Asymptotic Design and Analysis of Linear Detectors for Asynchronous CDMA Systems

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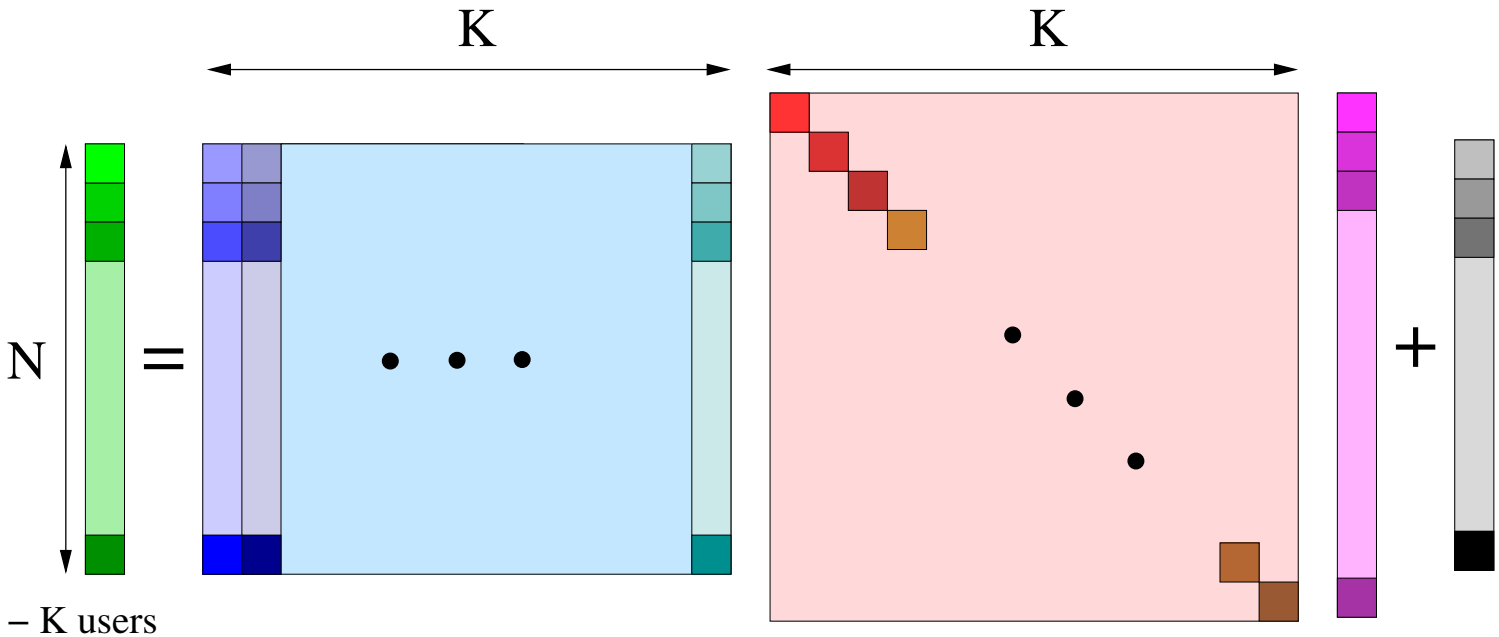
1. Problem Statement
2. LMMSE Detectors
3. Multistage Detectors
4. Performance Analysis
5. Summary

Synchronous CDMA Uplink System Model

$$y(m) = \underbrace{S(m)A(m)}_{H(m)} b(m) + n(m)$$



– K users
– Spreading factor: N

Linear Multiuser Detection

LMMSE Detector

$$\begin{aligned}\widehat{\mathbf{b}}_{\mathrm{LMMSE}} &= \left(\boldsymbol{H}^H\boldsymbol{H} + \sigma^2\boldsymbol{I}\right)^{-1}\boldsymbol{H}^H\boldsymbol{y} \\ &= \sum_{i=0}^{K-1} \tilde{w}_i \left(\boldsymbol{H}^H\boldsymbol{H}\right)^i \boldsymbol{H}^H\boldsymbol{y}\end{aligned}$$

With appropriate coefficients $(\tilde{w}_0, \tilde{w}_1, \ldots, \tilde{w}_{K-1})$.

