On Random CDMA with Constant Envelope

Ralf R. Müller
Department of Electronics and Telecommunications
The Norwegian University of Science and Technology, Trondheim, Norway.
Email: ralf@iet.ntnu.no

***Abstract*—This paper studies the design of random code-division multiple-access (CDMA) with continuous-time constant envelope. The proposed scheme is compatible with linear CDMA and allows for standard methods of linear multiuser detection while avoiding phase jumps at all times.**

The proposed algorithm finds a set of spreading waveforms with approximately rectangular power spectral density and stop-band attenuation of more than 60 dB at exactly constant envelope. Alternatively, the algorithm can provide perfect stop-band attenuation at a peak-to-average power ratio of 0.04 dB at spreading factor 512.

I. Introduction

Constant envelope (CE) modulation offers significant advantages compared to amplitude modulation with respect to the design and linearity demands of the radio-frequency (RF) hardware, in general, and of the power amplifier, in particular. These advantages made it the method of choice for the GSM (Groupe Spécial Mobile, later Global System for Mobile Communications) standard [1].

State-of-the-art CE modulation also brings along some disadvantages. On a single user communication link, many forms of CE modulation come with intrinsic intersymbol interference and the need for a trellis based equalizer at the receiver. However, there are ways to circumvent this drawback in practice with negligible loss, see e.g. [2].

Furthermore, CE modulation only allows for information coded in phase, but not in amplitude. On a single user channel, this makes spectral efficiency saturate at only half its potential value at high signal-to-noise ratio (SNR) since the amplitude is kept unused for information transfer. Aiming for higher spectral efficiency the successors of GSM like IMT-2000 (International Mobile Telecommunications-2000) and others discarded CE modulation and went for linear modulation despite the need for more expensive RF-hardware. However, on a multiple-access channel, the amplitude component is utilized by the superposition of the signals of various users [3]. For a large number of users, spectral efficiency is not impaired due to the restriction to phase modulation. In multiuser communication systems, the spectral efficiency of CE modulation was, in fact, found to be almost indistinguishable from the spectral efficiency of CDMA with linear quadrature amplitude modulation [4]. Unfortunately, the findings in [4] came at the cost of rather big spectral side-lobes that are incompatible with the spectral masks of most wireless communication standards.

This material is based upon work supported by the Research Council of Norway under Grant No. 171133/V30.

The combination of continuous phase (CP) modulation and CDMA was first addressed in [5] in a very general setting. In the sequel, the following two ways to combine CP modulation with CDMA were given further attention: 1) A traditional spread spectrum signal is feed into a CP modulator [6]. This allows for a simple implementation of the transmitter by means of a voltage controlled oscillator. In its general form it prohibits linear multiuser detection, as it does not allow for a description of the transmitted signal as linear or quasi-linear CDMA. To overcome this drawback, Lampe et al. [7] proposed to closely approximate the received signal by an offset quaternary phase-shift keying (OQPSK) signal. 2) The spreading sequence is fed into a CP modulator before the data sequence is modulated [8]–[10]. This method decouples the phase state of the CP modulator from the phase of the data signal and therefore constitutes a linear CDMA system with constant envelope. In its general form it creates phase jumps at the transitions between symbols. To overcome this drawback, [4] proposed to use particular frequency pulses that avoid those phase jumps.

The two ways to combine CDMA with CP modulation still have their drawbacks. The modulator by Lampe et al. [7] achieves acceptable stop-band attenuation, but the pass-band spectrum is severely colored. Furthermore, the question whether the CP modulation increases the correlation of the signature waveforms has not been addressed. Reference [4] has found ways to shape the pass-band spectrum while keeping the correlation of the signature waveforms unaffected. However, the attenuation in the stop-band remained unsatisfactory.

In this paper, a novel way of CE modulation for CDMA is proposed that avoids the problems with spectral side-lobes at the cost of only fractions of decibels. For that purpose an iterative algorithm is proposed to generate sets of random spreading sequences with CE such that each user is assigned a set of spreading sequences. The spreading sequences of a user are very similar and differ only at the very beginning and the very end. They are chosen in a data-dependent fashion to avoid phase jumps at symbol transitions. In order for the iterative algorithm to converge where it is intended to, it is initialized with random spreading sequences generated by a Markov chain. We note that spreading sequences generated by Markov chains were previously proposed in literature, e.g. [11], [12], though with other purposes in mind.

