On the Capacity Loss due to Separation of Detection and Decoding for ISI Channels

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***Abstract*—The performance loss due to separation of detection and decoding on the binary-input intersymbol interference (ISI) channel is quantified in terms of mutual information. It is shown that this performance loss can be calculated via the BCJR algorithm. Results are reported for several test channels.**

I. Introduction

Separation of detection and decoding, though suboptimum in general, is wide-spread in communications technology. This is due to the huge complexity which is required by the optimum joint approach. Examples are GSM where a soft-output Viterbi equalizer (BCJR algorithm [1]) is used for detection first and then another Viterbi algorithm is used for decoding, xDSL (digital subscriber lines), where separate decision–feedback equalization (e.g. implemented with equivalent precoding) or reduced-state equalization is performed before channel decoding, and UMTS, where state-of-the-art receivers rely on separate detection using a Rake receiver and turbo decoding.

While separate processing is used frequently, little is known about its loss in terms of channel capacity compared to optimum joint processing. In this paper, this loss is investigated for binary transmission. In Section II, we give a general formula for the capacity of separate processing. This formula is specialized to ISI channels in Section III, and it is shown that the BCJR algorithm [1] can be used to evaluate the loss of separate processing. Finally, numerical examples are given.

II. Capacity of Separate Processing

Consider a binary transmission with channel coding over a discrete–time channel, e.g. an ISI channel. As long as the user's information rate is lower than the capacity of the channel there is a coding scheme that enables perfect reconstruction of the transmitted symbols x_k from the knowledge of the received sequence.

With respect to the data processing lemma, it is not obvious whether this reconstruction can be handled by a structure with separate detection and decoding[1]. Hereby, the detector is allowed to be any algorithm that has no knowledge about the code laws while the independent decoder is assumed to have no knowledge about the channel.

[1] In this case additional interleaving after channel coding and deinterleaving in the receiver is assumed if the channel has memory.

Without loss of generality consider now the channel from x_1 to p_1, where p_1 is the detector output corresponding to x_1. Assume that the detector does its job in an optimum fashion. Thus, its output is canonically given by the a posteriori probability $p_1 \overset{\triangle}{=} \Pr(x_1 = 1|\boldsymbol{r})$, where vector $\boldsymbol{r}$ represents the received sequence.

For the further development, the following lemma will be essential:

Lemma 1: Consider an arbitrary memoryless weakly symmetric channel with binary input and output $Y \in \mathcal{Y}$. Let $P_\mathrm{e}(y)$ denote the minimum uncoded probability of detection error of this channel conditioned on the observation of the output symbol y. Then, the capacity of that channel is given by

$$C = 1 + \mathop{\mathrm{E}}_{y}\left[P_\mathrm{e}(y)\log_2 P_\mathrm{e}(y) + \big(1-P_\mathrm{e}(y)\big)\log_2\big(1-P_\mathrm{e}(y)\big)\right]. \tag{1}$$

For a proof of Lemma 1 we refer to [4]. Define the binary entropy function $\mathrm{e}_2(x) \overset{\triangle}{=} -x\log_2(x) - (1-x)\log_2(1-x)$. From Lemma 1, we have for the capacity of separated detection and decoding

$$C_\mathrm{sep} = \mathrm{I}(X_1; P_1) = 1 - \mathop{\mathrm{E}}_{p_1} \mathrm{e}_2\big(P_\mathrm{e}(p_1)\big). \tag{2}$$

Since p_1 is a deterministic function of $\boldsymbol{r}$ averaging over p_1 is equivalent to averaging over $\boldsymbol{r}$. This yields

$$C_\mathrm{sep} = 1 - \mathop{\mathrm{E}}_{\boldsymbol{r}} \mathrm{e}_2\big(P_\mathrm{e}(p_1)\big) = 1 - \mathop{\mathrm{E}}_{\boldsymbol{r}} \mathrm{e}_2(p_1). \tag{3}$$

Note that (3) is the capacity for "soft" detection. The capacity for hard detection is smaller and reads

$$C_\mathrm{sep}^\mathrm{hard} = \mathrm{I}\Big(X_1; \mathrm{sign}\left(P_1 - \tfrac{1}{2}\right)\Big) = 1 - \mathrm{e}_2\big(\mathop{\mathrm{E}}_{\boldsymbol{r}} P_\mathrm{e}(p_1)\big). \tag{4}$$

III. Application to ISI channels

For intersymbol interference (ISI) channels, the optimum approach of joint equalization and decoding using the super trellis diagram is not feasible, in general. Therefore, in most practical applications equalization and decoding are separated. E.g., in the GSM system for mobile communications a BCJR algorithm [1] or a suboptimum approximation thereof is used for calculation of a posteriori probabilities (APP's) of the transmitted symbols exploiting the entire received signal of a block, and subsequently a soft–input channel decoder is employed. Because unreliable equalizer outputs often occur in bursts, an interleaver is inserted after channel coding, and deinterleaving is performed at the equalizer output. Hence, the equivalent channel for decoding is a memoryless[2] fading channel, and the results of Section II can be applied in case of binary transmission for evaluation of the capacity of separate soft–output equalization and decoding. Note that neither numerical nor closed–form results on this capacity seem to be available in the literature on ISI channels yet. According to (3),