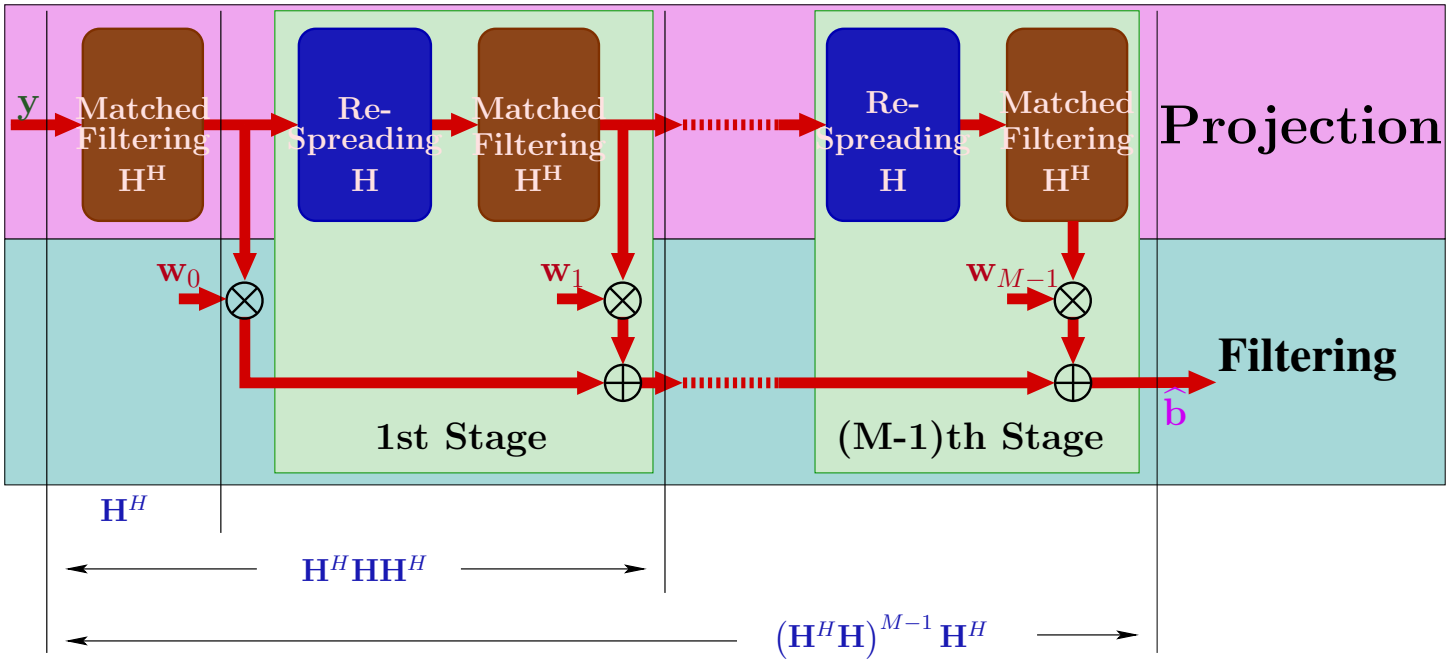
Multistage Detectors

$$\hat{\boldsymbol{b}}_{\mathrm{MS}} = \sum_{i=0}^{M-1} \boldsymbol{W}_i \left(\boldsymbol{H}^H\boldsymbol{H}\right)^i \boldsymbol{H}^H\boldsymbol{y} \qquad \text{for } M < K.$$

$\boldsymbol{W}_i$ diagonal matrix minimizing the mean square error

$$\mathrm{E}\left\{\left\|\mathbf{b} - \widehat{\mathbf{b}}_{\mathrm{MS}}\right\|^2\right\}$$

Multistage Detector Structure



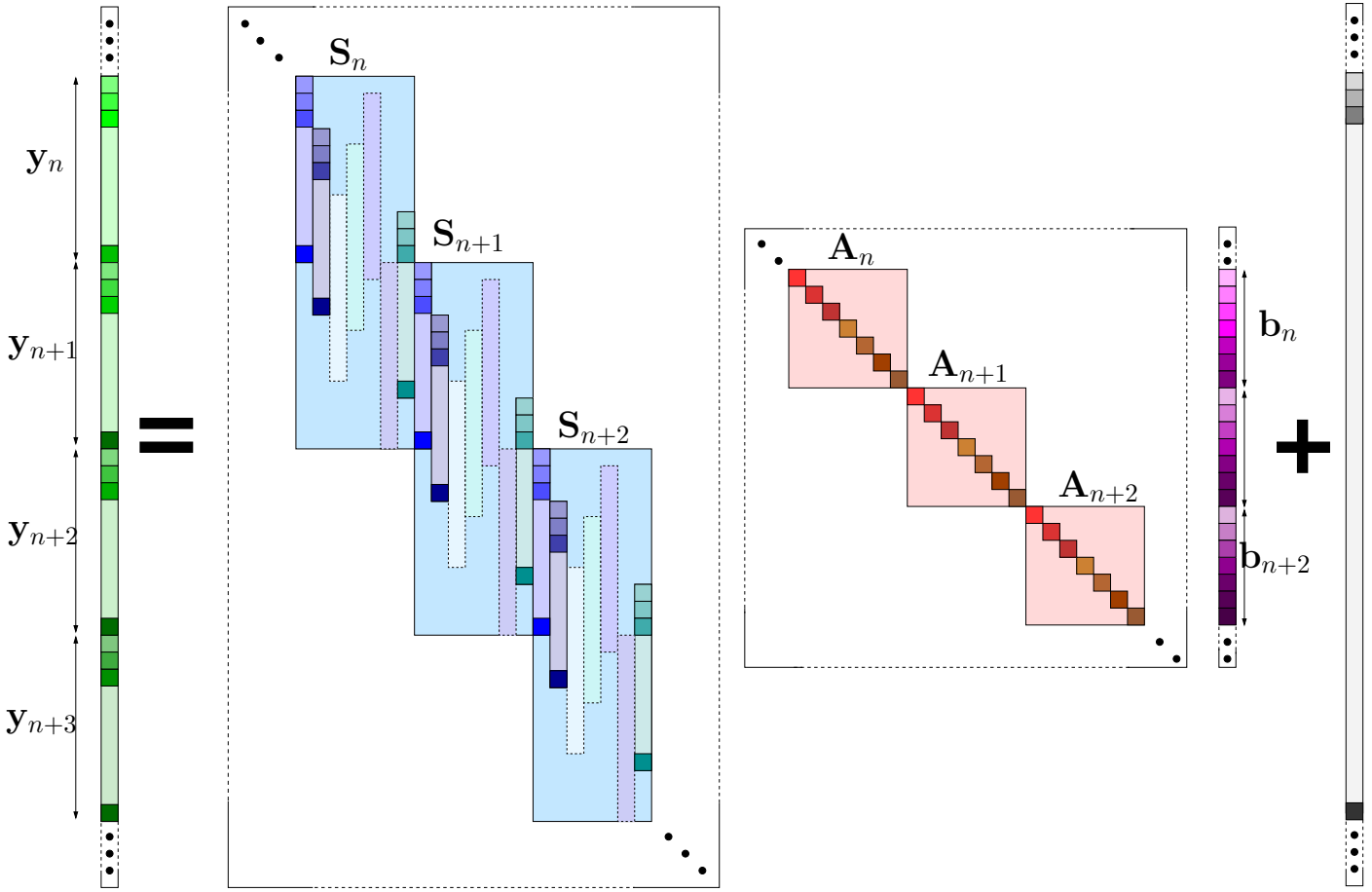
Linear Complexity/bit!
(like SUMF)

