A Generalized Resource Pooling Result for Correlated Antennas with Applications to Asynchronous CDMA

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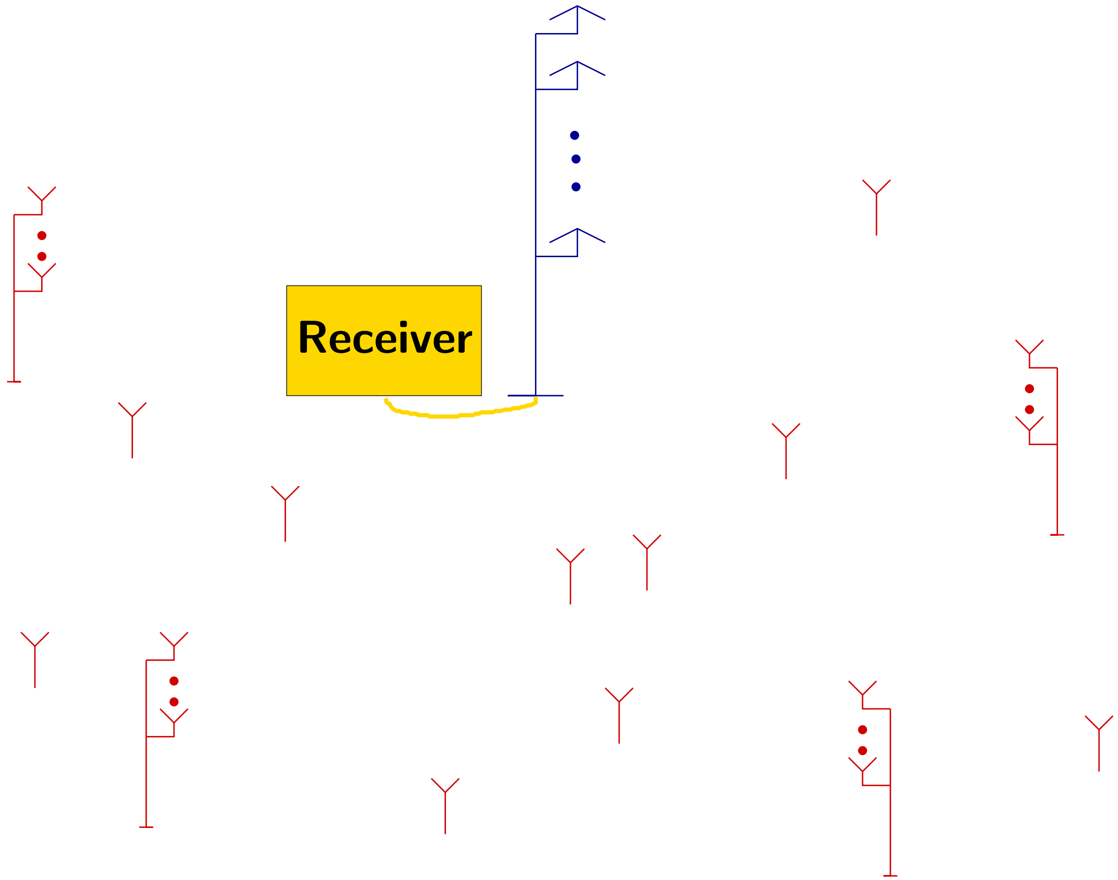
1. Problem Statement
2. Linear MMSE Detectors
3. Asymptotic Performance Analysis
4. Summary

