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A Generalized Resource Pooling Result for Correlated Antennas with Applications to Asynchronous CDMA

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Abstract

This contribution analyzes the behaviour of CDMA (code division multiple access) systems with correlated spatial diversity. The users transmit to one or more antenna arrays. The centralized receiver employs a linear minimum mean squared error (MMSE) detector. We derive the performance of the linear detector in a large system with random spreading sequences and weak assumptions on the flat fading channel gains—the fading may be correlated and contain line-of-sight components. We show that, as the number of users and the spreading factor grow large with fixed ratio, the performance of the system is fully characterized by a square matrix with size equal to the number of receiving antennas. This limiting square matrix can be regarded as a kind of multidimensional multiuser efficiency. Our general result includes as special cases the results discussed by Hanly and Tse in 2001 [1] for independent channel gains, gives stronger convergence results, and provides the rigorous proofs missing in [1].

As a byproduct of the analysis, we also find the large-system signal-to-interference and noise ratio of symbol-quasi-synchrounous but chip-asynchronous CDMA systems (with certain constraints on the chip waveform) for single-antenna systems.

1. INTRODUCTION

Modelling of spreading matrices in CDMA (code division multiple access) systems by random matrices has resulted extremely fruitful from both the theoretical prospective of system analysis, see [2], [3], and [4], and from the practical point of view of receiver design, e.g. [5]. In the large system limit, as both the transmitted signals K and the spreading factor N tend to infinity with a fixed ratio, the random matrices show self-averaging properties. These allow the description of the system in terms of few macroscopic system parameters and provide thus deep insights into the system behaviour. Modelling the spreading matrices as random matrices, Hanly and Tse [1] analyzed a CDMA system consisting of users transmitting to a multiuser receiver with spatial diversity. The spatial diversity can be obtained by multiple antenna elements at a single base station, or by combining of signals received at multiple base-stations. In [1], these two cases of spatial diversity are referred to as microdiversity and macrodiversity, respectively. In this celebrated work the performance of linear multiuser receivers are analyzed under the assumption that the spreading sequences are Gaussian and the random channel gains are circular symmetric and independent for all users and antennas, and for any user the gains to all antennas are identically distributed. The analysis does not span cases of practical interest like multi-antenna element systems with correlated channels and/or line-of-sight components.

The pioneering works in [6] and [7] on antenna arrays at the transmitter and the receiver promise huge increases in the throughput of wireless communication systems. Therefore, they promoted the blossoming of a rich production of works that study the capacities of such systems in more realistic situations. In this flow are works that analyze the effects of channel correlation [8], [9], [10], [11], [12], [13], [14], line-of-sight components [15], [16], multiple scattering [17], and keyholes [11] (this list does not claim to be comprehensive). Fading correlation and line-of-sight components were found to affect channel capacity severely. It is natural and of practical interest to consider their effects also in a CDMA system with spatial diversity.

In this paper we consider a general framework with one or more antenna arrays at the receive side including combined micro- and macro-diversity scenarios. The transmitting users may use multiple element antennas, but need not to do so. The channel gains satisfy very

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weak constraints. They may be correlated and contain line of sight components, i.e. their mean may be different from zero. The analysis underlies the assumption of independent random spreading. Our general result includes the results in [1] as special cases. Additionally, it leads to a stronger result (convergence in the mean square sense instead of convergence in probability) and provides a rigorous proof of the results for the macro-diversity case, only conjectured in [1].

In the microdiversity case with independent channel gains, the system behaviour is captured by the conditional multiuser efficiency, which converges to a deterministic constant in the large system limit. With correlated channel gains, we show that the large system behaviour is captured by a deterministic positive definite Hermitian matrix with size equal to the number of receive antennas.

In this contribution we consider the linear MMSE receiver. Thanks to the assumption of independence among the chips, the analysis shows that this linear receivers is not affected by channel correlation between transmitting antennas and suffers only from channel correlations among receiving antennas. For large CDMA systems without receive antenna diversity, the multiuser efficiency fully characterizes the system since it is identical for all users. In contrast, we show that the multiuser efficiency in CDMA systems with spatial diversity changes from user to user, in general. Additionally, we give sufficient conditions under which also a system with spatial diversity and correlated channel gains is characterized by a unique scalar multiuser efficiency.