The paper is composed of five more sections. Section II introduces the multiple-access channel considered in this paper. Section III introduces the novel method to generate

CE signature waveforms. Section IV addresses the issue of avoiding phase discontinuities at symbol transitions. Section V shows that the correlation properties of the proposed waveforms do not fall behind the ones of independent identically distributed (i.i.d.) random signatures. Conclusions are drawn in Section VI.

II. System Model

Consider an asynchronous CDMA system with K users in flat[1] Rayleigh fading. We focus on the uplink (reverse link) communication here, since the choice of CE modulation is motivated the by cost of transmitter hardware which is more relevant for mobile devices than for base stations.

The received signal at the base station is given by

$$y(t) = s(t) + w(t) \quad (1)$$
$$= \sum_{k=1}^{K} a_k s_k(t - \tau_k) + w(t) \quad (2)$$

in complex base-band notation. Here, $a_k \in \mathbb{C}$ is the received amplitude of user k accumulating the effects of transmitted amplitude, fading, and carrier phase offset; $\tau_k \in \mathbb{R}$ is the signal delay of user k; $w(t)$ is zero-mean additive white Gaussian noise with power spectral density N_0; and $s_k(t)$ is the spread signal of user k. It can be decomposed into

$$s_k(t) = \sum_{m=-\infty}^{+\infty} b_k[m] c_{k,m}(t - mT_\mathrm{s}) \quad (3)$$

where $b_k[m] \in \mathcal{B}$ is the symbol transmitted by user k at time instant m, $c_{k,m}(t)$ is the signature waveform of user k at time slot m, T_s is the symbol clock cycle, and $\mathcal{B} \subset \{z \in \mathbb{C} : |z| = 1\}$ is the symbol alphabet. Introducing the spreading factor N and the chip clock cycle

$$T_\mathrm{c} = \frac{T_\mathrm{s}}{N} \quad (4)$$

we represent the signature waveform

$$c_{k,m}(t) = \sum_{n=0}^{N-1} s_{k,m}[n] \psi(t - nT_\mathrm{c}) \quad (5)$$

by the discrete-time chip sequence $s_{k,m}[\cdot]$ and the chip waveform $\psi(t)$. Finally, we assume that $\tau_k \mathrm{mod} T_\mathrm{c}$ is uniformly distributed within the chip interval.

III. Generation of CE Signature Waveforms

In the following an iterative method to generate CE signature waveforms with high stop-band attenuation and suitable pass-band spectrum is proposed. For that purpose the signature waveforms at iteration $\ell = 0$ are represented as

$$c_k^{(0)}(t) = \begin{cases} \sum_{\tilde{n}=0}^{2N-1} \mathrm{j}^{(-1)^k \tilde{n}} \tilde{s}_k^{(0)}[\tilde{n}] \tilde{\psi}^{(0)} \left(t - \frac{\tilde{n} T_\mathrm{c}}{2}\right) & \forall 0 \le t < T_\mathrm{s} \\ 0 & \text{elsewhere} \end{cases} \quad (6)$$

[1]We restrict the consideration to flat fading for sake of simplicity. The effect of multipath can be easily incorporated into the shape of the chip waveform if the excess delay is much shorter than the symbol interval.

by means of two-fold oversampling and offset modulation. Following an idea of [4], the offset is chosen to be in opposite directions for even and odd user indices.

In order to ensure convergence of the iterative algorithm towards the desired fixed-point, the initial choices of $\tilde{s}_k^{(0)}[\cdot]$ and $\tilde{\psi}^{(0)}(\cdot)$ are important. The following initializations were found to lead to satisfactory results:

1) Create subsequent samples of $\tilde{s}_k^{(0)}[\cdot]$ by consecutively assigning the outputs of a binary symmetric antipodal Markov source with transition probability $\Pr(-1|+1) = \Pr(+1|-1) = p < \frac{1}{2}$.
2) Start with the half-sinewave pulse shape, i.e.

$$\tilde{\psi}^{(0)}(t) = \cos(\pi t / T_\mathrm{c}) \qquad \forall |t| < T_\mathrm{c}/2 \quad (7)$$

and 0 elsewhere.