Performance Degradation
w.r.t. LMMSE

Projection & optimum filtering

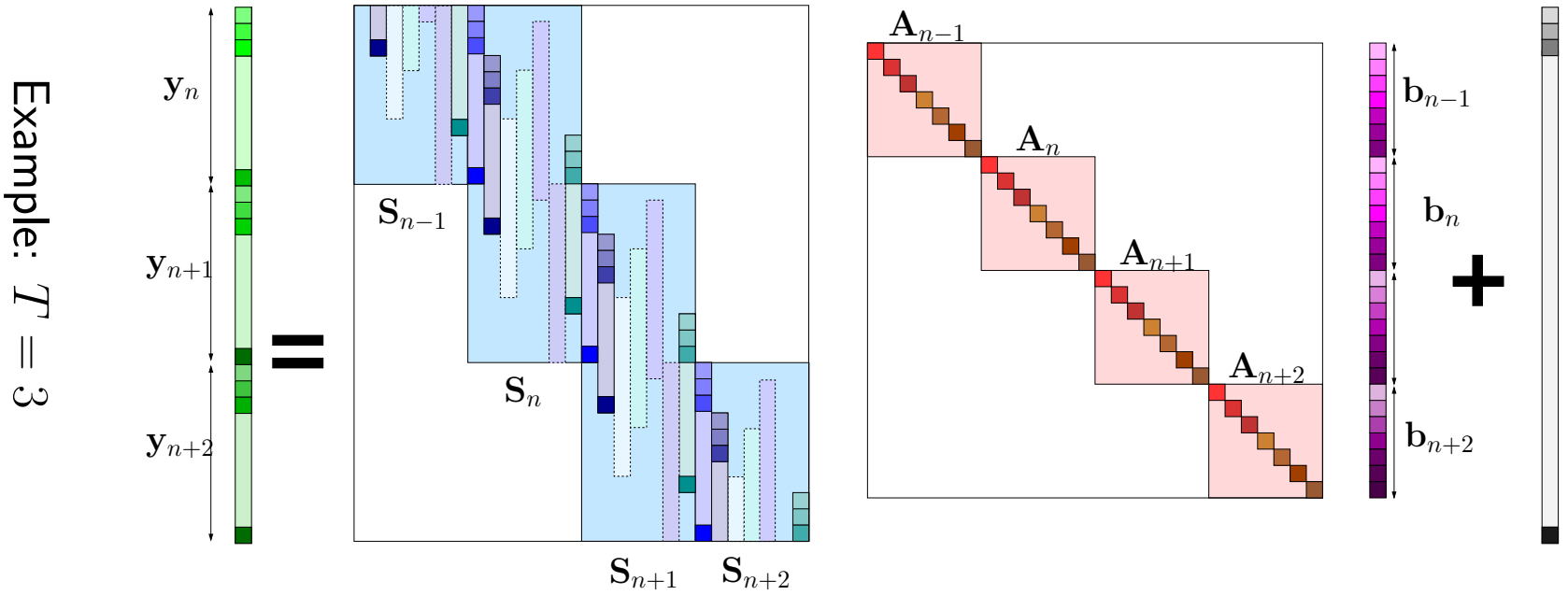
1. **Implementation:** $M-1$ **identical stages performing re-spreading and matched filtering.**
2. **Tracking signal subspace not required.**
3. **No scaling of the needed rank with the system size.**