The same mathematics developed for CDMA systems with correlated spatial diversity can also be applied to the analysis of a CDMA system with asynchronous chips, and maximum relative time shift non higher than the chip interval. Concerning this last restriction on the maximum time-shift we observe that the analysis of asynchronous CDMA systems is usually split into two sub-problems, e.g. [18] and [19]:

- Analysis of the asynchronicity effects due to signal shifts multiples of the chip interval;
- Analysis of the asynchronicity due to signal shifts smaller than the chip interval.

In [19] it is proven that the linear MMSE detector for CDMA systems with relative signal shifts multiple of the chip interval and observation window centered on the user of interest performs as well as a linear MMSE detector for synchronous systems as the observation window tends to infinity while it performs worse for finite observation windows. In [20] the performance analysis of linear MMSE detectors and multistage detectors in symbol asynchronous but chip synchronous systems is extended to all the users in the system and to any finite observation window. The effect of chip asynchronicity on multistage detectors has been analyzed in [21]. However, no theoretical analysis of linear MMSE detectors for systems with time shifts smaller than the chip interval was available yet. The analytical tools provided in this work solve the problem for linear MMSE detectors.

2. SYSTEM MODEL AND NOTATION

In the following we denotes with $\boldsymbol{I}_n$ the identity matrix of size $n \times n$. $\mathrm{E}\{\cdot\}$ is the expectation operator. $\delta(\lambda)$ is the Dirac's delta function. $\otimes$ denotes the Kronecker product. $\boldsymbol{e}_l$ is the L-dimensional unit vector whose elements are zero except the l-th that equals 1 ($\boldsymbol{e}_l = (\delta_{lj})_{j=1}^{L}$).

We consider a CDMA system with spreading factor N and K' users. Each user employs a transmit antenna array with N_{T} elements sending independent data streams through each of the elements. Thus, we may speak of a system with $K = K'N_{\mathrm{T}}$ virtual users. The signal is received by L receive antennas. These antennas can be part of an array or can be placed at different locations, but the received signals are processed jointly.

The baseband discrete-time system model, as the channel is flat fading, is given by

$$\boldsymbol{y} = \mathcal{H}\boldsymbol{b} + \boldsymbol{n} \tag{1}$$

where $\boldsymbol{y}$ is the NL-dimensional received vector, $\boldsymbol{b}$ is the K-dimensional transmitted vector, and $\boldsymbol{n}$ is discrete-time, circularly symmetric complex-valued white Gaussian noise with zero mean and variance σ^2. The influence of spreading, fading, and spatial diversity is described by the $NL \times K$ matrix

$$\mathcal{H} = \sum_{l=1}^{L} (\boldsymbol{S}\boldsymbol{T}\boldsymbol{\Lambda}_l) \otimes \mathbf{e}_l \tag{2}$$

where $\boldsymbol{S}$ is the $N \times K$ spreading matrix whose k-th column is the spreading sequence of the k^{th} virtual user. The diagonal square matrix $\boldsymbol{T} \in \mathbb{C}^{K \times K}$ contains the transmitted amplitudes of all virtual users such that its k^{th} diagonal element t_k is the amplitude of the signal transmitted by the virtual user indexed by k. The diagonal matrices $\boldsymbol{\Lambda}_1, \boldsymbol{\Lambda}_2, \ldots, \boldsymbol{\Lambda}_L \in \mathbb{C}^{K \times K}$ take into account the effect of the flat fading channel. The k^{th} diagonal element of $\boldsymbol{\Lambda}_l$ is the channel gain between the transmitting antenna element of the k^{th} virtual user and the l^{th} receive antenna and will be denoted by λ_{lk}

in the following. The channel gains can be, in general, correlated and contain line of sight components as in Rice channels.

Alternatively, (1) models a CDMA system with asynchronous chips, negligible interchip interference, and maximum time shift, for the k-th signal, $\tau_k \in \left[0, \frac{T_c}{2}\right)$. T_c is the chip interval. $\boldsymbol{S}$ is again the spreading matrix with random spreading sequences and spreading factor N. There are K users in the system and $\boldsymbol{T}$ is the matrix of received amplitudes that takes into account the transmitted amplitudes and the channel gains. L is the sampling rate normalized to the chip interval T_c. Let $\psi(t)$ be the chip waveform in the continuous-time domain. The matrices $\boldsymbol{\Lambda}_1, \boldsymbol{\Lambda}_2, \ldots, \boldsymbol{\Lambda}_L$ are obtained sampling $\psi(t)$. The k-th element of $\boldsymbol{\Lambda}_l$ is given by $\lambda_{kl} = \psi\left(\frac{(2l-1-L)T_c}{2L} - \tau_k\right)$, $k = 1, \ldots, K$, $l = 1, \ldots, L$. In this case the matrices $\boldsymbol{\Lambda}_1, \boldsymbol{\Lambda}_2, \ldots, \boldsymbol{\Lambda}_L$ are completely correlated (i.e. given one of them the others are deterministically determined) and independent of $\boldsymbol{T}$. Let us notice that with this model we assume $\psi(t-\tau_k) \approx 0$, $k = 0, \ldots, K$, for $t \notin [-\frac{T_c}{2} + \tau_k, \frac{T_c}{2} + \tau_k)$.

