Asymptotic Design and Analysis of Multistage Detectors with Unequal Powers

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Abstract — **In this work we provide equations to precisely calculate the asymptotic weighting of multistage detectors satisfying the individually and jointly LMMSE criteria in the projection subspace for scenarios with unequal powers. Additionally, a general expression of the SINR achievable at the filter output as system size grows large is derived. Such equation can be applied to any multistage detector. We specialize this result to both the individually and jointly LMMSE multistage detector with asymptotic weighting. We show that the individually LMMSE detector outperforms the other detector in the case of unequal received powers while both the detectors are equivalent in the case of equal received powers.**

I. Introduction

There has recently been an increasing interest in the asymptotic analysis of multistage detectors under the assumption of random spreading. This has been due to at least two reasons. Firstly, closed-form solutions for the signal-to-interference-and-noise ratio (SINR) of various multiuser detectors have been derived assuming random spreading sequences of length $N \to \infty$ while keeping the system load $\beta = \frac{K}{N}$, i.e. the number of physical users K per chip, fixed [1, 2, 3]. Secondly, the demand of detectors able to efficiently mitigate crosstalk for large scale systems has led to the analysis of reduced-rank detectors, especially multistage detectors [4, 5, 6, 7]. As shown in [5, 6], for this class of detectors, a low number of stages is sufficient for near-LMMSE (linear minimum-mean square error) performance, even as system size grows large. The practical usefulness of this class of detectors derives from the drastic reduction in complexity that can be achieved by approximating optimum weighting by asymptotic weighting [7], at the cost of a slight degradation in performance [8]. Several different criteria for weight optimization have been proposed and their asymptotic performance has been analyzed in the case of equal received powers. However, asymptotic weighting is unknown for unequal received powers and performance has been analyzed only for the optimality criterion proposed in [6].

In this work we provide an equation to precisely calculate the asymptotic weighting required for the optimality criteria proposed in [4] and in [6] in the case of unequal received powers and also a general equation for the limiting SINR achievable at the filter output, as $N \to \infty$ and β is constant, applicable to any multistage detector.

For the true LMMSE detector it is well known that the minimization of the MSE per user is equivalent to the minimization of their sum and also to the maximization of the SINR at the output of the filter for each physical user. This is due to the fact that no constraint is enforced on the space where the matrix of the filter lies. This property does not hold if the detector is enforced to lie in a specific subspace as in the case of multistage detectors. In such a case the difference between the joint minimization of the MSE proposed in [4] and the minimization of the MSE for each user proposed in [6] appears and the maximization of the SINR is achieved only in the second case.

We rewrite the detector for the optimality criterion proposed in [4] as $\mathbf{L} = \sum_{l=0}^{L-1} w_l \mathbf{R}^l \mathbf{A}^H \mathbf{S}^H$ and the detector proposed in [6] as $\mathbf{M} = \sum_{l=0}^{L-1} \mathbf{W}_l \mathbf{R}^l \mathbf{A}^H \mathbf{S}^H$ with $\mathbf{W}$ being diagonal. By calculating precisely the asymptotic weights for $\mathbf{L}$ and $\mathbf{M}$ and specializing the general equation for the limiting SINR to the two receivers under consideration we can compare them. While their performance coincides in the case of equal powers it differs for unequal powers and the detector $\mathbf{M}$ outperforms the detector $\mathbf{L}$. The interest in an exact comparison is due to the fact that the complexity of weight computation for the detector $\mathbf{L}$ is substantially lower than the corresponding complexity for $\mathbf{M}$.

II. Definitions and Results

Let $\mathbf{r}$ be the N-dimensional received vector. For example, the elements of $\mathbf{r}$ may be samples at the output of a chip-matched filter for synchronous CDMA or of an antenna array channel. We assume

$$\mathbf{r} = \mathbf{S}\mathbf{A}\mathbf{b} + \mathbf{n} \tag{1}$$

where $\mathbf{b}$ is the K-dimensional vector of transmitted symbols, $\mathbf{S}$ is the $N \times K$ matrix of signature sequences and $\mathbf{A}$ is the $K \times K$ diagonal matrix of amplitudes. $\mathbf{n}$ is the Gaussian vector noise with covariance matrix $\sigma^2\mathbf{I}$.