Symbol-Async. and Chip-Sync. Systems Model



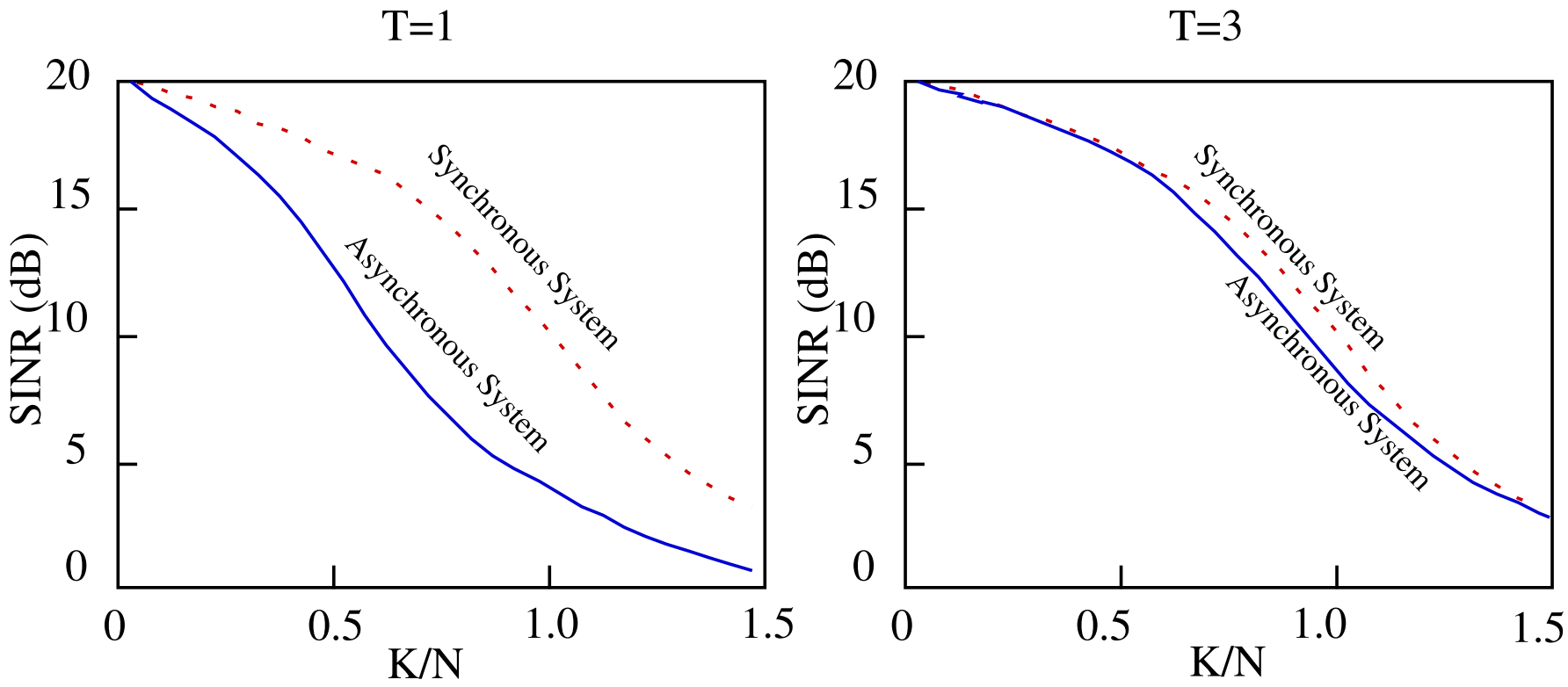
$$\mathcal{Y} = \underbrace{\mathcal{S}\mathcal{A}\mathcal{B}}_{\mathcal{H}} + \mathcal{N}$$

LMMSE Detector: Truncation



$$\mathcal{Y}_T(n) = \underbrace{\mathcal{S}_T(n)\mathcal{A}_T(n)}_{\mathcal{H}_T(n)}\mathcal{B}_T(n) + \mathcal{N}_T(n)$$

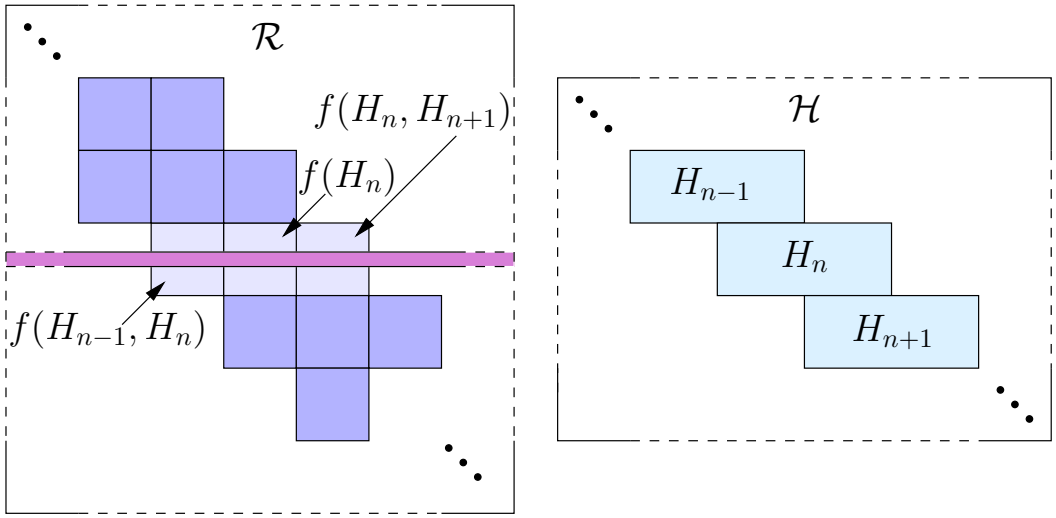
LMMSE Detector: Truncation Effects



As the window size $T \to \infty$, the effect of asynchronicity vanishes.

