Efficient Implementation of Iterative Multiuser Decoding

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I. Introduction

Iterative multiuser decoding based on the sum-product algorithm can give excellent performance [1]. To reduce complexity, inherent maximum-aposteriori detection is often replaced by linear MMSE detection combined with parallel successive cancellation.

The powers of the users are often unbalanced, either by design to improve performance as proposed in [2] or due to independent fading in the reverse link. During iterations, the power distribution of the residual interference evolves. This can lead to a severely mismatched MMSE detector after few iterations unless it is re-calculated for every iteration as in [3]. Here, a computationally less costly approach is presented.

II. Proposed Scheme

Let the synchronous Gaussian CDMA channel in vector notation [4] be given as $\boldsymbol{y}_\mu = \boldsymbol{S}\sqrt{\boldsymbol{W}}\boldsymbol{x}_\mu + \boldsymbol{n}_\mu$ where $\boldsymbol{n}_\mu$ is unit variance, zero-mean AWGN, $\boldsymbol{x}_\mu$ is the vector of transmitted symbols at time instant μ, $\boldsymbol{y}_\mu$ is the vector of chip samples at time instant μ, the $L \times K$ matrix $\boldsymbol{S}$ denotes the spreading sequences $\boldsymbol{s}_1, \ldots, \boldsymbol{s}_K$, and the diagonal matrix $\boldsymbol{W}$ is composed of the users' powers $w_1, \ldots, w_K$. W.l.o.g., let $\mathrm{E}\boldsymbol{x}\boldsymbol{x}^{\mathrm{H}} = \mathbf{I}$.

Each user's signal is encoded and interleaved in discrete time μ. Consider now a parallelly operating iterative multiuser decoder based on interference cancellation as considered in [2]. Then, the signal of user k fed into the decoding unit at iteration number m is given by

$$d_{k,\mu}^m = (\boldsymbol{f}_k^m)^{\mathrm{H}} \left(\boldsymbol{y}_\mu - \boldsymbol{S}\sqrt{\boldsymbol{W}}\hat{\boldsymbol{x}}_\mu^m + \boldsymbol{s}_k\sqrt{w_k}\hat{x}_{\mu,k}^m\right) \tag{1}$$

here $\boldsymbol{f}_k^m$ should be chosen appropriately, e.g. according to the unconditional MMSE criterion, cf. [2].

The unconditional MMSE detector leads to $(\boldsymbol{f}_k^m)^{\mathrm{H}} = \sqrt{w_k}\boldsymbol{s}_k^{\mathrm{H}}(\boldsymbol{S}\boldsymbol{W}\boldsymbol{V}^m\boldsymbol{S}^{\mathrm{H}} + \mathbf{I} - w_k v_k^m \boldsymbol{s}_k\boldsymbol{s}_k^{\mathrm{H}})^{-1}$ with the error covariance matrix $\boldsymbol{V}^m = \mathrm{E}(\boldsymbol{x}_\mu - \hat{\boldsymbol{x}}_\mu^m)(\boldsymbol{x}_\mu - \hat{\boldsymbol{x}}_\mu^m)^{\mathrm{H}}$. We assume that the estimates $\hat{\boldsymbol{x}}_\mu^m$ result from extrinsic information only and the girth of the code's graph is greater than m. Thus, $\boldsymbol{V}^m$ is diagonal, its entries are denoted by v_k^m.

Let the users' powers be quantized in $J < K$ levels and $\mathcal{K}_j$ denote the set of users belonging to class j. W.l.o.g., we assume that the powers of users in class j are greater than the powers of users in class $j+1$, $\forall j$. Now, we propose a suboptimum approach that calculates only J different detectors. Detector #j assumes the powers in classes $i < j$ to be zero, and the powers of in classes $i \geq j$ to be $w_k v_k^m$ for $k \in \mathcal{K}_i$. That means that only users that have not (almost) been canceled out are suppressed by linear detection.

Define the diagonal matrix $\boldsymbol{Z}_j$ such that $z_{j,k}$ is zero if k belongs to a class smaller than j and one otherwise. Decompose the spreading matrix into $\boldsymbol{S} = \boldsymbol{S}\boldsymbol{Z}_j + \boldsymbol{S}\left(\mathbf{I} - \boldsymbol{Z}_j\right)$. Note that the exact MMSE detector, depends on the iteration index m via the matrix $\boldsymbol{V}^m$. This is overcome by the approximation $\boldsymbol{V}^m \approx \rho^m \boldsymbol{V}_j\boldsymbol{Z}_j$ with $\boldsymbol{V}_j$ denoting the matrix $\boldsymbol{V}^{m_0}$ frozen at iteration m_0 when the last switch of detectors took place and ρ^m being an arbitrary real-valued *scalar* depending on the iteration index m.