Hereinafter we assume the following:

A: the sequence of the empirical eigenvalue distribution of $\mathbf{A}\mathbf{A}^H$ converges almost surely, as $K \to \infty$, to a non-random distribution function with upper bounded support;

B.1: the elements of the matrix $\mathbf{S}$, s_{ij}, are i.i.d. with $|s_{11}| \leq \frac{\log N}{\sqrt{N}}$, $\mathrm{E}\{s_{11}\} = 0$, $\mathrm{E}\left\{|s_{11}|^2\right\} = \frac{1}{N}$ and finite fourth moment or

B.2: the elements of the matrix $\mathbf{S}$ are i.i.d., Gaussian with $\mathrm{E}\{s_{11}\} = 0$ and $\mathrm{E}\left\{|s_{11}|^2\right\} = \frac{1}{N}$.

Let us denote by $\chi_L(\mathbf{SA})$ the subspace in the space of the $K \times N$ matrices in $\mathbb{C}$ defined by

$$\chi_L(\mathbf{SA}) = \mathrm{span}\{\mathbf{R}^l\mathbf{A}^H\mathbf{S}^H\}|_{l=0}^{L-1} \tag{2}$$

where $\mathbf{R} = \mathbf{A}^H\mathbf{S}^H\mathbf{SA}$. As shown in [4], the true LMMSE detector is in $\chi_K(\mathbf{SA})$. We refer to $\mathbf{L} = \sum_{l=0}^{L-1} w_l\mathbf{R}^l\mathbf{A}^H\mathbf{S}^H$ such that $\mathrm{E}\{\|\mathbf{Lr}-\mathbf{b}\|^2\}$ is minimized as the jointly LMMSE detector in $\chi_L(\mathbf{SA})$. Defining $\mathbf{W}_l = \mathrm{diag}(w_{1l}, w_{2l}, \dots w_{Kl})$, the individually LMMSE detector in $\chi_L(\mathbf{SA})$ is the linear detector $\mathbf{M} = \sum_{l=0}^{L-1} \mathbf{W}_l\mathbf{R}^l\mathbf{A}^H\mathbf{S}^H$ such that $\mathrm{E}\left\{\|\mathbf{Mr}-\mathbf{b}\|^2\right\}$ is minimum. This is equivalent to the minimization of the MSE for each component b_i of $\mathbf{b}$ in the correspondent subspace $\chi_{L,i}(\mathbf{SA}) = \mathrm{span}\{\mathrm{row}(\mathbf{R}^l, i)\mathbf{A}^H\mathbf{S}^H\}|_{l=0}^{L-1}$ where $\mathrm{row}(\mathbf{X}, l)$ denotes the operator that maps the matrix $\mathbf{X}$ into its l-th row. Note that all the subspaces $\chi_{L,i}(\mathbf{SA})$ are, in general, distinct.

The optimum scalar weights w_l for the jointly LMMSE detector in $\chi_L(\mathbf{SA})$ are given by [4]

$$\mathbf{w} = \mathbf{\Phi}^{-1}\mathbf{c} \tag{3}$$

where the L-dimensional vector $\mathbf{c}$ and the elements of the $L \times L$ matrix $\mathbf{\Phi}$ can be expressed in terms of the traces of the powers of $\mathbf{R}$ as $c_i = \mathrm{trace}(\mathbf{R}^i)$ and $\Phi_{ij} = \mathrm{trace}(\mathbf{R}^{i+j}) + \sigma^2\mathrm{trace}(\mathbf{R}^{i+j-1})$.

The jointly LMMSE detector in $\chi_L(\mathbf{SA})$ for the asymptotic case, as $N, K \to \infty$ with β constant, satisfies

$$\mathbf{w}^\infty = (\mathbf{\Phi}^\infty)^{-1}\mathbf{c}^\infty \tag{4}$$

where $c_i^\infty = \mathrm{E}\left\{\lambda^i\right\}$, $\Phi_{ij}^\infty = \mathrm{E}\left\{\lambda^{i+j}\right\} + \sigma^2\mathrm{E}\left\{\lambda^{i+j-1}\right\}$ and λ are the eigenvalues of $\mathbf{R}$. Under the assumptions we made on $\mathbf{S}$ and $\mathbf{A}$, the sequence of the empirical eigenvalue distribution of $\mathbf{R}$ converges almost surely, as $N, K \to \infty$ with $\frac{K}{N} = \beta$, to a deterministic distribution [9]. For $\mathbf{AA}^H = \mathcal{P}\mathbf{I}$ a closed-form expression of the eigenvalue moments is in [10]. The weighting (4) was proposed first in [7] as the one minimizing the ratio between the total useful power and the total noise and interference power for $\mathbf{AA}^H = \mathcal{P}\mathbf{I}$. The same expression can be obtained without any constraint on $\mathbf{A}$ minimizing the quantity:

$$\lim_{\substack{N,K\to\infty \\ \frac{K}{N}=\beta}} \frac{\sum_{k=1}^{K} \mathrm{MSE}_k}{K} = \lim_{\substack{N,K\to\infty \\ \frac{K}{N}=\beta}} \frac{\mathrm{trace}\left(\mathbf{C}_\epsilon\right)}{K} \tag{5}$$

where MSE_k denotes the MSE for the k-th symbol and $\mathbf{C}_\epsilon$ is the error covariance matrix.

A recursive structure of the individually LMMSE detector in $\chi_L(\mathbf{SA})$ is provided in [6][1] and it is equivalent to the following:

$$\mathbf{w}_i = \mathbf{\Phi}_i^{-1}\mathbf{c}_i \tag{6}$$

where[2] $(\mathbf{w}_i)_l = (\mathbf{W}_l)_{ii} = w_{il}$, $(\mathbf{c}_i)_l = (\mathbf{R}^{l+1})_{ii}$ and $(\mathbf{\Phi}_i)_{lm} = (\mathbf{R}^{l+m})_{ii} + \sigma^2(\mathbf{R}^{l+m-1})_{ii}$.

The expression of the individually LMMSE detector in $\chi_L(\mathbf{SA})$ for the asymptotic case requires the existence and the knowledge of the following limits:

$$\lim_{\substack{N,K\to\infty \\ \frac{K}{N}=\beta}} (\mathbf{R}^l)_{ii} = R_{ii,\infty}^l \qquad \forall i \in \mathbb{N}, \qquad 1 \leq l \leq L^2. \tag{7}$$

To that end we can use the following theorem:

THEOREM 1 *Let $\mathbf{A}$ be a $K \times K$ diagonal matrix in $\mathbb{C}$ with bounded elements satisfying hypothesis A and let $\mathbf{S}$ be an $N \times K$ matrix in $\mathbb{C}$ satisfying hypothesis B.1. If a_{ii} is the i-th element of $\mathbf{A}$, then, for any $i, l \in \mathbb{N}$, $(\mathbf{R}^l)_{ii}$ converges almost surely, as $N, K \to \infty$, with $\frac{K}{N}$ constant, to the deterministic quantity $R_{ii,\infty}^l$ depending on $|a_{ii}|^2$*

$$R_{ii,\infty}^l = \sum_{\substack{(i_0,i_1,\dots i_{l-1}): \\ i_0+\sum_{j=1}^{l-1} ji_j = l \\ \sum_{j=1}^{l-1}(j+1)i_j \leq l}} \varphi(i_0, i_1, \dots i_{l-1})|a_{ii}|^{2i_0} \prod_{k=1}^{l-1} (m_{\mathbf{T}}^k)^{i_k} \tag{8}$$

where $m_{\mathbf{T}}^k$ denotes the eigenvalue moment of order k of the matrix $\mathbf{T} = \mathbf{SAA}^H\mathbf{S}^H$ and $(i_0, i_1, \dots i_{l-1})$ is an l-tuple of nonnegative integers satisfying the constraints $i_0 + \sum_{j=1}^{l-1} ji_j = l$ and $\sum_{j=1}^{l-1}(j+1)i_j \leq l$. $\varphi(i_0, i_1, \dots i_{l-1})$ are nonnegative integer coefficients given in (9). α and γ in (9) are defined by $\alpha = \sum_{j=1}^{l-1} i_j$ and $\gamma = l - \sum_{j=1}^{l-1}(j+1)i_j$ respectively. $\forall r,s > 0$ $g_r[s]$ in (9) satisfies the recursive equation

$$g_r[s] = g_{r-1}[s] + g_r[s-1] \tag{10}$$

with initializing values

$$g_1[0] = 1, \qquad g_r[0] = 0, \qquad g_1[s] = 1. \tag{11}$$

Furthermore, by convention, it is stated $\sum_{\substack{r=1 \\ t\geq 1}}^{t} g_{t+1-r}[-1]r = 1$ and $\sum_{\substack{r=1 \\ t\geq 1}}^{t} g_{t+1-r}[s]r = 0$ for $s < -1$. □