Multistage Detectors: No Truncation Effects

1-st Stage: $\mathcal{R}\mathcal{H}^H$

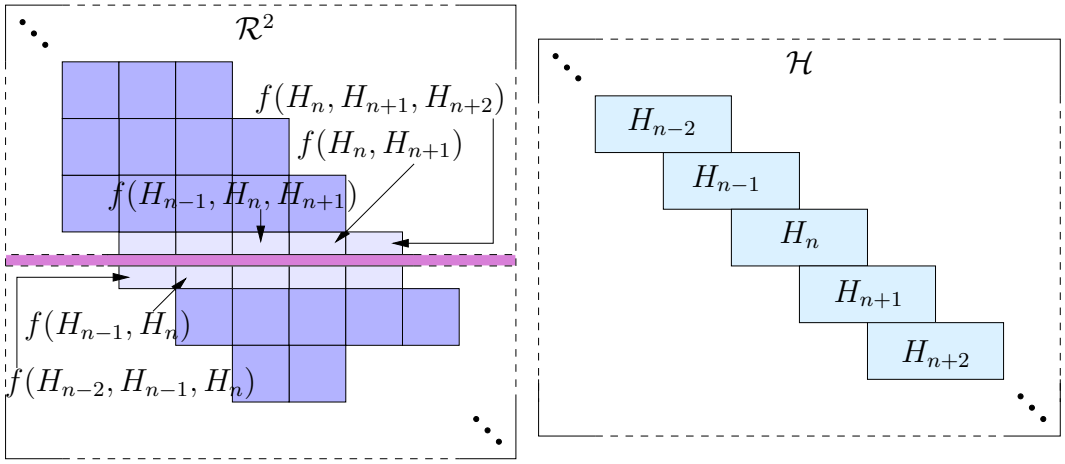


$$\mathcal{R} = \mathcal{H}^H\mathcal{H}$$

The projector onto $\mathcal{R}\mathcal{H}^H$ for the estimation of $b_i(n)$ depends only on H_{n-1}, H_n, H_{n+1}!

Multistage Detectors: No Truncation Effects

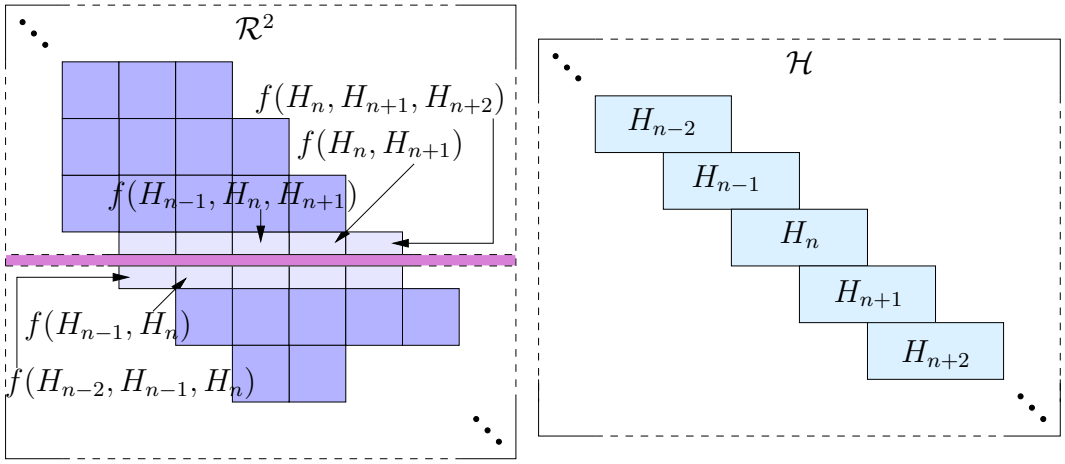
2-nd Stage: $\mathcal{R}^2\mathcal{H}^H$



The projector onto $\mathcal{R}^2\mathcal{H}^H$ for the estimation of $b_i(n)$ depends only on H_{n-2}, H_{n-1}, H_n, H_{n+1}, H_{n+2}!

