Performance Analysis of Large Dual Antenna Array Systems with Binary Modulation

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Abstract — **Channel capacity and minimum probability of error for commuication via large dual antenna arrays is calculated for binary modulation taking into account statistical dependencies among channel coefficients using the correlation model introduced in [1]. Calculations rely upon the replica method developed in statistical physics [2].**

I. Channel Model

Consider a channel with K transmit and N receive antennas $\boldsymbol{r} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{n}$ where the vectors $\boldsymbol{x}$ and $\boldsymbol{r}$ contain the transmit and receive signals, $\boldsymbol{H}$ is the channel matrix, and $\boldsymbol{n}$ is additive white Gaussian noise with variance σ_0^2.

Following [1], $\boldsymbol{H}$ can be decomposed into $\boldsymbol{H} = \boldsymbol{\Phi}^\dagger \boldsymbol{A} \boldsymbol{\Theta}$ where the $S \times K$ and $S \times N$ steering matrices $\boldsymbol{\Theta}$ and $\boldsymbol{\Phi}$ account for the propagation from the transmit array to the S scattering objects and from there to the receive array, respectively, and the $S \times S$ matrix $\boldsymbol{A}$ accounts for propagation between scattering objects. Moreover, the entries of the matrices $\boldsymbol{\Theta}$ and $\boldsymbol{\Phi}$ are independent identically distributed random variables with vanishing odd order moments and variances $1/N$ and $1/S$, respectively, the sizes of the matrices grow large with load $\beta = \frac{K}{N}$ and richness $\rho = \frac{S}{N}$ remaining fixed, and the matrix $\boldsymbol{A}\boldsymbol{A}^\dagger$ has an asymptotic eigenvalue distribution as $S \to \infty$.

In contrast to [1], the channel is assumed to be real-valued. It is conjectured that the results on a real-valued channel for binary phase-shift keying generalize to the complex-valued channel with quaternary phase-shift keying and Gray mapping in the natural way, i.e. the bit error probability and the capacity per real dimension are identical, as the number of antennas grows large.

II. Replica Analysis

Using the replica method, the free energy is shown to be

$$\mathcal{F} = \int_{\mathrm{IR}} \log\left[\cosh\left(\sqrt{E}z + E\right)\right] Dz - \frac{E}{2}\left(1+m\right) - \frac{1}{2\beta}\left[1 + \int \log[1 + \lambda \frac{\beta}{\sigma_0^2}\left(1-m\right)]\, dF_\lambda(\lambda)\right] \qquad (1)$$

with the macroscopic parameters m and E being defined by

$$m = \int_{\mathrm{IR}} \tanh\left(\sqrt{E}z + E\right) Dz, \quad E = \int \frac{\lambda dF_\lambda(\lambda)}{\sigma_0^2 + \lambda\beta(1-m)}, \qquad (2)$$

the Gaussian measure $Dz \triangleq e^{-z^2/2} dz/\sqrt{2\pi}$, and $F_\lambda(x)$ denoting the limiting distribution of the eigenvalues of $\boldsymbol{\Phi}^\dagger \boldsymbol{A}\boldsymbol{A}^\dagger \boldsymbol{\Phi}$. If the system of equations (2) has multiple solutions, the correct solution is those one for which the free energy (1) is larger. For the particular case of $\boldsymbol{A} = \mathbf{I}$, the integrals over the eigenvalue distributions in (1) and (2) can be evaluated explicitly.

III. Channel Capacity

Channel capacity in nats, for channel state information at receiver side only, relates to the free energy like $C = -\mathcal{F} - \frac{1}{2\beta}$ as $K \to \infty$ [3]. The limited input entropy of binary signaling affects the optimal partitioning of antennas into transmit and receive antenna. For rich scattering, it is beneficial to have more transmit antennas than receive antennas given a fixed total number of antennas at both ends altogether, see Fig. 1, as entropy per transit antenna is upper bounded by 1 bit, while the entropy per receive antenna is unlimited.

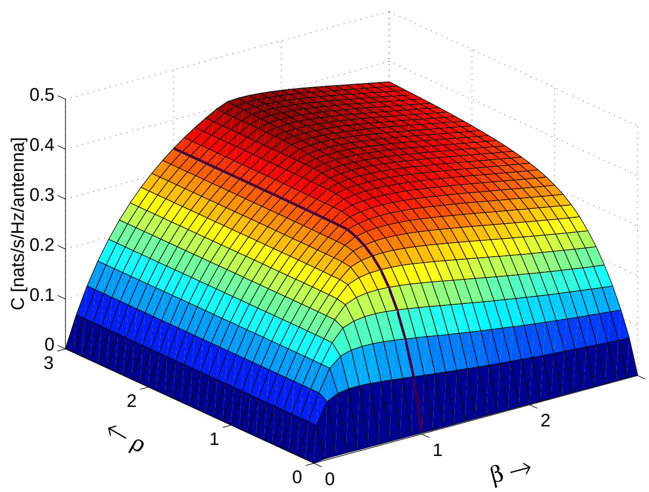

Fig. 1: Channel capacity per total number of antennas vs. load and richness for binary input alphabet, 9 dB SNR, and $\boldsymbol{A} = \mathbf{I}$.

IV. Bit Error Probability

The bit error probability of the individually optimum detector can be shown to converge to $\int_{\sqrt{E}}^{\infty} Dz$ as the number of antennas grows large [3]. It shows a *waterfall* behavior (phase transition) for poor scattering and/or strongly overloaded systems.

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References

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