In the following, the spreading matrix is modelled as a random matrix whose elements are independent[1] with zero mean, variance $\frac{1}{N}$, and fourth moment such that there exists a $\gamma > 1$ for which $\mathrm{E}\{|s_{11}|^4\} \leq \frac{1}{N^\gamma}$. This condition is satisfied by all practically relevant choices of chips. Moreover, we assume the transmitted symbols to be uncorrelated and identically distributed random variables with zero mean and unit variance, i.e. $\mathrm{E}\{\boldsymbol{b}\boldsymbol{b}^H\} = \boldsymbol{I}_K$. In order to simplify notation, it will be helpful in the following to define the L-dimensional vectors of the received amplitudes of the virtual user k, $\boldsymbol{l}_k = t_k[\lambda_{1k}, \lambda_{2k}, \ldots, \lambda_{Lk}]^T$, $k = 1, \ldots, K$.

3. Linear MMSE Receiver

Throughout we adopt the following notation

- $\boldsymbol{h}_k$ denotes the k-th column of $\mathcal{H}$;
- $\mathcal{H}_k$ is the $NL \times (K-1)$ matrix obtained from $\mathcal{H}$ suppressing the k-th column $\boldsymbol{h}_k$.

From the Wiener-Hopf theorem [22] for the estimation of zero-mean random variables, the linear MMSE receiver for the transmitted signal k is given by

$$\boldsymbol{c}_k = \mathrm{E}\{\boldsymbol{y}\boldsymbol{y}^H\}^{-1}\mathrm{E}\{b_k^*\boldsymbol{y}\} \tag{3}$$

with the expectation taken over the transmitted symbols $\boldsymbol{b}$ and the noise. Specializing the Wiener-Hopf equation to the system model (1) yields

$$\boldsymbol{c}_k = (\mathcal{H}\mathcal{H}^H + \sigma^2\boldsymbol{I})^{-1}\boldsymbol{h}_k \tag{4}$$

$$= c \cdot (\mathcal{H}_k\mathcal{H}_k^H + \sigma^2\boldsymbol{I})^{-1}\boldsymbol{h}_k \tag{5}$$

for some $c \in \mathbb{R}$. The second step follows from the matrix inversion lemma. The linear MMSE detector generates a soft decision $\widehat{\boldsymbol{b}}_k = \boldsymbol{c}_k^H\boldsymbol{y}$ based on the observation $\boldsymbol{y}$. Its performance is measured by the signal-to-interference-and-noise ratio SINR_k at its output which is well-known (e.g. [22]) to be given by

$$\mathrm{SINR}_k = \boldsymbol{h}_k^H(\mathcal{H}_k\mathcal{H}_k^H + \sigma^2\boldsymbol{I})^{-1}\boldsymbol{h}_k. \tag{6}$$

The SINR can be conveniently expressed in terms of the multiuser efficiency

$$\mathrm{SINR}_k = \frac{||\boldsymbol{l}_k||^2}{\sigma^2}\,\eta_k. \tag{7}$$

The multiuser efficiency η_k is a useful measure, since it is identical to all users in special cases [23].