The proof of this theorem is in [11]. It can be shown [11] that also the following recursive equation holds for $R_{ii,\infty}^l$

$$R_{ii,\infty}^l = \mathcal{P}_i \sum_{k=0}^{l-1} R_{ii,\infty}^k \beta m_{\mathbf{R}}^{l-k-1}. \tag{12}$$

[1] In [6] the individually LMMSE detector in $\chi_L(\mathbf{SA})$ is referred to as MultiStage Wiener Filter.

[2] If not differently specified, $(.)_i$ denotes the i-th component of the vector argument and $(.)_{ij}$ denotes the element ij of the matrix argument.

$$\varphi(i_0,i_1,\dots i_{l-1}) = \begin{cases} 1 & \text{for } i_0=1,\ i_0+\sum_{j=1}^{l-1} j i_j = l \text{ and } \sum_{j=1}^{l-1}(j+1)i_j \le l \\ \frac{\left(\sum_{j=1}^{l-1} i_j\right)!}{\prod_{j=1}^{l-1} i_j!}\left(\sum_{r=1}^{\alpha-1} g_{\alpha-r}[\gamma-1]r + 2\sum_{r=1}^{\alpha} g_{\alpha+1-r}[\gamma-2]r + \sum_{r=1}^{\alpha+1} g_{\alpha+2-r}[\gamma-3]r\right) & \\ & \text{for } i_0>1,\ i_0+\sum_{j=1}^{l-1} j i_j = l \text{ and } \sum_{j=1}^{l-1}(j+1)i_j \le l \\ 0 & \text{else} \end{cases} \tag{9}$$

For the moments $m^k_{\mathbf{T}}$ and $m^k_{\mathbf{R}}$, where $m^k_{\mathbf{R}}$ denotes the k-th eigenvalue moment of the matrix $\mathbf{R}$, the relation $m^k_{\mathbf{T}} = \beta m^k_{\mathbf{R}}$ holds and the moments $m^k_{\mathbf{R}}$ can be derived recursively according to the following theorem:

THEOREM 2 *Let* $\mathbf{A}$ *and* $\mathbf{S}$ *be as in Theorem 1 and let* $m^l_{|\mathbf{A}|^2}$ *be the eigenvalue moments of the diagonal matrix* $\mathbf{A}\mathbf{A}^H$*, then the asymptotic eigenvalue moments of* $\mathbf{R}$ *are given by*

$$m^l_{\mathbf{R}} = \sum_{\substack{(i_0,i_1,\dots i_{l-1}):\\ i_0+\sum_{j=1}^{l-1} j i_j = l \\ \sum_{j=1}^{l-1}(j+1)i_j \le l}} \varphi(i_0,i_1,\dots i_{l-1}) m^{i_0}_{|\mathbf{A}|^2} \prod_{k=1}^{l-1}\left(\beta m^k_{\mathbf{R}}\right)^{i_k} \tag{13}$$

where the l*-tuple* $(i_0,i_1,\dots i_{l-1})$ *and the coefficients* $\varphi(i_0,i_1,\dots i_{l-1})$ *are defined as in Theorem 1. For the initializing moment it results* $m^1_{\mathbf{R}} = m^1_{|\mathbf{A}|^2}$. □

Theorem 13 is proven in [11].

Noting that for signals having the same power, i.e. $\mathbf{A}\mathbf{A}^H = \mathcal{P}\mathbf{I}$, $(\mathbf{R}^k)_{ii}$ converges almost surely to a value that does not depend on the index i, we obtain the following corollary:

COROLLARY 1 *Let* $\mathbf{S}$ *be as in Theorem 1 and let the* $K\times K$ *diagonal matrix* $\mathbf{A}$ *be such that* $\mathbf{A}\mathbf{A}^H = \mathcal{P}\mathbf{I}$*, then, for any* $i,k\in\mathbb{N}$*,* $(\mathbf{R}^l)_{ii}$ *converges almost surely, as* $N,K\to\infty$ *with* $\frac{K}{N}$ *constant to the non-random quantity*

$$(\mathbf{R}^l)_{ii} \xrightarrow{a.s.} m^l_{\mathbf{R}}. \tag{14}$$

□

This corollary can also be derived, for $\mathbf{A}$ and $\mathbf{S}$ with elements in $\mathbb{R}$, from Theorem 2 in [12] as proposed in [7].