Multistage Detectors: No Truncation Effects

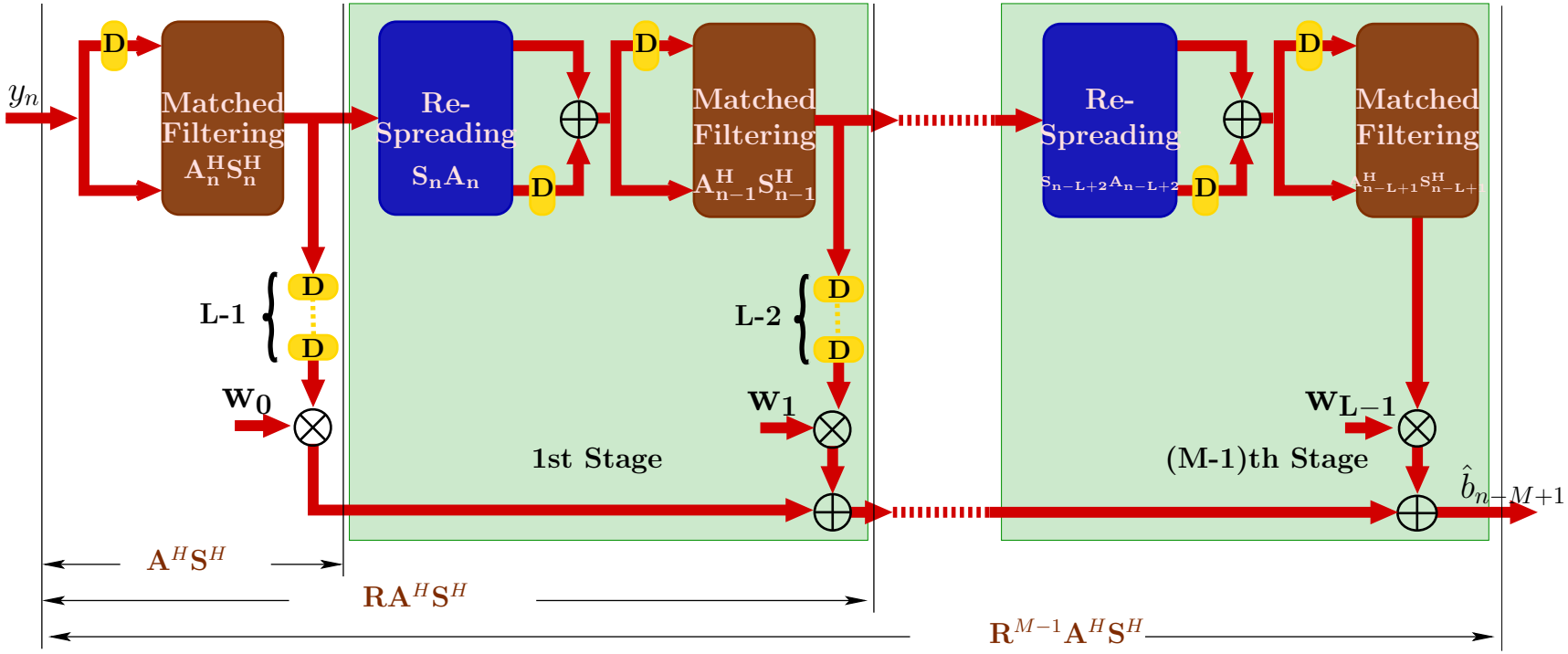
2-nd Stage: $\mathcal{R}^2\mathcal{H}^H$



The projector onto $\mathcal{R}^2\mathcal{H}^H$ for the estimation of $b_i(n)$ depends only on H_{n-2}, H_{n-1}, H_n, H_{n+1}, H_{n+2}!

The signal projection onto $\mathcal{H}^H$, $\mathcal{R}\mathcal{H}^H$, $\mathcal{R}^2\mathcal{H}^H$... can be computed exactly with finite delay!

Asynchronous Detector Structure



1. **No truncation effects.**
2. **Enlargement of the observation window with number of stages.**

Linear Detectors: A Comparison

- **Linear MMSE Detector:**
  - Optimization in the full signal space.
  - Truncation effects!
- **Multistage Detector:**
  - Optimization in a subspace.
  - No truncation effect.

WHICH ONE PERFORMS BETTER?

