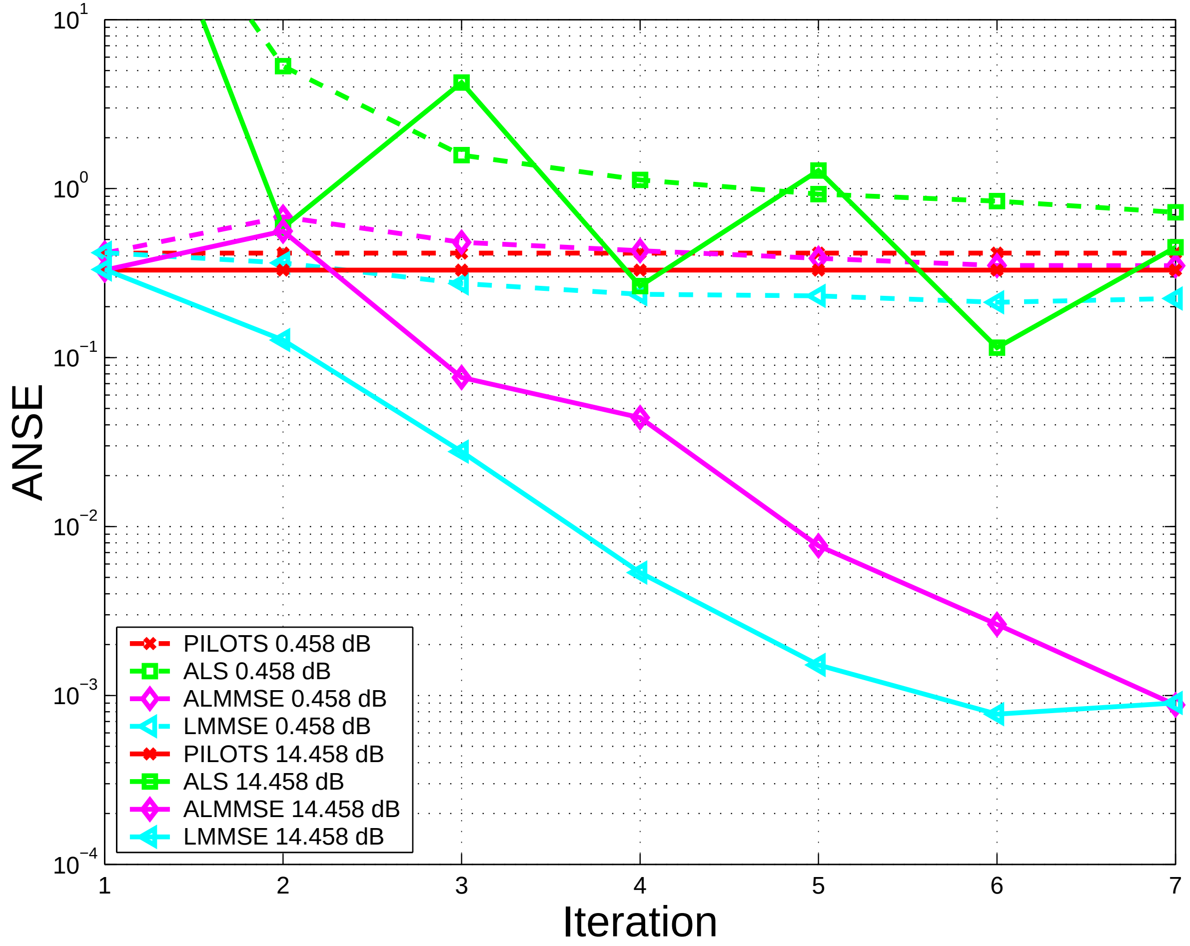
BER after 7th Iteration



Averaged Normalized Squared Error

Realizations $R = 100$

$$\text{ANSE} \triangleq \frac{1}{R}\sum_{i=1}^{R}\frac{\|\boldsymbol{h}-\hat{\boldsymbol{h}}\|^2}{\|\boldsymbol{h}\|^2}$$



Conclusions

- Soft symbols in feedback substantially improve channel estimation and overall receiver performance.
- Extrinsic info shows better performance than APP info in feedback.
- Improved LMMSE estimator “weights” reliability best.
- ANSE decreases monotonically.