Scenarios with Spatial Diversity: Micro-diversity

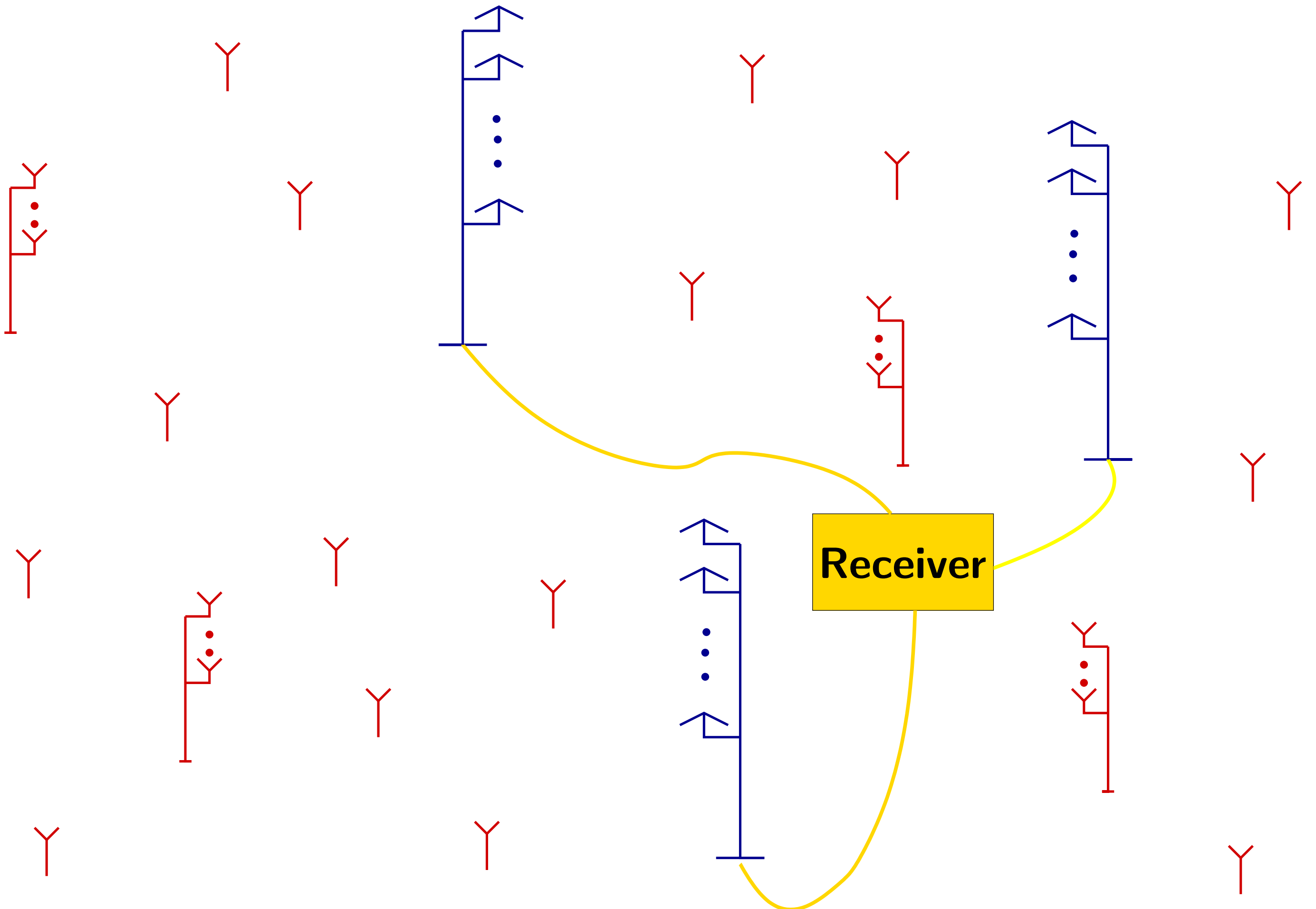


- L receiving antennas
- K' users with N_{T_k} antennas

$$\Downarrow$$

$K = \sum_{k=1}^{K'} N_{T_k}$ virtual users

Scenarios with Spatial Diversity: Macro-diversity

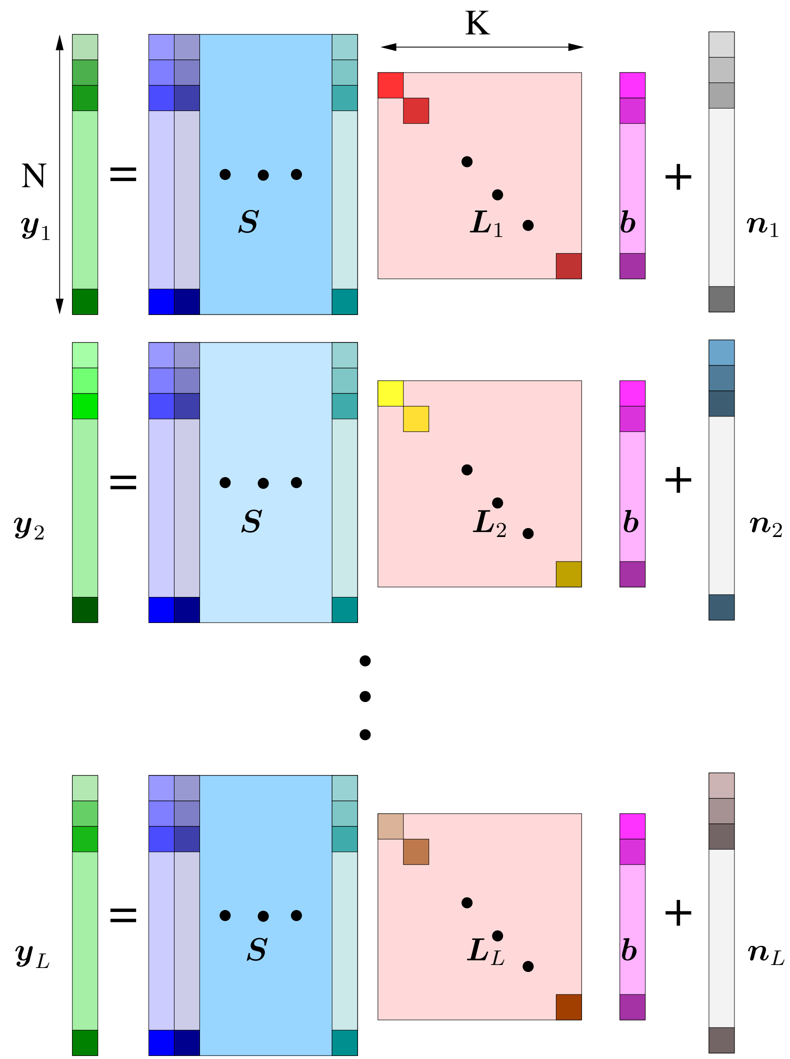


Synchronous CDMA Uplink System Model