Performance Analysis

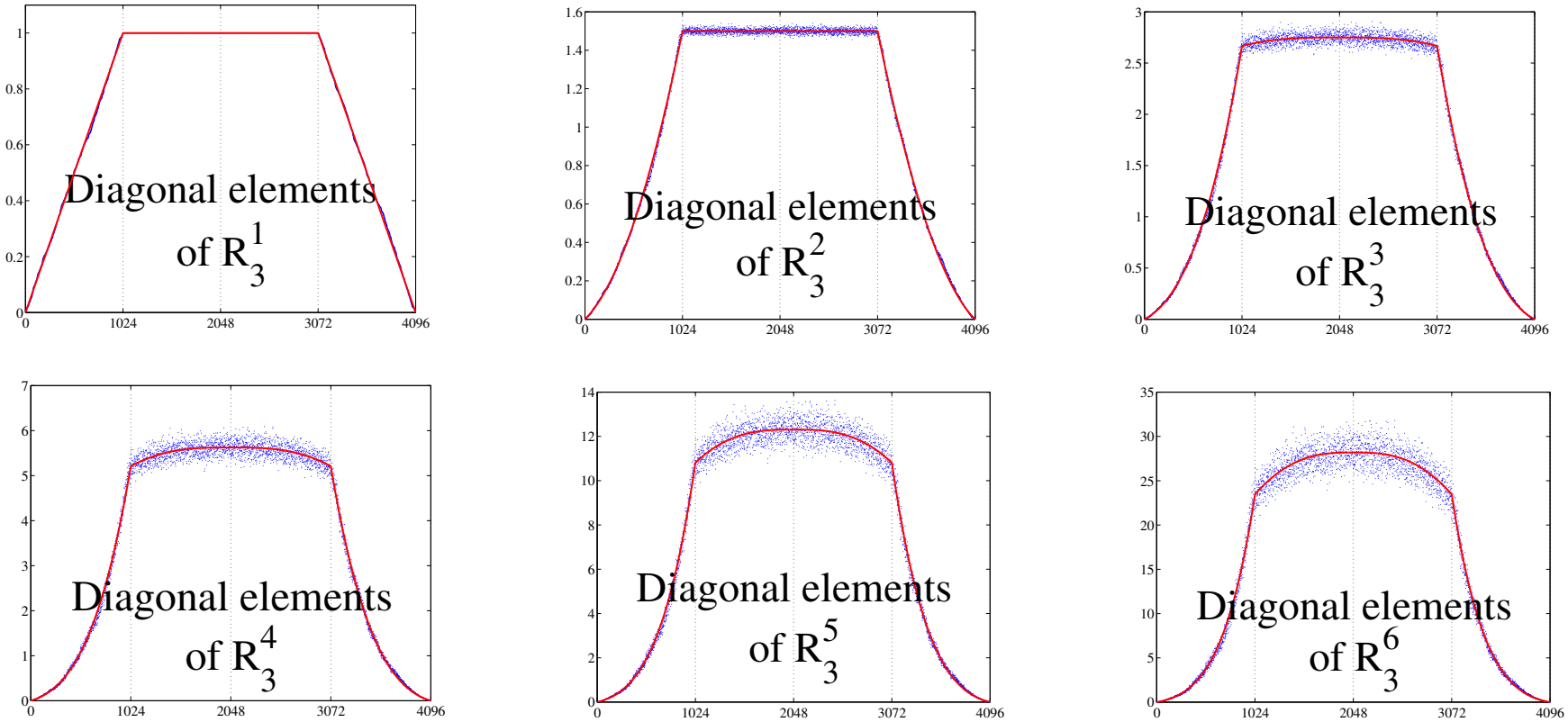
$\mathrm{SINR}[b_k(n)]$ depends on

$$\begin{cases} \mathcal{R}_{kk}(n),\ \mathcal{R}_{kk}^2(n), \ldots, \mathcal{R}_{kk}^{2M}(n) & M-\text{stage detector} \\ \mathcal{R}_{T,kk}(n),\ \mathcal{R}_{T,kk}^2(n), \ldots & \text{LMMSE detector} \end{cases}$$

$\mathcal{R}_{T,kk}^m(n)$ converges almost surely to a deterministic constant*
as $K, N \to \infty$ with $\frac{K}{N} \to \beta$

* see Theorem in the full version of the paper

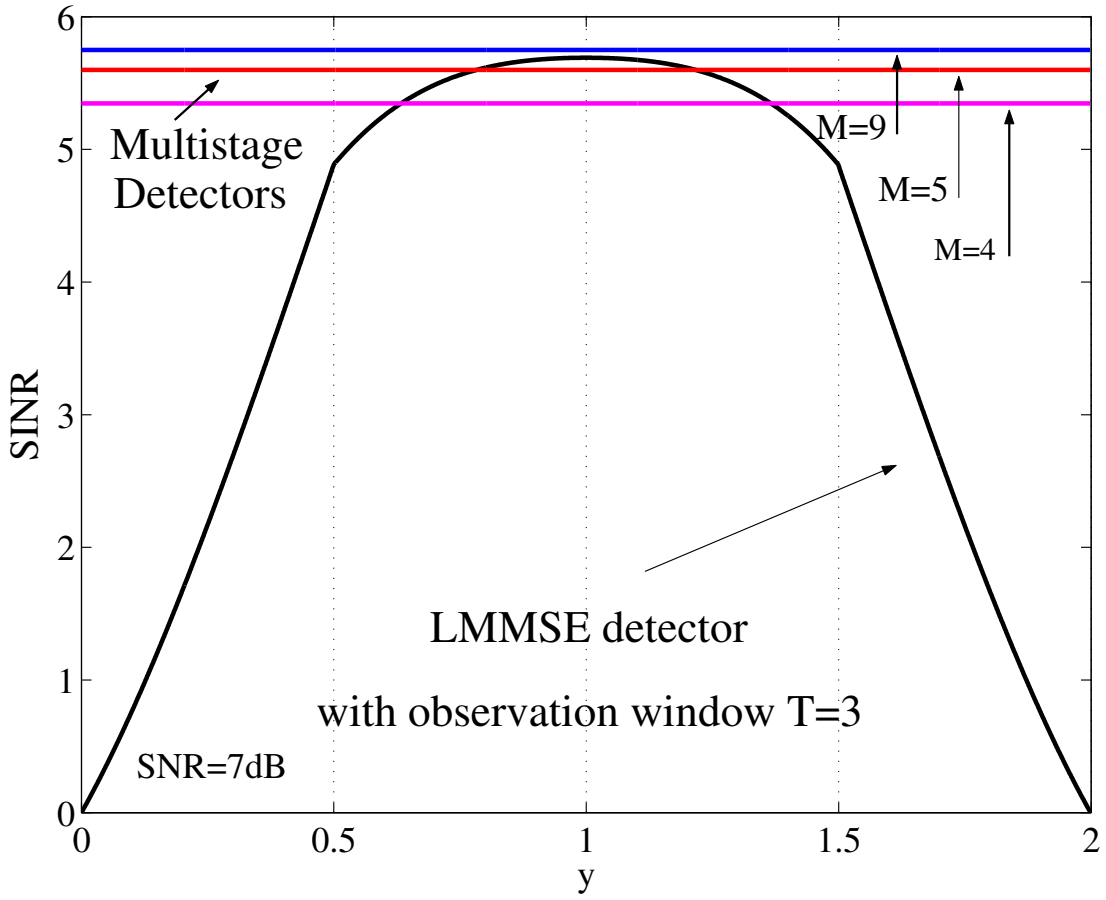
$\mathcal{R}^m_{T,kk}(n)$: Asymptotic versus Finite Systems



