Vector Precoding for a Single-User MIMO Channel: Matched Filter vs. Distributed Antenna Detection

R. de Miguel and Ralf R. Müller
Department of Electronics and Telecommunications
Norwegian University of Science and Technology
7491 Trondheim, Norway

***Abstract*— Convex vector precoding is proposed for interference-free reception in single user MIMO channels using either spatial matched filtering or distributed antenna detection at the receiver. We use the replica method of statistical physics to compare the large system performance (in terms of transmitted energy vs. uncoded transmission rate) of both the matched filter and the distributed detection schemes. Using a convex quaternary alphabet we find that distributed detection is most advantageous only when the ratio of antennas at the receiver to that at the transmitter is either smaller than 0.5 or between 1 and 1.22. Matched filter detection is most advantageous for ratios greater than 2. In the two remaining regions the choice of precoding for matched filter or distributed detection results in a tradeoff between energy and transmission rate.**

***Index Terms*— Single user MIMO, channel inversion, zero-forcing, non-linear vector precoding, singular channels, asymptotic analysis, replica method.**

I. INTRODUCTION

In multiple-input/multiple-output (MIMO) channels information is conveyed simultaneously from a group of transmitting antennas to a group of receiving antennas. As these transmissions are not orthogonal, yet they occur over the same physical medium and bandwidth, crosstalk becomes unavoidable. As a result signal processing needs to be done at the receiver and/or transmitter side of the channel if significant data rates are to be achieved. In MIMO broadcast channels the transmitting antennas are collocated and they can jointly generate and pre-process the data streams to be transmitted. In contrast to multiuser channels, in a single-user MIMO channel the receiving antennas are also collocated and they do have the possibility of jointly processing data.

However, in the context of low cost receivers with limited processing power, it might be advantageous to shift most of the signal processing to the transmitter side. One technique which might be employed by the transmitter in order to keep the receivers from doing any signal processing is channel inversion before transmission (provided that the transmitter has complete channel state information). If the number of transmit antennas is larger than the number of receive antennas, direct channel inversion is not posssible. However, this problem can be overcome modifying the channel inversion process and applying a spatial matched filter at the receiver.

Regardless of whether a spatial matched filter or distributed antenna detection is applied at the receiver, channel inversion at the transmitter comes at an increased transmission energy cost. One technique which may be used to contain the transmit power while inverting the channel is non-linear vector precoding (henceforth vector precoding) [1], [2], [3], [4]. The vector precoding technique, outlined in Section II, consists of extending the input alphabets representing different information states; this permits the search for symbols which draw less energy when transmitted with channel inversion. In [5], [6] we used the replica method of statistical physics to analyze different vector precoding techniques in asymptotic multiuser MIMO broadcast channels. In [7] we described a channel inversion technique which allows for simple matched filter detection in single-user MIMO channels; this technique was proposed as an alternative to conventional channel inversion for cases when the number of antennas at the receiver exceeds that at the transmitter.

In this contribution we compare vector precoding for matched filter and distributed antenna detection by a single multiantenna user; we consider all transmitter to receiver antenna ratios, including those resulting in singular channels [8]. We use the replica method of statistical physics to analyze the performance (in terms of transmitted energy vs. uncoded transmission rate) of both the matched filter and the distributed detection schemes. Using the convex extension for QPSK alphabets proposed in [5] we find that distributed detection is most advantageous only when the ratio of antennas at the receiver to that at the transmitter is either smaller than 0.5 or between 1 and 1.22. Matched filter detection is most advantageous for ratios greater than 2. In the two remaining regions the choice of precoding for matched filter or distributed detection results in a tradeoff between energy and transmission rate.

This paper is organized as follows. The vector precoding technique is presented in Section II. The asymptotic methods used to find the transmitted energy in the many antenna limit are described in Section III. The results are finally presented and discussed in Section IV.

This work was supported by the Research Council of Norway (Grant 171133/V30).

II. VECTOR PRECODING

The single-user MIMO channel may be represented by the following vector equality:

$$\mathbf{r} = \mathbf{H}\mathbf{t} + \mathbf{n}, \tag{1}$$

where $\mathbf{t}$ is the N-dimensional input to the channel, $\mathbf{r}$ is a vector containing the K received signals, $\mathbf{n}$ is a random vector containing additive noise components, and the complex channel matrix $\mathbf{H}$ has independent and identically distributed entries with zero mean and unit variance.