This choice is motivated as follows: The offset modulation together with the half-sinewave pulse shape ensures CE of the signature waveforms by means of the well-known identity $\sin^2(x) + \cos^2(x) = 1 \forall x$. The spectrum of the half-sinewave pulse has a low-pass characteristic with severely colored passband and many sidelobes. The offset modulation shifts the spectrum of the signature waveform compared to the spectrum of the pulse shape to the right and to the left by $1/(2T_\mathrm{c})$[2] for even and odd user indices, respectively. For independent identically distributed chips, the resulting cumulated spectrum of all users $\Phi_{ss}(f)$ (power spectral density of the signal $s(t)$) is therefore, an M-shaped spectrum with sidelobes where low (and high) frequencies are underrepresented compared to medium frequencies around $\pm 1/(2T_\mathrm{c})$. To amplify the low-frequencies without affecting the CE, the chip sequence is given low-pass properties. This is implemented by means of the Markov source with appropriate transition probability. The Markov source amplifies the low-frequency components and attenuates the high-frequency components. By this means, the signals of all users maintain CE and the power spectral density of the superposition signal $s(t)$ achieves almost rectangular shape in the passband. However, the stop-band properties are still unsatisfactory. This is to be overcome by the following iterations.

The signature waveform is iterated as follows

$$\tilde{c}_k^{(\ell)}(t) = c_k^{(\ell)}(t) \overset{T_\mathrm{s}}{*} r(t) \quad (8)$$
$$c_k^{(\ell+1)}(t) = \frac{\tilde{c}_k^{(\ell)}(t)}{|\tilde{c}_k^{(\ell)}(t)|} \quad (9)$$

where $\overset{T_\mathrm{s}}{*}$ denotes cyclic convolution (efficiently implemented by means of oversampling and fast Fourier transform) with respect to T_s and $r(t)$ is a root-raised cosine filter with roll-off factor $\alpha = 0.1$ and Nyquist frequency $(1+\alpha)/(2T_\mathrm{c})$ windowed to the length of the symbol interval T_s. The convolution with the root-raised cosine filter serves the purpose of reducing the sidelobes in the stop-band. One would wish to use an ideal

[2]At several places a factor of 2 is lacking in the published manuscript. In this corrected version, the missing factors are marked in red.

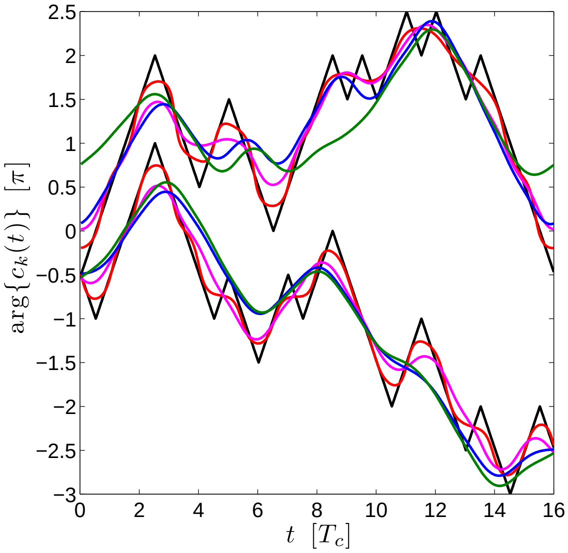

Fig. 1. The phases of two signature waveforms of length $16T_c$ after 0, 1, 10, 100, and 1000 iterations shown by the black, red, magenta, blue, and green lines, respectively. The amplitude, since exactly equal to one, is not shown.

