Asymptotic Design and Analysis of Linear Detectors for Asynchronous CDMA Systems

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Abstract — **The asymptotic performance of the linear MMSE detector for any finite observation window and any symbol impinging the observed signal is derived for asynchronous but chip synchronous CDMA systems with random spreading. Additionally, a multistage detector that does not suffer from windowing effects and performs as well as the correspondent detector in synchronous systems is proposed. In contrast to the synchronous case, considering a sufficient large delay, the proposed multistage detector can even outperform the full rank linear MMSE detector constrained to a finite fixed observation window.**

Summary

We consider an asynchronous CDMA system with K users and a spreading factor N. For analysis tractability we constraint, as in [1], the relative time shift T_k between the signal of user k and the reference user 1 to be a multiple of the chip interval T_c. Without loss of generality we assume that the time shift between any user and user 1 is, at most, one symbol and the users are ranked in ascending order of time shift T_k. Let $\mathbf{y}(m) \in \mathbb{C}^N$ and $\mathbf{b}(m) \in \mathbb{C}^K$ be the observed column vector synchronized to the reference user and the column vector of transmitted symbols at time m, respectively. $\mathbf{S}(m) \in \mathbb{C}^{2N\times K}$ is the spreading matrix containing the appropriately shifted spreading sequences at time m by columns. For notation reasons we split the matrix $\mathbf{S}(m)$ into two matrices $\mathbf{S}_u(m), \mathbf{S}_d(m) \in \mathbb{C}^{N\times K}$ such that $\mathbf{S}(m) = [\mathbf{S}_u^T(m), \mathbf{S}_d^T(m)]^T$. The asynchronous system is then described by

$$\mathcal{Y} = \mathcal{S}\mathcal{A}\mathcal{B} + \mathcal{N} \tag{1}$$

where $\mathcal{Y} = [\ldots, \mathbf{y}^T(m-1), \mathbf{y}^T(m), \mathbf{y}^T(m+1)\ldots]^T$, $\mathcal{B} = [\ldots, \mathbf{b}^T(m-1), \mathbf{b}^T(m), \mathbf{b}^T(m+1)\ldots]^T$, $\mathcal{A}$ is a block diagonal matrix with all blocks equal to the matrix of complex amplitudes $\mathbf{A} = \mathrm{diag}(a_1, a_2, \ldots a_K)$. $\mathcal{N}$ is the additive white gaussian noise with variance σ^2. The matrix $\mathcal{S}$ is a bi-diagonal block matrix, infinite in both directions and with block row

$$\mathcal{S} = \left[\begin{array}{cccccc} \cdots & \mathbf{0} & \mathbf{S}_d(m-1) & \mathbf{S}_u(m) & \mathbf{0} & \cdots \end{array}\right] \tag{2}$$

The multistage detector $\mathcal{M} = \sum_{n=0}^{M-1} \mathcal{W}_n \mathcal{R}^n \mathcal{A}^H \mathcal{S}^H$ is optimized such that $\mathrm{E}\{\|\mathcal{M}\mathcal{Y} - \mathcal{B}\|^2\}$ is minimum. $\mathcal{R} = \mathcal{A}^H\mathcal{S}^H\mathcal{S}\mathcal{A}$ and $\mathcal{W}_n$, $n = 0, \ldots, M-1$, is a block diagonal matrix with blocks equal to $\mathbf{W}_n(m) = \mathrm{diag}(w_{n1}(m), w_{n2}(m), \ldots, w_{nK}(m))$. An additional feature of the multistage detector, peculiar to asynchronous systems, is that it can be implemented without truncation, with a finite delay equal to MT_s (see Figure 1 in [3]). Under some technical constraint on the random entries of $\mathbf{S}$, (see [2]), the limiting distribution of the eigenvalues of $\mathbf{A}\mathbf{A}^H$, $F_{\mathbf{A}\mathbf{A}^H}(\lambda)$, and of $\tau_k = \frac{T_k}{T_s}$, the following results holds:

1. Multistage detector. We show that

$$\lim_{K=\beta N\to\infty} (\mathcal{R}^n(m))_{kk} = R^n_{kk,\infty} \qquad \forall\, 1 \le n \le M^2. \tag{3}$$

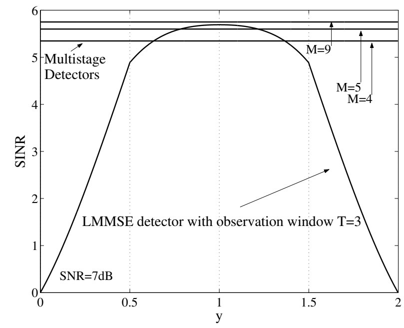

Fig. 1: Asymptotic SINR of linear MMSE and multistage detectors for varying M ($T = 3$, $\frac{E_b}{N_0} = 7$ dB)

$(\mathcal{R}^n(m))_{kk}$ is the diagonal element of $\mathcal{R}^n$ corresponding to the user k at time m and $R^n_{kk,\infty}$ is the limit value of the corresponding synchronous system with the same $F_{\mathbf{A}\mathbf{A}^H}(\lambda)$, and received power $|a_k|^2$. Recursive and closed form expressions for $R^n_{kk,\infty}$ are in [2]. Therefore, $\mathbf{W}^\infty_m = \lim_{K=\beta N\to\infty} \mathbf{W}_m$ is given by $\mathbf{w}^\infty_k = (\mathbf{\Phi}^\infty_k)^{-1}\mathbf{c}^\infty_k$ where $(\mathbf{w}^\infty_k)_n = (\mathbf{W}^\infty_n)_{kk}$, $\mathbf{c}^\infty_k$ is an M-dimensional column vector, $\mathbf{\Phi}^\infty_k \in \mathbb{R}^{M\times M}$, $(\mathbf{c}^\infty_k)_n = R^{n+1}_{kk,\infty}$, $(\mathbf{\Phi}^\infty_k)_{lm} = R^{l+m}_{kk,\infty} + \sigma^2 R^{l+m-1}_{kk,\infty}$. The output SINR of user k is given by

$$\mathrm{SINR}_k(n) \quad = \quad \frac{(\mathbf{c}^\infty_k)^T(\mathbf{\Phi}^\infty_k)^{-1}\mathbf{c}^\infty_k}{1-(\mathbf{c}^\infty_k)^T(\mathbf{\Phi}^\infty_k)^{-1}\mathbf{c}^\infty_k}. \tag{4}$$

2. Linear MMSE detector. The linear LMMSE detector suffers from windowing effects [1]. In the full version of this paper, we provide a theorem to quantify the windowing effects on the asymptotic ($K, N \to \infty$) performance of the linear LMMSE detector with finite observation window T for all the transmitted symbols that impinge the received signal.

Figure I shows the asymptotic SINR of a linear MMSE detector with observation window $T = 3$, $\beta = \frac{1}{2}$, τ uniformly distributed in $[0, 1]$, and $\frac{E_b}{N_0} = 7$dB as function of the impinging transmitted signal $y = \lim_{N\to\infty} \frac{k}{N}$ and $k = 1, \ldots, 4K$. The performance of the MMSE is compared with the asymptotic SINR multistage detectors for increasing number of stages ($M = 4, \ldots, 9$) .

References

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