Let us notice that SINR_k depends on the spreading sequences and the channel parameters of all users. To get deeper insights on the LMMSE behaviour it is convenient to analyze the performance, as $K, N \to \infty$ with constant ratio $\beta = \frac{K}{N}$. To this aim, we have to define how the matrices $\boldsymbol{T}, \boldsymbol{\Lambda}_1, \boldsymbol{\Lambda}_2, \ldots, \boldsymbol{\Lambda}_L$ behave as the system grows large. We assume that the joint empirical distribution of their diagonal elements converges almost surely to a limit distribution function $F_{\boldsymbol{T},\boldsymbol{\Lambda}_1,\boldsymbol{\Lambda}_2,\ldots,\boldsymbol{\Lambda}_L}(t, \lambda_1, \lambda_2, \ldots, \lambda_L)$ with bounded support[2], as $K \to \infty$. In the following, notation will simplify referring to the limiting joint distribution $F_{\boldsymbol{l}}(l_1, l_2, \ldots, l_L)$ rather than the limit distribution $F_{\boldsymbol{T},\boldsymbol{\Lambda}_1,\boldsymbol{\Lambda}_2,\ldots,\boldsymbol{\Lambda}_L}(t, \lambda_1, \lambda_2, \ldots, \lambda_L)$. $F_{\boldsymbol{l}}$ is obtained by the projection $(l_1, l_2, \ldots, l_L) = (t\lambda_1, t\lambda_2, \ldots, t\lambda_L)$. Under these assumptions the asymptotic performance will depend on a small set of parameters, as shown by the following theorem.

Theorem 1 *Let $\boldsymbol{S}$ be an $N \times K$ random matrix with independent entries. Let its elements s_{ij} be zero*

[1] Note that the random variables s_{nk} are not required to be identically distributed

[2] Let us make some considerations in order to clarify the concept of joint limit distribution for the matrices $\boldsymbol{T}, \boldsymbol{\Lambda}_1, \boldsymbol{\Lambda}_2, \ldots, \boldsymbol{\Lambda}_L$. Consider the virtual user index as ensemble index. Then define the $(L+1)$-tuples $(t_k, \lambda_{1,k}, \ldots, \lambda_{L,k})$, $\forall k = 1, \ldots, K$ as realizations of an $(L+1)$-dimensional random vector with a given distribution $F_{\boldsymbol{T},\boldsymbol{\Lambda}_1,\boldsymbol{\Lambda}_2,\ldots,\boldsymbol{\Lambda}_L}(t, \lambda_1, \lambda_2, \ldots, \lambda_L)$. For a given number of virtual users K we can consider the empirical probability density function $F^K_{\boldsymbol{T},\boldsymbol{\Lambda}_1,\boldsymbol{\Lambda}_2,\ldots,\boldsymbol{\Lambda}_L}(t, \lambda_1, \lambda_2, \ldots, \lambda_L) = \frac{\sum_{i=1}^K \delta(\|(t,\lambda_1,\lambda_2,\ldots,\lambda_L) - (t_k,\lambda_{1k},\lambda_{2k},\ldots,\lambda_{Lk})\|)}{K}$. We assume that there exists the limit $\lim_{K\to\infty} F^K_{\boldsymbol{T},\boldsymbol{\Lambda}_1,\boldsymbol{\Lambda}_2,\ldots,\boldsymbol{\Lambda}_L}(t, \lambda_1, \lambda_2, \ldots, \lambda_L) = F_{\boldsymbol{T},\boldsymbol{\Lambda}_1,\boldsymbol{\Lambda}_2,\ldots,\boldsymbol{\Lambda}_L}(t, \lambda_1, \lambda_2, \ldots, \lambda_L)$

mean, with variance $\mathrm{E}\{|s_{ij}|^2\} = \frac{1}{N}$ *and forth moment* $\mathrm{E}\{|s_{ij}|^4\} \leq \frac{1}{N^\gamma}$ *and* $\gamma > 1$. *Let* $\boldsymbol{l}_k$ *be the vector of received amplitudes of the virtual user k. Let us assume that, almost surely, the empirical joint distribution of* $\boldsymbol{l}_1, \boldsymbol{l}_2, \ldots, \boldsymbol{l}_K$ *converges to some limiting joint distribution* $F_{\boldsymbol{l}}(l_1, l_2, \ldots, l_L)$ *with bounded support. Then, as* $N, K \to \infty$ *with* $\frac{K}{N} \to \beta$ *and L fixed, the SINR of virtual user k, given the fading amplitudes* $\boldsymbol{l}_k$, *converges in the mean square sense to the value*

$$\underset{K,N\to\infty}{\mathrm{l.i.m.}}\ \mathrm{SINR}_k = \frac{\boldsymbol{l}_k^H \boldsymbol{A} \boldsymbol{l}_k}{\sigma^2} \tag{8}$$

where $\boldsymbol{A}$ *is the unique deterministic* $L \times L$ *matrix solution to the matrix-valued fixed point equation*

$$\boldsymbol{A}^{-1} = \boldsymbol{I}_L + \beta \int \frac{\boldsymbol{l}\boldsymbol{l}^H}{\sigma^2 + \boldsymbol{l}^H \boldsymbol{A} \boldsymbol{l}} \, \mathrm{d}F_{\boldsymbol{l}}(l_1, l_2, \ldots, l_L) \tag{9}$$

such that $\boldsymbol{A} > 0$ *for* $\sigma^2 > 0$.