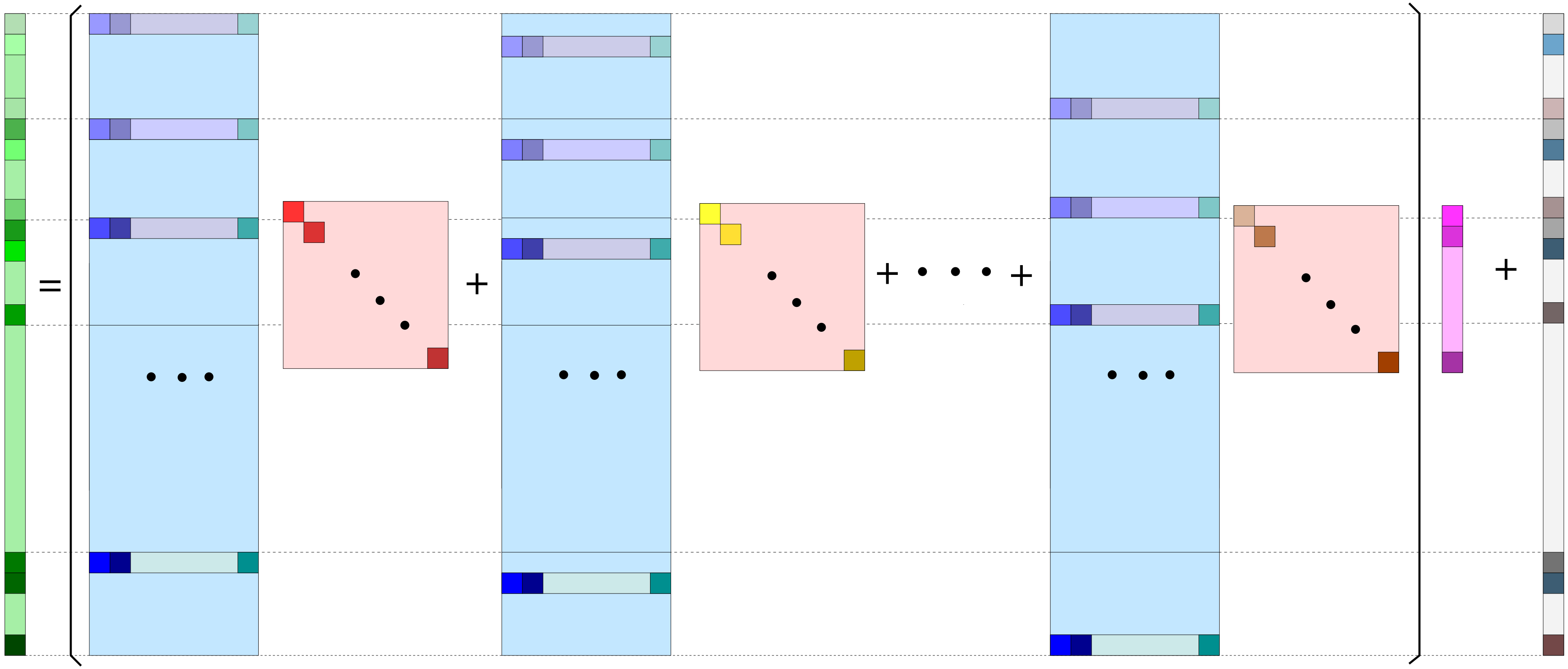
Receiving Antenna 1

Receiving Antenna 2

Receiving Antenna L



Synchronous CDMA Uplink System Model



$$\mathcal{Y} = \mathcal{H}b + \mathcal{N}$$

Linear MMSE detector

$$b_{\mathrm{LMMSE},k} = \boldsymbol{h}_k^H (\boldsymbol{\mathcal{H}}\boldsymbol{\mathcal{H}}^H + \sigma^2 \boldsymbol{I})^{-1} \boldsymbol{\mathcal{Y}}$$