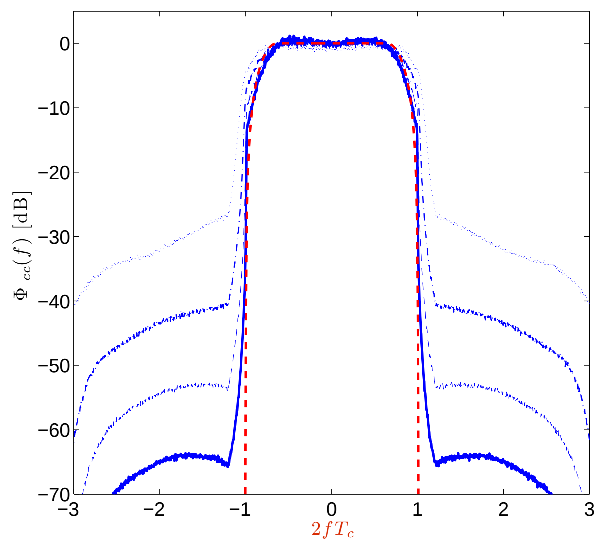

Fig. 2. Average power density spectrum of the proposed signature waveforms for $p = \frac{1}{3}$, and spreading factor $N = 512$ after 10, 10^2, 10^3, and 10^4 iterations (blue lines). For comparison, we also show the power spectral density of i.i.d. random CDMA with a root-raised cosine pulse with roll-off factor 0.22 and Nyquist frequency $1/(2.44T_c)$ (red line).

low-pass filter (zero roll-off) instead of the root-raised cosine filter. However, iterations did not converge in that case. A small, but non-zero roll-off (or something similar) seems to be necessary[3]. The normalization in (9) restores the CE after the filtering. The cyclic convolution was not primarily preferred over linear convolution to simplify implementation by means of fast Fourier transform, though it is certainly helpful in this respect, but to ensure that the signature waveform converges to a cyclic signal.

An example of two signature waveforms for spreading factor $N = 16$ is shown in Fig. 1. It is observed that the more iterations the smoother is the phase trajectory.

Fig. 2 shows the average power density spectrum for spreading factor $N = 512$ after various numbers of iterations. Depending on the number of iterations, a stop-band attenuation of more than 60 dB can be reached while maintaining exactly constant envelope in continuous time[4]. Even after 10^4 iterations, no fixed point was reached and further improvements of the stop-band attenuation are possible. The figure indicates that every tenfold increase of the number of iterations approximately buys additional 10 decibels of stop-band attenuation.

The proposed algorithm can also produce a perfectly band-limited signal at the expense of small concessions with respect to CE. For that purpose, we only need to stop the iterations after (8) and (11) instead of (9) and (12), respectively. The signal is then perfectly band-limited due to the filtering with $r(t)$ within the limits of the spectral properties of time-limited signals. The peak-to-average power ratios (PAPRs) are given in

TABLE I

PEAK-TO-AVERAGE POWER RATIO FOR VARIOUS NUMBERS OF ITERATIONS WHEN THE SIGNATURE WAVEFORMS ARE PERFECTLY BAND-LIMITED ($p = \frac{1}{3}$). THE PAPR OF I.I.D. RANDOM CDMA WITH THE ROOT-RAISED COSINE PULSE WITH ROLL-OFF FACTOR 0.22 THAT APPEARS IN THE IMT-2000 STANDARD IS SHOWN FOR COMPARISON.

	10 iter.	10^2 iter.	10^3 iter.	10^4 iter.	i.i.d.
$N = 8$	1.6 dB	0.5 dB	0.4 dB	0.4 dB	5.3 dB
$N = 32$	1.6 dB	0.4 dB	0.3 dB	0.3 dB	5.3 dB
$N = 128$	1.7 dB	0.5 dB	0.1 dB	0.07 dB	5.3 dB
$N = 512$	1.8 dB	0.5 dB	0.1 dB	0.04 dB	5.3 dB

Table I for various numbers of iterations. The PAPR converges slower for large spreading factors, but saturates at a lower level. Further experiments showed that the saturation level can be lowered by the choice of a larger roll-off factor for the root-raise cosine filter $r(t)$.

IV. GUARD CHIPS

To ensure that the spectral properties of the signature waveforms also hold for the spectrum of the spread signal $s_k(t)$, phase jumps at the symbol transition points (integer multiples of T_s) must be avoided. In fact, CP is only a necessary, but not sufficient condition. To ensure high stop-band attenuation even first and higher order derivatives of the phase should vanish or be sufficiently small.

Phase jumps do not occur at symbol transitions where the data symbols do not change, as the signature waveform is cyclic due to the cyclic convolution in the iterative process that created it. However, symbol transitions with changing symbols pose an obstacle. A way to overcome this problem by means of *guard chips* will be proposed in this section.

[3]We found empirically that a reduced roll-off reduces the achievable stop-band attenuation. The choice of $\alpha = 0.1$ appeared as a sensible trade-off between stop-band and pass-band properties of the spectrum.