The proof of this theorem can be found in [24].

It is clear from (7) that the matrix $\boldsymbol{A}$ is a matrix-valued generalization of multiuser efficiency. Let us analyze under which conditions on the limiting joint distribution $F_{\boldsymbol{l}}(l_1, l_2, \ldots, l_L)$ or, equivalently, on the corresponding limiting probability density function $f_{\boldsymbol{l}}(l_1, l_2, \ldots, l_L)$ the generalized multiuser efficiency $\boldsymbol{A}$ is diagonal. In fact, for diagonal $\boldsymbol{A}$, the general result in Theorem 1 simplifies to the system of fixed-point equations in [1], Theorem 3. The following corollary summarizes some sufficient conditions that yield a diagonal structure of $\boldsymbol{A}$.

Corollary 1 *Let* $\boldsymbol{S}$ *and* $\boldsymbol{l}_k$ *be as in Theorem 1. If the joint probability density function* $f_{\boldsymbol{l}}(l_1, l_2, \ldots, l_L)$, *for any k, is an even function of* $\mathrm{Re}(l_k)$ *and* $\mathrm{Im}(l_k)$ *for any value of the parameters* $(l_1, \ldots, l_{k-1}, l_{k+1}, \ldots l_L)$ *then as* $N, K \to \infty$ *with* $\frac{K}{N} \to \beta$ *and L fixed, the SINR of virtual user k, given the fading amplitudes* $\boldsymbol{l}_k$, *converges in the mean square sense to the value*

$$\underset{K,N\to\infty}{\mathrm{l.i.m.}}\ \mathrm{SINR}_k = \frac{|t_k|^2}{\sigma^2} \sum_{\ell=1}^{L} a_\ell |\lambda_{\ell k}|^2 \tag{10}$$

where a_ℓ, $\ell = 1 \ldots L$, *are the unique positive solutions to the system of fixed-point equations*

$$a_\ell = \frac{1}{1 + \beta \int \frac{|l_\ell|^2}{\sigma^2 + \sum_{n=1}^{L} a_n |l_n|^2} f_{\boldsymbol{l}}(l_1, \ldots, l_L) \mathrm{d}l_1 \ldots \mathrm{d}l_L} \tag{11}$$

for $\ell = 1 \ldots L$.

Proof: In order to verify that system (9) is equivalent to system (11) under the above mentioned conditions on $f_{\boldsymbol{l}}(l_1, l_2, \ldots, l_L)$ it is sufficient to verify that, for all $i, j = 1, \ldots, L$, with $i \neq j$, the off-diagonal elements of $\boldsymbol{A}$ are zero. The uniqueness of the solution for system (9) guarantees that the constants a_ℓ are the desired ones. In fact, $\forall i, j = 1, \ldots, L$

$$\int \frac{l_i l_j \, f_{\boldsymbol{l}}(l_1, l_2, \ldots, l_L)}{\sum_{\ell=1}^{L} a_\ell |l_\ell|^2} \mathrm{d}l_1 \mathrm{d}l_2 \ldots \mathrm{d}l_L = \int l_i \mathrm{d}l_1, \ldots, \mathrm{d}l_{j-1} \cdot \mathrm{d}l_{j+1}, \ldots \mathrm{d}l_L \int \frac{l_j f_{\boldsymbol{l}}(l_1, \ldots, l_L)}{\sum_{\ell=1}^{L} a_\ell |l_\ell|^2} \mathrm{d}l_j.$$

Since the function $l_j / \sum_{\ell=1}^{L} a_\ell |l_\ell|^2$ is an odd function of $\mathrm{Re}(l_j)$ and $\mathrm{Im}(l_j)$, the integral with respect to l_j will be always null if $f_{\boldsymbol{l}}(l_1, l_2, \ldots, l_L)$ is an even function in $\mathrm{Re}(l_j)$ and $\mathrm{Im}(l_j)$ for all possible values of l_j, $(j = 1, \ldots, L) \bigcap (j \neq i)$.