$\boldsymbol{A = I}$ $\quad T = 3$ $\quad N = 2048$ $\quad K = 1024$

LMMSE versus Multistage Detectors: Asymptotic Performance

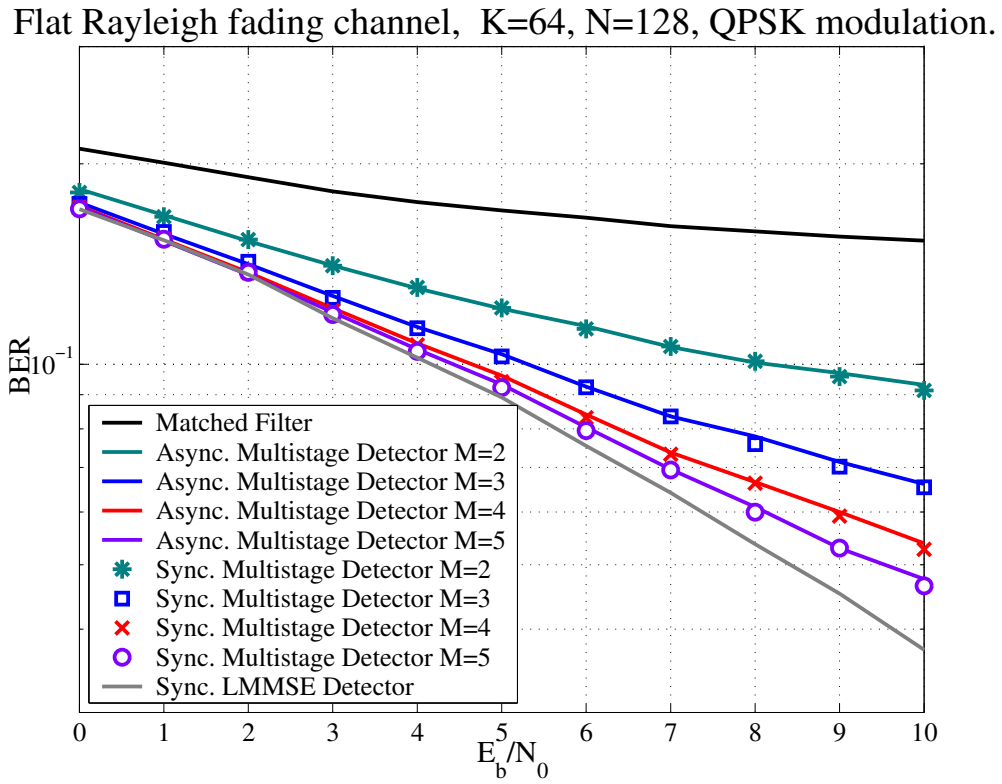
- Channel matrix $\boldsymbol{A} = \boldsymbol{I}$.
- Observation window $T = 3$.
- $\mathrm{SNR} = 7$ dB.
- $\beta = \frac{1}{2}$.



$\mathcal{R}^m_{T,kk}(n)$ as $T \to \infty$: Effect on Multistage Detectors

As $T \to \infty$ the diagonal elements of $\mathcal{R}_T$ equal the diagonal elements of $\mathcal{R}$ for synchronous systems.

Multistage detectors for large synchronous and asynchronous systems have:
– Same performance.
– Same weights.



Summary

- **A multistage detector structure for chip synchronous but symbol-asynchronous systems.**
- **An algorithm to determine the diagonal elements of $\mathcal{R}_T^m$ $\Rightarrow$ large system performance.**

In contrast to LMMSE detectors, multistage detectors have:

No truncation effects!

Same performance as for synchronous systems.

SINR independent of the time shift of the detected symbol.

Same complexity order per user as the matched filter.

For a sufficient large number of stages they can outperform the MMSE detectors with fixed observation window.

Remarks

Effect of chip asynchronicity:

Multistage detectors:

Laura Cottatellucci and Ralf R. Müller. Multistage detectors for asynchronous CDMA. *International Zurich Seminar on Communications (IZS)*, Zurich, Feb. 2004.

Linear MMSE detector:

Laura Cottatellucci and Ralf R. Müller. A generalized resource pooling result for correlated antennas or asynchronous CDMA systems. *Proc. of International Symposium on Information Theory and its Applications (ISITA)*, Parma, Oct. 2004.