[4]To emulate continuous time signals, the chip rate was 32-fold oversampled.

The general idea to overcome the problem of phase jumps is to use a set of several signature waveforms for each user. Assume that the data symbols stem from a regular M-ary phase shift keying, i.e. $\mathcal{B} = \{z : z^M = 1\}$. Then, we only need $|\mathcal{B}|^2 = M^2$ signature sequences per user. One of these is the signature waveform produced by the iterative algorithm in Section III. It will be used in the present time slot if the data symbols of the present, the past and the future time slot are identical.

In order not to jeopardize linear multiuser detection all M^2 signature waveforms of a given user should be very similar, e.g. to differ only at the very first and very last chips, called the *guard chips* in the sequel. These guard chips can be discarded by the multiuser detector just like the cyclic prefix is discarded in orthogonal frequency-division multiplexing. A more sophisticated version of linear multiuser detection, based on expected spreading waveforms which partially utilizes the guard chips is discussed in [13].

The M^2 signature waveforms per user are created by means of secondary iterations with use the outcome of the primary iterations (8) to (9) as starting point. First, we concatenate two signature sequences where the phase of one of them is shifted by one of the phase-shift-keying constellation points d

$$c_k^{(0)}(t,d) = \begin{cases} c_k^{(\infty)}(t+T_\mathrm{s})/d & -T_\mathrm{s} \le t < 0 \\ c_k^{(\infty)}(t) & 0 \le t < T_\mathrm{s} \\ 0 & \text{elsewhere} \end{cases} . \quad (10)$$

Then, for all constellation points $d \in \mathcal{B}$ we iterate the concatenation of the two signature sequences

$$\tilde{c}_k^{(\ell)}(t,d) = c_k^{(\ell)}(t,d) \overset{2T_\mathrm{s}}{*} r(t) \quad (11)$$

$$c_k^{(\ell+1)}(t,d) = \frac{\tilde{c}_k^{(\ell)}(t,d)}{|\tilde{c}_k^{(\ell)}(t,d)|} \quad (12)$$

until the phase jumps have smoothened out and we have re-established the desired spectral properties while preserving CE. Now, we can assign the data-dependent spreading sequences by cutting out the middle part of the concatenation and undoing the phase shift:

$$c_{k,m}(t) = \begin{cases} c_k^{(\infty)}\left(t, \frac{b_k[m]}{b_k[m-1]}\right) & 0 \le t < T_\mathrm{s}/2 \\ c_k^{(\infty)}\left(t-T_\mathrm{s}, \frac{b_k[m+1]}{b_k[m]}\right) \frac{b_k[m+1]}{b_k[m]} & T_\mathrm{s}/2 \le t < T_\mathrm{s} \\ 0 & \text{elsewhere} \end{cases} . \quad (13)$$

With these data-dependent spreading sequences, all symbol transitions are smooth.

The proposed method works excellently for large spreading factors. For small spreading factors, the method suffers from the well-known time-frequency uncertainty. Since the signals are strictly time-limited to preserve CE, their spectral properties deteriorate the shorter the signature waveform. Fig. 3 shows the spectra for various spreading factors after 10^4 primary iterations and 10^3 secondary iterations. For $N \ge 32$, the spectral regrowth due to CE dominates. For $N \le 16$, spectral regrowth is dominated by the time-limitation of the spreading sequences.

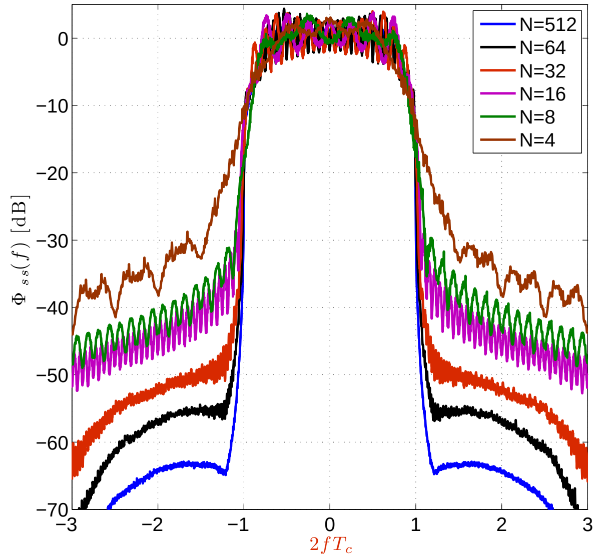

Fig. 3. Cumulated power density spectrum for $p = \frac{1}{3}$, 10^4 primary and 10^3 secondary iterations and spreading factors $N = 4, 8, 16, 32, 64, 512$ shown by the brown, green, purple, red, black, and blue lines, resp.