Performance

$$\mathrm{SINR}_k = \boldsymbol{h}_k^H (\boldsymbol{\mathcal{H}}\boldsymbol{\mathcal{H}}^H + \sigma^2 \boldsymbol{I})^{-1} \boldsymbol{h}_k$$

Dependence on the spreading sequences and channel gains of all users!

State of Art: Micro-diversity

Hanly and Tse, *Resource pooling and effective bandwidth in CDMA networks with multiuser receivers and spatial diversity*, 2001.

Hypotheses

- i.i.d. circular symmetric complex Gaussian random chips;
- Channel gains:
  - Independent for all users and antennas;
  - Identically distributed to all antennas, given a user.

Thesis

Limiting system performance characterized by a constant a solution of a fixed point equation

$$\lim_{K=\beta N\to\infty} \frac{\mathrm{SINR}_k}{\sum_{i=1}^{L} |\ \ell_{k,i}\ |^2} \overset{\mathcal{P}}{=} a$$

Resource pooling interpretation: Same limiting performance as a system with a single antenna and processing gain NL.

State of Art: Macro-diversity

Hanly and Tse, 2001.

Hypotheses

- i.i.d. circular symmetric complex Gaussian random chips;
- Channel gains independent for all users and antennas.

Conjecture Limiting system performance characterized by L constants a_i solutions to a system of L fixed point equations

$$\lim_{K=\beta N \to \infty} \mathrm{SINR}_k \stackrel{\mathcal{P}}{=} \sum_{i=1}^{L} a_i \mid \ell_{k,i} \mid^2$$

Performance for Correlated Channel Gains

Empirical joint distribution of the channel gains:

$$F^{(K)}(\ell_1, \ell_2, \dots, \ell_L) = \frac{1}{K} \sum_{k=1}^{K} 1(\ell_{1,k} \leq \ell_1) 1(\ell_{2,k} \leq \ell_2) 1(\ell_{L,k} \leq \ell_L)$$

Hypotheses

- Independent random chips;
- The sequence of the empirical joint distribution converges to a limiting joint distribution with bounded support

$$\lim_{K \to \infty} F^{(K)}(\ell_1, \ell_2, \dots, \ell_L) = F(\ell_1, \ell_2, \dots, \ell_L)$$

Thesis

Given the channel gains of user k, $\boldsymbol{\ell}_k$

$$\underset{K=\beta N \to \infty}{\text{l.i.m.}} \text{SINR}_k = \frac{\boldsymbol{\ell}_k^H \boldsymbol{A} \boldsymbol{\ell}_k}{\sigma^2}$$

with

$$\boldsymbol{A}^{-1} = \boldsymbol{I}_L + \beta \int \frac{\boldsymbol{\ell}\boldsymbol{\ell}^H}{\sigma^2 + \boldsymbol{\ell}^H \boldsymbol{A} \boldsymbol{\ell}} \mathrm{d}F(\ell_1, \ell_2, \dots, \ell_L)$$

Asymptotic performance is characterized by an $L \times L$ matrix $\boldsymbol{A}$

Revisiting the Known Results

Micro-diversity in Hanly et al., 2001

- Channel gains independent for all users and antennas;
- Identically distributed to all antennas, given a user.

$$\boldsymbol{A} = a\boldsymbol{I}$$

Macro-diversity in Hanly et al., 2001

- Channel gains independent for all users and antennas;

$$\boldsymbol{A} = \begin{bmatrix} a_1 & 0 & \ddots & \ddots \\ 0 & a_2 & 0 & \ddots \\ \ddots & \ddots & \ddots & \ddots \\ \ddots & \ddots & 0 & a_L \end{bmatrix}$$

Can we find more general conditions under which the resource pooling effect arises?

Generalized Resource Pooling

Hypothesis A The joint probability density function $f(\ell_1, \ell_2, \dots, \ell_L)$ is an even function of $\mathrm{Re}(\ell_i)$ and $\mathrm{Im}(\ell_i)$ for any i.