The transmitted vector $\mathbf{t}$ is a linear transformation of the information symbols (contained in $\mathbf{x}$) intended for the K antenna elements at the receiver, thus we might write

$$\mathbf{t} \equiv \mathbf{T}\mathbf{x}. \tag{2}$$

In order to guarantee simple detection by the user-receiver, the transmitter, who has complete channel state information, might construct $\mathbf{T}$ such that the information symbols in $\mathbf{x}$ can be received interference free (up to additive noise) by a simple linear operation $\hat{\Omega}$ on $\mathbf{r}$:

$$\hat{\Omega}\mathbf{r} = \mathbf{x} + \hat{\Omega}\mathbf{n}. \tag{3}$$

Using this transmission scheme, the transmitted energy per symbol is given by

$$K^{-1}\mathbf{t}^{\dagger}\mathbf{t} = K^{-1}\mathbf{x}^{\dagger}\mathbf{E}\mathbf{x}, \tag{4}$$

where the energy metric $\mathbf{E}$ is given by

$$\mathbf{E} = \mathbf{T}^{\dagger}\mathbf{T}. \tag{5}$$

A. Channel inversion

The $K \times K$ matrix $\mathbf{H}\mathbf{H}^{\dagger}$ and the $N \times N$ matrix $\mathbf{H}^{\dagger}\mathbf{H}$ have rank given by $\min\{N, K\}$. Even if the matrices are rank deficient, channel inversion techniques might be used to transmit complex data vectors which can be constructed in the $\min\{N, K\}$-dimensional space spanned by either of these matrices. In order to allow for the possibility of inverting rank deficient channel-based matrices, the transmitter might employ the generalized channel inversion technique outlined in the following.

When a matrix $\mathbf{M}$ is hermitian, as are both matrices under consideration, we might write

$$\mathbf{M} = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^{\dagger}, \tag{6}$$

where $\mathbf{U}$ is unitary and $\mathbf{\Lambda} \equiv \mathrm{diag}\,(\lambda_1, \lambda_2, \cdots, \lambda_T)$ is a diagonal matrix containing the T eigenvalues of $\mathbf{M}$. We might then define the pseudoinverse of $\mathbf{M}$ as

$$\mathbf{M}^{\widetilde{-1}} = \mathbf{U}\mathbf{\Lambda}^{\widetilde{\mathbf{-1}}}\mathbf{U}^{\dagger}, \tag{7}$$

where

$$\mathbf{\Lambda}^{\widetilde{\mathbf{-1}}} \equiv \lim_{\epsilon \to \mathbf{0}} \mathrm{diag}\left(\frac{\mathbf{1} - \delta_{\lambda_1,0}}{\lambda_1 + \epsilon}, \frac{\mathbf{1} - \delta_{\lambda_2,0}}{\lambda_2 + \epsilon}, \cdots, \frac{\mathbf{1} - \delta_{\lambda_T,0}}{\lambda_T + \epsilon}\right) \tag{8}$$

and $\delta_{i,j}$ is the Kronecker delta. Note that $\mathbf{M}^{\widetilde{-1}}$ is nothing but the Moore-Penrose inverse of the square matrix $\mathbf{M}$.

1) Distributed antenna detection: Distributed antenna detection at the receiver may be carried out if the matrix $\mathbf{T}$ and the operator $\hat{\Omega}$ are constructed as follows [5], [6]:

$$\mathbf{T} \rightarrow \mathbf{T}_{\mathcal{D}} \equiv \frac{\mathbf{H}^{\dagger}}{\sqrt{N}} \left(\frac{\mathbf{H}\mathbf{H}^{\dagger}}{N}\right)^{\widetilde{-1}}, \tag{9}$$

$$\hat{\Omega} \rightarrow \hat{\Omega}_{\mathcal{D}} \equiv \frac{1}{\sqrt{N}}\mathbf{I}. \tag{10}$$