We introduce the singular value decomposition (SVD) $\boldsymbol{S}\sqrt{\boldsymbol{W}\boldsymbol{V}_j\boldsymbol{Z}_j} = \boldsymbol{T}_j\boldsymbol{D}_j\boldsymbol{U}_j^{\mathrm{H}}$, such that $\boldsymbol{T}_j$ and $\boldsymbol{U}_j$ are unitary and $\boldsymbol{D}_j$ is diagonal up to some additional columns or rows which are all zero. Define $\boldsymbol{Q}_j = \boldsymbol{T}_j^{\mathrm{H}}\boldsymbol{S}$. With (1), we find

$$\hat{d}_{k,\mu}^m = \frac{\boldsymbol{q}_{j,k}^{\mathrm{H}}\left(\rho^m\boldsymbol{D}_j\boldsymbol{D}_j^{\mathrm{H}} + \mathbf{I}\right)^{-1}\left(\boldsymbol{T}_j^{\mathrm{H}}\boldsymbol{y}_\mu - \boldsymbol{Q}_j\sqrt{\boldsymbol{W}}\hat{\boldsymbol{x}}_\mu^m\right)}{\boldsymbol{q}_{j,k}^{\mathrm{H}}\left(\rho^m\boldsymbol{D}_j\boldsymbol{D}_j^{\mathrm{H}} + \mathbf{I}\right)^{-1}\boldsymbol{q}_{j,k}} + \sqrt{w_k}\hat{x}_{\mu,k}^m$$

Given $\boldsymbol{Q}_j$ and the SVD, this involves only $\mathrm{O}(K)$ multiplications/additions per user, symbol, and iteration, as the calculation of $\boldsymbol{T}_j^{\mathrm{H}}\boldsymbol{y}_\mu - \boldsymbol{Q}_j\sqrt{\boldsymbol{W}}\hat{\boldsymbol{x}}_\mu^m$ is common to all users and the other operations are inner products of vectors with diagonal kernels. Only when a switch from detector j to detector $j+1$ takes place, a SVD and a matrix multiplication are needed.

III. Switching Criterion

Assume i.i.d. random spreading and $K \gg 1$ enabling use of large system results. To decide if time has come to switch, the multiuser efficiencies at the outputs of the actual and the alternative detector are calculated. Then, that detector promising better performance is chosen for the next iteration.

Assume the residual interference power of user k at iteration m is p_k^m. Making use of some results in [5, 6], it can be shown that the multiuser efficiency converges almost surely to the large-system limit

$$\kappa^m = \left(1 - \sum_{k=1}^K \frac{(p_k^m\eta^m)^2}{(1+p_k^m\eta^m)^2}\right)\left(1 + \sum_{k=1}^K \frac{w_k v_k^m}{(1+p_k^m\eta^m)^2}\right)^{-1}$$

where η^m is the unique positive solution to the fixed point equation $(\eta^m)^{-1} = 1 + \sum_{k=1}^K p_k^m(1+\eta^m p_k^m)^{-1}$ and the interference power is approximated by

$$p_k^m = \begin{cases} w_k v_k^m & k \in \bigcup_{i=j+1}^J \mathcal{K}_i \\ 0 & \text{otherwise} \end{cases}, \quad p_k^m = \begin{cases} w_k v_k^{m_0}\rho^m & k \in \bigcup_{i=j}^J \mathcal{K}_i \\ 0 & \text{otherwise} \end{cases}$$

in case the detector is switched, not switched, respectively.

Certainly, one would like to chose ρ^m in such a way as to maximize multiuser efficiency, i.e. $\rho^m = \mathrm{argmax}\,\kappa^m$. A suboptimum, but computationally less costly alternative achieving satisfying results is $\rho^m = \sum_{k=1}^K v_k^m / \sum_{k=1}^K v_k^{m_0}$.

References

[1] J. Boutros, G. Caire. Iterative multiuser joint decoding: Unified framework and asymptotic analysis. *IEEE Trans. Inf. Th.*, Submitted.

[2] G. Caire, R. Müller. The optimal received power distribution for IC–based iterative multiuser joint decoders. In *Proc. of 39th An. Allerton Conf. on Commun., Contr. & Comp.*, Monticello, IL, Oct. 2001.

[3] X. Wang, V. Poor. Iterative (turbo) soft interference cancellation and decoding for coded CDMA. *IEEE Trans. Comm.*, 47(7):1046-1061, 1999.

[4] S. Verdú. *Multiuser Detection.* Cambridge University Press, NYC, 1998.

[5] D. Tse, S. Hanly. Linear multiuser receivers: Effective interference, effective bandwidth and user capacity. *IEEE Trans. Inf. Th.*, 45(2):641–657, 1999.

[6] J. Zhang, E. Chong, D. Tse. Output MAI distributions of linear MMSE multiuser receivers in DS–CDMA systems. *IEEE Trans. Inf. Th.*, 47(3):1128–1144, 2001.