On the Capacity Loss due to Separation of Detection and Decoding in Large CDMA Systems

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Abstract — **The performance loss due to separation of detection and decoding on the binary-input Gaussian CDMA channel is calculated in the large system limit. It is shown that a previous result in [1] found for Gaussian input alphabet holds also for binary input alphabet.**

I. Introduction

Separation of detection and decoding, though suboptimum in general, is wide-spread in communications technology. This is due to the huge complexity which is required by the optimum joint approach. Here, we analyze the loss incurred by this suboptimum approach on the real-valued binary-input symbol-synchronous Gaussian CDMA channel with random spreading and flat fading with arbitrary distribution.

II. Random CDMA

Proposition 1 (Tanaka [2][1]) Let the symbol alphabet of all users be $\{+a(t); -a(t)\}$ with $\mathrm{E}_t\, a^2(t) \le P$, the signature sequences be independent identically distributed (i.i.d.) Gaussian random variables, the frequency flat fading of user i be $g_i \in \mathbb{R}$ with $\mathrm{E}_i\, g_i^2 = 1$, and the number of users K and the spreading factor N grow to infinity, but the load $\beta = K/N$ remain fixed. Moreover, assume that the replica method may be applied[2] and the capacity for joint decoding converges to a non-random limit. Then, in presence of AWGN of unit variance, the multiuser efficiency of the individually optimum detector is a solution to the fixed point equation

$$\frac{1}{\eta} = 1 + P\beta\left[1 - \mathop{\mathrm{E}}_{x,g}\, g^2 \tanh\left(\sqrt{P\eta}gx + P\eta g^2\right)\right] \qquad (1)$$

with $x \sim \mathcal{N}(0,1)$ and the capacity for joint decoding in nats is given in terms of the multiuser efficiency as

$$C_{\mathrm{J}}(P) = P\eta + \frac{\eta - 1 - \log\eta}{2\beta} - \mathop{\mathrm{E}}_{x,g} \log\cosh\left(\sqrt{P\eta}gx + P\eta g^2\right)$$

In case the fixed point equation (1) has multiple solutions, the correct one is that solution for which C_{J} is smallest.

Consider now separated detection and decoding where detection is performed without knowledge of the code laws but individually optimum within the limits of this contraint. That is, the subsequent K independent decoders are provided with memoryless non-linear MMSE estimates.

[1] The generalization to flat fading is due to [3].
[2] See [2, 4] for further details about this condition.

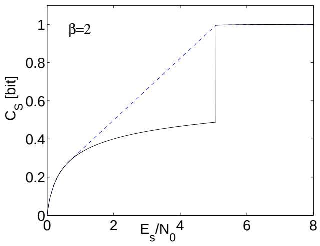

Figure 1: Capacity without fading (w.f.) over signal-to-noise ratio in linear scale. The dashed line refers to the convex hull.

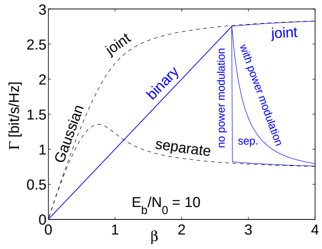

Figure 2: Spectral efficiency w.f. vs. load for $E_{\mathrm{b}}/N_0 = 10$ dB, binary (solid lines) vs. Gaussian (dashed lines) input alphabet.

Proposition 2 (new) Consider the same conditions as in Proposition 1. Then, the capacity for separated detection and decoding in nats is

$$C_{\mathrm{S}} = \max_{p(t):\mathop{\mathrm{E}}_t p \le P} \mathop{\mathrm{E}}_t \left\{ C_{\mathrm{J}}(p) - \frac{\eta(p) - 1 - \log\eta(p)}{2\beta} \right\}.$$

This is a striking analogy to Theorem IV.1 in [1] which states the same loss due to separation of detection and decoding for Gaussian instead of binary inputs.

It is illustrated in Fig. 1 that with constant power capacity is not a concave function of the signal-to-noise ratio. The capacity for optimum scheduling of the signal power is given by the convex hull of the capacity for constant power.

It is also remarkable to observe from Fig. 1 that without power scheduling channel capacity is not a continuous function of the signal-to-noise ratio. Such effects are well-known in statistical mechanics and called *phase transitions*. In context of CDMA, the discontinuity can be understood considering that there is a waterfall region in the bit error rate [2].

It is also insightful to compare the spectral efficiency of binary signaling to that of Gaussian signaling. An example is given in Fig. 2.

References

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