Then the transmitted energy per symbol is given by

$$K^{-1}\mathbf{t}^{\dagger}\mathbf{t} = K^{-1}\mathbf{x}^{\dagger}\mathbf{E}_{\mathcal{D}}\mathbf{x}, \tag{11}$$

where

$$\mathbf{E}_{\mathcal{D}} \equiv \mathbf{T}_{\mathcal{D}}^{\dagger}\mathbf{T}_{\mathcal{D}} = \left(\frac{\mathbf{H}\mathbf{H}^{\dagger}}{N}\right)^{\widetilde{-1}}. \tag{12}$$

2) Matched filter detection: When the state of the channel is known at both ends, then a complex data vector $\mathbf{x}$ containing N entries might be detected interference free with a matched filter if the matrix $\mathbf{T}$ and the operator $\hat{\Omega}$ are constructed in an alternative fashion [7]:

$$\mathbf{T} \rightarrow \mathbf{T}_{\mathcal{M}} \equiv \left(\frac{\mathbf{H}^{\dagger}\mathbf{H}}{K}\right)^{\widetilde{-1}}, \tag{13}$$

$$\hat{\Omega} \rightarrow \hat{\Omega}_{\mathcal{M}} \equiv \frac{1}{K}\mathbf{H}^{\dagger}. \tag{14}$$

Under this transmission scheme, the transmitted energy per symbol is given by

$$K^{-1}\mathbf{t}^{\dagger}\mathbf{t} = K^{-1}\mathbf{x}^{\dagger}\mathbf{E}_{\mathcal{M}}\mathbf{x}, \tag{15}$$

where

$$\mathbf{E}_{\mathcal{M}} \equiv \mathbf{T}_{\mathcal{M}}^{\dagger}\mathbf{T}_{\mathcal{M}} = \left(\frac{\mathbf{H}^{\dagger}\mathbf{H}}{K}\right)^{\widetilde{-2}}. \tag{16}$$

B. Minimizing the transmitted energy

Although channel inversion by the transmitter eases the task of detection at the receiver, it might come at the cost of a high transmission energy. The goal of the vector precoding technique is minimizing the cost of the channel inversion process, *i.e.* minimizing Eq. (4). For this purpose, it is agreed between the transmitter and the receiver that, although there must be a minimum distance between any two symbols representing different information states, each state might be represented by more than one symbol. This gives the transmitter greater freedom to construct the information vector $\mathbf{x}$ with symbols which faithfully represent the intended information, yet they are chosen so as to minimize Eq. (4).

The i^{th} component of the vector $\mathbf{x}$ represents the information state s_i. The symbols which might represent the state s_i are those contained in the set $\mathcal{A}_{s_i}$. Then the information vector $\mathbf{x}$ is constructed such that $\mathbf{x} \in \mathcal{A}$, where $\mathcal{A} = \prod_i \mathcal{A}_{s_i}$. The transmitter chooses a symbol

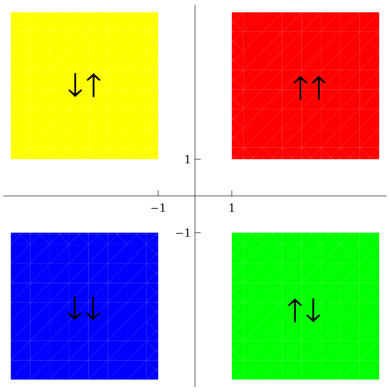

Fig. 1. Convex alphabet relaxation for QPSK.

representation in the space $\mathcal{A}$ which can be transmitted free of interference at a minimum energy cost:

$$\mathbf{x} \equiv \arg \min_{\tilde{\mathbf{x}} \in \mathcal{A} \cap \mathcal{S}} K^{-1} \tilde{\mathbf{x}}^{\dagger} \mathbf{E} \tilde{\mathbf{x}}, \tag{17}$$

where $\mathcal{S}$ denotes the span of the relevant matrix for the chosen transmission scheme, *i.e.* the span of $\mathbf{H}\mathbf{H}^{\dagger}$ for distributed detection, or the span of $\mathbf{H}^{\dagger}\mathbf{H}$ for matched filter detection.

If the symbol alphabets are discrete, then $\mathbf{x}$ can only be found performing an exhaustive search, which becomes prohibitively expensive when the number of users is large or the alphabet contains many symbols. However, if the alphabets representing the different information states are convex, then efficient algorithms might be used to find the optimal $\mathbf{x}$.