$$C_{\rm sep}^{\rm ISI} = 1 - \mathop{\rm E}_{\boldsymbol{r}} {\rm e}_2(p_k), \qquad (5)$$

where p_k denotes the a posteriori probability of the kth transmitted symbol when the entire received signal is known, $p_k \triangleq \Pr(x_k = 1|\boldsymbol{r})$, with $\boldsymbol{r} \triangleq [r_0 \quad r_1 \quad \ldots \quad r_{L+q_h-1}]$. Here, L and q_h denote the number of transmitted coded symbols per block and the channel order, respectively, and the received signal samples are given by $r_k = \sum_{\kappa=0}^{q_h} g_\kappa \, x_{k-\kappa} + n_k$, with binary antipodal i.i.d. transmit symbols x_k, additive white Gaussian noise n_k with variance σ^2, and the discrete–time channel impulse response g_κ. We assume that q_h known tailing symbols are transmitted after each block of coded symbols for trellis termination in the BCJR algorithm for soft–output equalization.

Hence, according to (5), capacity may be evaluated numerically by calculating APP's via the BCJR algorithm and averaging $1 - {\rm e}_2(p_k)$ over a sufficiently long burst.

Of course, it is interesting to compare this capacity to the capacity of optimum joint processing. For (discrete–time) ISI channels with non–Gaussian i.i.d. inputs, no closed–form results are available in general on capacity but only bounds [2]. Recently, a numerical method has been provided by Arnold et al. in [3]. There, capacity is evaluated using

$$C_{\rm joint}^{\rm ISI} = \lim_{L\to\infty} \frac{1}{L} \underbrace{h(R_0\, R_1\, \ldots\, R_{L-1})}_{\triangleq\, h(R)} - h(N), \qquad (6)$$

where $h(\cdot)$ denotes differential entropy, and $h(N) = \frac{1}{2}\log_2(2\,\pi\, e\, \sigma^2)$. For long blocks it can be shown that $-\frac{1}{L} \cdot \log_2(p_r(\tilde{\boldsymbol{r}}))$ ($p_r(\cdot)$: pdf of received vector), where $\tilde{\boldsymbol{r}}$ results from $\boldsymbol{r}$ by truncating the last q_h entries, converges to $h(R)$ [3]. On the other hand, $p_r(\tilde{\boldsymbol{r}})$ can be obtained by marginalizing the joint probabilities obtained in the forward recursion of the BCJR algorithm over the trellis states in the Lth recursion step (summing up all final α's [1] of the BCJR algorithm)[3]. Hence, as in the case of separate equalization and decoding, capacity can be determined from a run of the BCJR algorithm over a long block.

[2] Ideal interleaving is assumed.

[3] In order to avoid numerical problems, a slightly different approach should be preferred [3], rescaling the α's in each recursion step and using the scale factors for calculation of $h(R)$.

In the following, numerical results are shown for the test channels DICODE ($q_h = 1$), EPR4 ($q_h = 3$), E2PR4 ($q_h = 4$), and CH6 ($q_h = 6$), whose impulse responses are given e.g. in [3]. All channels have zeros only on the unit circle. Fig. 1 shows $C_{\rm joint}^{\rm ISI}$ according to [3] and $C_{\rm sep}^{\rm ISI}$ versus E_s/N_0 (E_s: average received energy per symbol, N_0: single–sided power spectral density of noise process) for all channels. In principle, the loss due to separation of equalization and channel decoding increases with increasing channel order / channel distortion. The smallest loss results for the DICODE channel, whereas the loss is significant for the CH6 channel. Hence, for severely distorting channels, the code rate should be adapted to the separated receiver structure. Furthermore, the significant loss of separate equalization and decoding for severely distorting channels suggests iterative equalization and decoding in order to approach the capacity of joint processing.

For wireless channels, the BCJR algorithm has to be executed for a sufficient number of random channel realizations and the capacities have to be averaged.

In [4], results on the capacity loss due to separation of detection and decoding are given for large CDMA systems with random signature sequences.

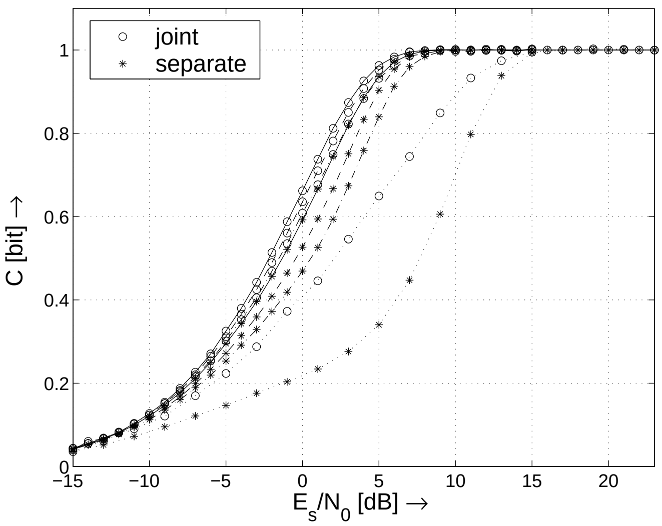

Fig. 1. Channel capacity for optimum joint processing ('∘') and separate optimum equalization and decoding ('∗'). Channels: DICODE (solid), EPR4 (dashed), E2PR4 (dash–dotted), CH6 (dotted).

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