If the signal set is not a regular M-ary phase shift keying, there are, in general, more than M phase differences. Then, more than M^2 signatures sequences per user are required and (10) to (13) must be generalized accordingly

The various signature waveforms of each user should be very similar in order to apply standard algorithms for linear multiuser detection. Due to their design, they show significant deviations from each other only at the very beginning and at the very end. In the example shown in Fig. 4, they differ significantly only within the first two and last two chip intervals. The deviation is insignificant. In fact, the correlation coefficient between the two sequences was measured as 0.998. In practice, mismatch due to channel estimation errors is much larger. Thus, this small deviation should not pose any practical concern for high spreading factors.

V. Correlation Properties

The initial generation by a Markov source and the iterative processing of the random signature waveforms leads to signature sequences with correlated chips. This raises the concern whether the signature waveforms proposed in the two previous sections are more correlated than signature waveforms formed by i.i.d. random samples. We do not have a proof to debunk such concern, but we will give strong numerical evidence that this concern is unfounded.

The deleterious impact of correlations among signature sequences becomes visible at the spread of the singular values of the matrix of signature sequences. Note that those singular values are the only way the signature sequences affect the total

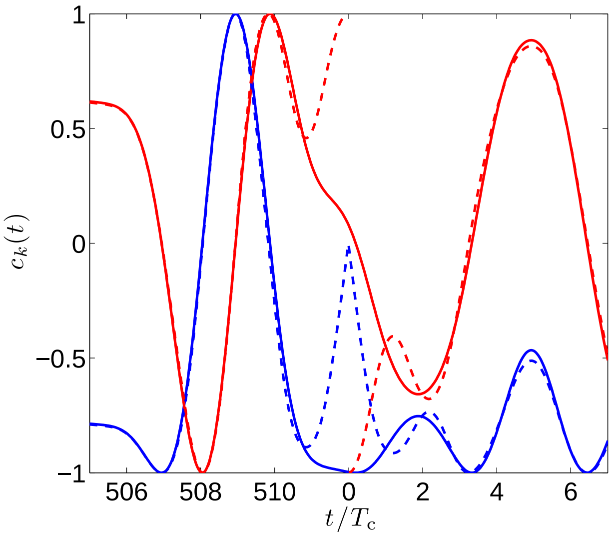

Fig. 4. First and last chips of a random set of CE signature waveforms of user k for spreading factor $N = 512$ and binary phase-shift keying. The blue and red lines refer to real and imaginary parts, resp. The solid and dashed lines refer to no symbol switching and symbol switching resp.

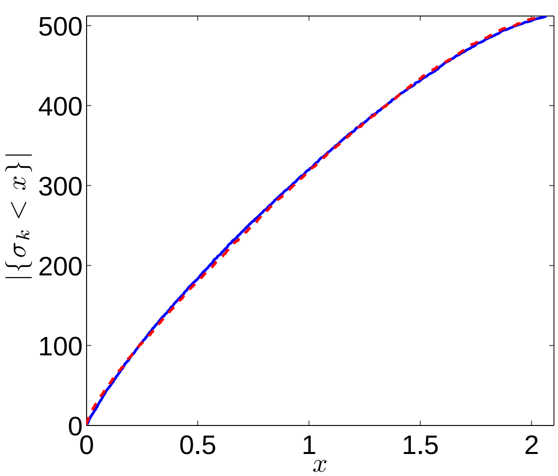

Fig. 5. Empirical distribution of the singular values σ_k for CE random waveforms with $p = \frac{1}{3}$ after 10^4 iterations (blue solid line) and i.i.d. random waveforms with root-raised cosine chip pulses with roll-off 0.22 (red dashed line). Both cases for $K = 512$ users and spreading factor $N = 512$.

capacity of the Gaussian multiple-access channel with channel state information at the receiver.