C. Alphabet relaxation for quaternary phase shift keying

We consider now a source of information consisting of four equiprobable states: ↑↑, ↑↓, ↓↓ and ↓↑. When no vector precoding is employed the entries in $\mathbf{x}$ are usually selected from the unit QPSK alphabets $\mathcal{A}_{\uparrow\uparrow} = \{1 + j\}$, $\mathcal{A}_{\uparrow\downarrow} = \{1 - j\}$, $\mathcal{A}_{\downarrow\downarrow} = \{-1 - j\}$ and $\mathcal{A}_{\downarrow\uparrow} = \{-1 + j\}$. We consider the convex alphabet relaxation shown in Fig. 1, which was proposed in [5]. For an analysis of the probability that a typical such relaxed vector can be found in $\mathcal{A} \cap \mathcal{S}$ the reader is referred to [8].

III. THE TRANSMITTED ENERGY

The technique outlined in Section II describes how to minimize the transmitted energy while achieving simple and interference free detection by the receiver. In order to obtain an expression for the transmitted energy using this transmission technique one might first note that, if $\mathbf{x}$ is T-dimensional, the minimum transmitted energy per symbol $\mathcal{E}$ might be written as

$$\mathcal{E} \equiv -\beta^{-1} T^{-1} \lim_{\beta \to \infty} \ln \sum_{\mathbf{x} \in \mathcal{A} \cap \mathcal{S}} \mathrm{e}^{-\beta \mathbf{x}^{\dagger} \mathbf{E} \mathbf{x}}. \tag{18}$$

The argument of the limit in (18) has the same form as the expression for the Helmholtz free energy of a thermodynamic ($T \to \infty$) system with temperature $1/\beta$ [9]. We might take advantage of this fact by using a thermodynamic approximation, *i.e.* assuming that K and N are infinitely large, yet they have a finite ratio $\alpha = K/N$. This approximation allows us to make use of mathematical tools imported from the statistical physics literature.

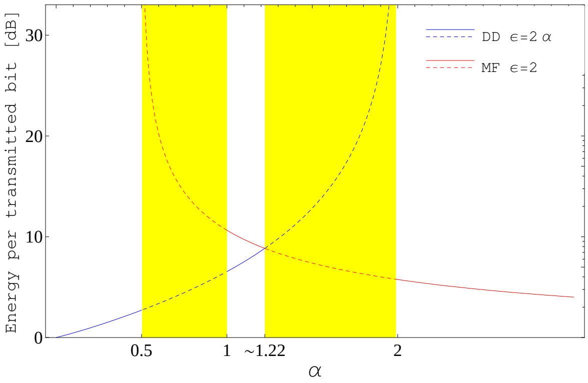

Fig. 2. Energy per transmitted bit vs. ratio of transmitter to receiver antennas, for distributed antenna detection (blue curve) and matched filter detection (red curve). In the shaded regions there is a tradeoff between spectral efficiency per transmitting antenna and energy per transmitted bit.

In the thermodynamic limit the eigenvalue distribution of $\mathbf{E}$ is well defined and fully determined by the statistics of the channel matrix $\mathbf{H}$. A function which fully describes the eigenvalue distribution of $\mathbf{E}$ is its R-transform, denoted $R_{\mathbf{E}}(\cdot)$. The R-transform of $\mathbf{E}_{\mathcal{D}}$ was derived in [5]. Due to space limitations, the R-transform of $\mathbf{E}_{\mathcal{M}}$ will be presented in a later contribution. Using the replica method of spin glass theory, they showed in [5] that the energy (18) is fully determined by the eigendistribution of $\mathbf{E}$ and the information symbol alphabets; the reader is referred to Proposition 1 in [5].

An important assumption which was made in [5] was that known as the *replica symmetry ansatz* [10], which we have shown to mimic finite size results for the convex alphabet considered in this work [6], [7]. One should note, however, that although replica symmetry yields asymptotically accurate results for convex alphabets, it fails to produce accurate results for alphabets relaxed onto a superlattice. While convex alphabets yield a single energy minimizing state $\mathbf{x}$, extended lattice (or otherwise non-convex) alphabets may exhibit complex energy landscapes with many peaks and valleys. This might cause different identical copies (*replicas*) of the system to become trapped in different energy wells at low temperature. The replica method must then be employed invoking Parisi's *replica symmetry breaking* scheme [11]. For a recent and novel analysis of lattice alphabets based on replica symmetry breaking the reader is referred to [12]. For a thorough discussion of replica symmetry, the reader is referred to comprehensive literature on disordered systems [13], [14].