Hypothesis B $f(\ell_1, \ell_2, \dots, \ell_L)$ is exchangeable, i.e. for any permutation π of $\{1, 2, \dots, L\}$

$$f(\ell_1, \ell_2, \dots, \ell_L) = f(\ell_{\pi(1)}, \ell_{\pi(2)}, \dots, \ell_{\pi(L)})$$

Hypothesis A $\Rrightarrow$ $\boldsymbol{A}$ **is diagonal**

Hypothesis A and B $\Rrightarrow$ $\boldsymbol{A} = a\boldsymbol{I}$

The resource pooling effect arises under hypotheses A and B

Remarks

The correlations of the channel gains at the transmitters do not affect the asymptotic performance of the LMMSE receiver.

The multiuser efficiency varies from user to user.

The SINR is maximum if $\boldsymbol{\ell}_k$ has the same direction as the eigenvector corresponding to the maximum eigenvalue of $\boldsymbol{A}$.

For channels with Gaussian limiting p.d.f, matrix $\boldsymbol{A}$ has the same eigenvectors of the covariance matrix $\mathrm{E}\{\boldsymbol{\ell}\boldsymbol{\ell}^H\}$.

Asymptotic versus Finite Performance

– Receiving antennas L=3.

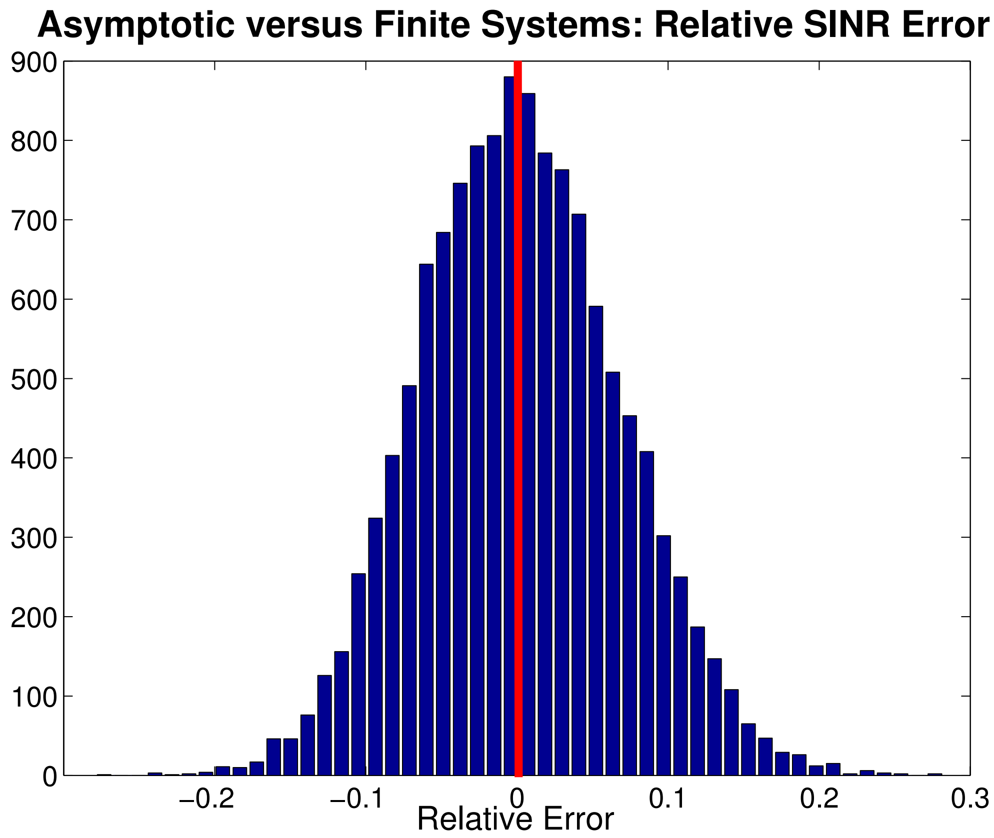
– Spreading factor N=256.

– Users K=128.

– $\mathrm{SNR} = 10$ dB.

– Gaussian channel gains with covariance matrix

$$\begin{bmatrix} 1 & 0.5 & 0.2 \\ 0.5 & 1 & 0.5 \\ 0.2 & 0.5 & 1 \end{bmatrix}$$



Summary

- **Asymptotic performance of linear MMSE detectors for CDMA systems with spatial diversity and correlation of the channel gains.**
- **Rigorous proof of Hanly and Tse's results for macro-diversity with stronger convergence (in mean square vs probability).**
- **Generalization of the conditions for the resource pooling effect.**

↓

System performance characterized by an $L \times L$ matrix.

Varying multiuser efficiency.

Performance depends on the direction of the channel gain vector w.r.t. the eigenvectors of A .