Fig. 5 shows the empirical distribution of singular values for both the proposed CE signature waveforms and a set of signature waveforms of i.i.d. random chips. In order to allow for a fair comparison, the i.i.d. random waveforms are modulated with a root-raise cosine pulse with roll-off factor 0.22. This results in almost identical power density spectra, cf. Fig. 2. It is evident from Fig. 5 that the singular value distributions of CE and i.i.d. signature waveforms differ insignificantly. Thus, we conclude that CE random waveforms show the same properties with respect to multiuser detection and channel capacity than CDMA with i.i.d. random signature waveforms.

VI. Conclusions

On the CDMA channel with moderate to high spreading factor, CE modulation comes at small to no cost in performance compared to conventional linear pulse shaping. It allows for the use of less expensive and more efficient power amplifiers at the transmitters. At the same time, the CE modulation proposed in this work is compatible with off-the-shelf linear multiuser detection due to its quasi-linear design. In multirate systems, users with small spreading factors can employ conventional spreading with root-raised cosine pulse shapes and co-exist with users with large spreading factors and CE signature waveforms. A stopband attenuation of 30 dB and 55 dB can be reached by spreading factors $N = 8$ and $N = 64$, respectively.

Acknowledgments

The author would like to thank Johannes Huber as well as Lutz and Alexander Lampe for insightful discussions.

References

[1] ETSI, "GSM specifications, series 01–12," 1991.

[2] J. Huber and W. Liu, "An alternative approach to reduced–complexity CPM–receivers," *IEEE Journal on Selected Areas in Communications*, vol. 7, pp. 1437–1449, Dec. 1989.

[3] L. Duan, B. Rimoldi, and R. Urbanke, "Approaching the AWGN channel capacity without active shaping," in *Proc. of IEEE International Symposium on Information Theory (ISIT)*, Ulm, Germany, Jun./Jul. 1997, p. 374.

[4] R. R. Müller and A. Lampe, "Spectral efficiency of random CDMA with constant envelope modulation," *International Journal of Electronics and Communications*, vol. 65, pp. 701–706, 2011.

[5] A. Svensson, C.-E. Sundberg, and G. Lindell, "On direct sequence spread spectrum with continuous phase modulation," in *Proceedings of Conference on Information Science and Systems (CISS)*, 1985, pp. 526–531.

[6] F. Giannetti, M. Luise, and R. Reggiannini, "Chip timing recovery in digital modems for continuous-phase CDMA radio communications," *IEEE Transactions on Communications*, vol. 43, no. 2/3/4, pp. 762–766, Feb./Mar./Apr. 1995.

[7] L. Lampe, R. Schober, R. Tzschoppe, and J. B. Huber, "Noncoherent continuous-phase modulation for DS-CDMA," in *Proceedings of IEEE International Conference on Communications (ICC)*, Anchorage, AK, USA, May 2003, pp. GC15–3.

[8] A. T. McDowell and J. S. Lehnert, "Phase-independent continuous phase modulation for bandwidth efficient multiple-access communications," in *Proceedings of Military Communications Conference (MILCOM)*, 1992, pp. 104–107.

[9] M. A. Landolsi and W. E. Stark, "Performance of DS/CDMA communications with continuous-phase signals," in *Proceedings of Military Communications Conference (MILCOM)*, 1993, pp. 748–752.

[10] R. T. Hsu and J. S. Lehnert, "The performance of continuous-phase-coded DS/SSMA communications," *IEEE Transactions on Communications*, vol. 46, no. 4, pp. 533–543, Apr. 1998.

[11] G. Mazzini, G. Setti, and R. Rovatti, "Chaotic complex spreading sequences for asynchronous DS-CDMA— Part I: System modeling and results," *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications*, vol. 44, no. 10, pp. 937–947, Oct. 1997.

[12] H. Fujisaki, "Design of optimum M-phase spreading sequences of Markov chains," *IEICE Transactions on Fundamentals*, vol. E90-A, no. 10, pp. 2055–2065, 2007.

[13] R. R. Müller, "Multiuser detection for continuous phase CDMA," in *Proc. of IEEE Topical Conference on Antennas and Propagation for Wireless Communications*, Torino, Italy, Sep. 2011.