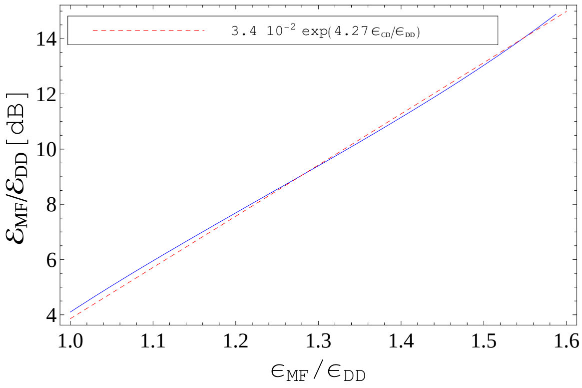

Fig. 3. Energy per bit vs. spectral efficiency tradeoff for $1 > \alpha > 0.625$. As α gets close to 1 DD is evidently more advantageous as it allows the transmitter to use less energy at no cost in spectral efficiency.

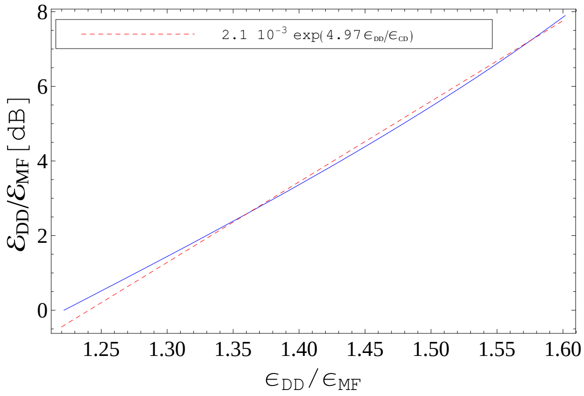

Fig. 4. Energy per bit vs. spectral efficiency tradeoff for $1.22 < \alpha < 1.6$. As α gets close to 1.22 DD is evidently more advantageous as it allows for higher spectral efficiency at no additional per-bit cost.

IV. RESULTS

In order to properly compare, from the point of view of the transmitter, the matched filter and distributed detection schemes proposed in Section II, we introduce the uncoded spectral efficiency ϵ, which is the number of bits per transmitting antenna. While for matched filter detection (MF) ϵ always equals 2, for distributed antenna detection (DD) ϵ equals 2α. Figure 2 shows the energy per transmitted bit (18) for both schemes as a function of α. When the ratio of antennas at the transmitter to that of the receiver is smaller than 0.5 only the DD scheme allows for the possibility of constructing a typical vector in $\mathcal{S}$; analogously, only the MF scheme might be employed when $\alpha > 2$. This issue is explored in detail in [8] where it is shown that, for the alphabet under consideration, the energy metric (5) must be at least half-rank.

When the ratio of antennas at the receiver to that at the transmitter is between 1 and 1.22 greater ϵ is achieved by the DD scheme at a lower cost in energy per transmitted bit. However, in this range, the DD scheme has a slight probability of failing to find a vector $\mathbf{x}$ in $\mathcal{A} \cap \mathcal{S}$ [8]; nevertheless, this probability is small and perhaps tolerable in systems with subsequent error control coding.

In the two remaining regions, namely $0.5 < \alpha < 1$ and $1.22 < \alpha < 2$, a tradeoff situation occurs in terms of energy per transmitted bit vs. ϵ. However, one must be careful when α is too close to 0.5 (only DD is viable in realistic finite channels) or too close to 2 (only MF is viable) [8]. When these two critical subregions are avoided, the $\mathcal{E}$ to ϵ tradeoff, shown in Figs. 3 and 4, is roughly exponential.

Depending on the priorities and resources of transmitter and receiver, other tradeoff situations might be of interest. For instance, looking at Fig. 2 we can see that, if the receiver has only slightly more than twice as many antennas as the transmitter, then some rate might be gained by turning off roughly half of the receiving antennas, such that $\alpha \in (1, 1.22)$, and switching to DD mode; this gain in rate would come, however, at a moderate additional energy penalty per bit for the transmitter.

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