Following the same derivation used for Corollary 4 in [1] we obtain:

Corollary 2 *Let* $\boldsymbol{S}$, $f_{\boldsymbol{l}}(l_1, l_2, \ldots, l_L)$ *be as in Corollary 1. If the limiting probability density function* $f_{\boldsymbol{l}}(l_1, l_2, \ldots, l_L)$ *is exchangeable, i.e. for any permutation* π *of* $\{1, \ldots, L\}$

$$f_{\boldsymbol{l}}(l_1, l_2, \ldots, l_L) = f_{\boldsymbol{l}}(l_{\pi(1)}, l_{\pi(2)}, \ldots, l_{\pi(L)})$$

then, as $N, K \to \infty$ *with* $\frac{K}{N} \to \beta$ *and L fixed, the SINR of virtual user k, given the fading amplitudes* $\boldsymbol{l}_k$, *converges in the mean square sense to the value*

$$\underset{K,N\to\infty}{\mathrm{l.i.m.}}\ \mathrm{SINR}_k = \frac{P_k}{\sigma^2} \eta \tag{12}$$

with

$$P_k = ||\boldsymbol{l}_k||^2,$$

and the unique scalar multiuser efficiency η *being solution to the fixed point equation*

$$\eta = \frac{1}{1 + \frac{\beta}{L} \int \frac{P}{\sigma^2 + \eta P} \mathrm{d}\, F_P(P)}$$

where P is the random variable defined by $P = ||\boldsymbol{l}||^2$ *and* $F_P(P)$ *is its distribution.*

Remarks:

- Corollaries 1 and 2 imply Theorem 3 and Corollary 4 in [1] and provide a rigorous proof for them. Additionally, the convergence is here shown to be the mean square sense.
- Channels with line of sight components do not satisfy the hypotheses of Corollaries 1 and 2: the limit matrix $\boldsymbol{A}$ is not diagonal and Theorem 1 has to be applied.

- A CDMA system with spatial diversity can be considered as a system with only one receive antenna and spreading factor NL, eventually with block-wise correlated chips.

- The asymptotic behaviour of the general system is completely described by an $L \times L$ matrix $\boldsymbol{A}$. In contrast to the case of single receive antenna, the multiuser efficiency of the LMMSE receiver varies from user to user. In particular, for user k, it depends on the direction of the channel gains, $\boldsymbol{l}_k$, with respect to the eigenvectors of $\boldsymbol{A}$: The multiuser efficiency is maximum if $\boldsymbol{l}_k$ has the same direction as the eigenvector corresponding to the maximum eigenvalue of $\boldsymbol{A}$.

- The extension of the concept of effective interference is not straightforward since the system behaviour is caught by a matrix. However, given the strong analogy with the results in [1] we can define the *matrix of the effective interference* of user k on the system as $\mathcal{I}(\boldsymbol{l}_k) = \frac{1}{N} \frac{\boldsymbol{l}_k \boldsymbol{l}_k^H}{1+\boldsymbol{l}_k^H \boldsymbol{A} \boldsymbol{l}_k/\sigma^2}$.

- Thanks to the independence of the spreading sequences the correlations of the channel gains at the transmitter side do not affect the asymptotic performance of the LMMSE receiver.

4. CONCLUSIONS

In this contribution we determined the asymptotic performance of a CDMA system with random spreading and spatial diversity in the general case as the channel gains are correlated and with line of sight components. This result includes as special cases the results in [1] derived under the constraints of independence of the channel gains and uniformly distributed phases. Deriving the results in [1] from the general equations (8) and (9) we could prove the results for the macrodiversity case, which was only conjectured in [1]. Additionally, we show that the convergence of the SINR to a deterministic constant, as the system grows large, holds in the mean square sense and not only in probability as shown in [1].

Our general Theorem 1 shows that the system is asymptotically described by the $L \times L$ matrix $\boldsymbol{A}$ characterizing completely the effects of channel correlation and line of sight components. The efficiency of the system in recovering the symbol transmitted by the physical user k depends deeply on the direction of the channel gain vector $\boldsymbol{l}_k$ with respect to the eigenvectors of $\boldsymbol{A}$. In particular, the spectral efficiency is maximum if $\boldsymbol{l}_k$ has the same direction of the eigenvector associated to the maximum eigenvalue of $\boldsymbol{A}$. Conditions under which the resource pooling result shows up, or, equivalently, the generalized multiuser efficiency matrix reduces to a scalar, in the general case, have been given.

The same mathematics can be used for the analysis of an asynchronous CDMA system.

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