Theorems 1 and 2 and Corollary 1 can be extended to the case of Gaussian distribution for the elements of $\mathbf{S}$ and can be rewritten with hypothesis B.2 instead of hypothesis B.1 using the following lemma:

LEMMA 1 *Let* $\mathbf{S}$ *be an* $N\times K$ *matrix with statistically independent and identically Gaussian distributed elements with zero mean and variance* $\mathrm{E}\left\{|s_{11}|^2\right\} = \frac{1}{N}$*. Then, as* $N,K\to\infty$ *with* $\frac{K}{N}$ *constant,*

$$\Pr\left\{\bigvee_{i=1}^{N}\bigvee_{j=1}^{K}\left(|S_{ij}| > \frac{\log N}{\sqrt{N}}\right)\right\} \to 0 \tag{15}$$

i.e. almost surely all the elements of $\mathbf{S}$ *satisfy the condition B.1 asymptotically.* □

Proof: Let us denote with $\mathcal{E}$ the following event:

$$\mathcal{E} = \bigvee_{i=1}^{N}\bigvee_{j=1}^{K}\left(|S_{ij}| > \frac{\log N}{\sqrt{N}}\right). \tag{16}$$

The probability of the event $\mathcal{E}$ is upper bounded as follows:

$$\begin{aligned} \Pr\{\mathcal{E}\} &= 1-\Pr\left\{\bigwedge_{i=1}^{N}\bigwedge_{j=1}^{K}\left(|S_{ij}| \le \frac{\log N}{\sqrt{N}}\right)\right\} \\ &= 1-\left[1-2\mathrm{Q}(\log N)\right]^{\beta N^2} \\ &\overset{(a)}{\le} 1-\left[1-\exp\left(-\frac{1}{2}\log^2 N\right)\right]^{\beta N^2} \\ &= \sum_{l=1}^{\beta N^2}\binom{\beta N^2}{l}(-1)^{l+1}\left(\frac{1}{N^{\frac{1}{2}\log N}}\right)^l \end{aligned} \tag{17}$$

For the inequality (a) we make use of the bound [13] $\mathrm{Q}(x) \le \frac{1}{2}\exp^{-\frac{x^2}{2}}\ \forall x \ge 0$. Since the previous series is lower bounded by zero, in order to prove its convergence to zero it is sufficient to show that it converges absolutely to zero:

$$\begin{aligned} \sum_{l=1}^{\beta N^2}\binom{\beta N^2}{l}\frac{1}{N^{\frac{1}{2}l\log N}} &\le \sum_{l=1}^{\beta N^2}\beta^l N^{\left(2-\frac{1}{2}\log N\right)l} \\ &= \frac{K^{\beta N^2+1}+K}{1-K} \end{aligned} \tag{18}$$

with $K = \beta N^{2-\frac{1}{2}\log N}$. As $N\to\infty$ $K\to 0$. Thus the absolute sum of the series converges to zero. □

The MSE and the SINR of user i for any multistage detector with weight vector $\overline{\mathbf{w}}_i$ are given respectively by

$$\begin{aligned} \mathrm{MSE}_i &= 1-2\mathrm{Re}\left(\mathbf{c}_i^T\overline{\mathbf{w}}_i\right)+\overline{\mathbf{w}}_i^T\mathbf{\Phi}_i\overline{\mathbf{w}}_i && (19) \\ \mathrm{SINR}_i &= \frac{\overline{\mathbf{w}}_i^H\mathbf{c}_i\mathbf{c}_i^T\overline{\mathbf{w}}_i}{\overline{\mathbf{w}}_i^H\left(\mathbf{\Phi}_i-\mathbf{c}_i\mathbf{c}_i^T\right)\overline{\mathbf{w}}_i}. && (20) \end{aligned}$$

where Re(.) is the real part operator. Specializing the previous equations to the case of the individually LMMSE detector in $\chi_L(\mathbf{SA})$ we obtain:

$$\begin{aligned} \mathrm{MSE}_{ind,i} &= 1-\mathbf{c}_i^T\mathbf{\Phi}_i^{-1}\mathbf{c}_i && (21) \\ \mathrm{SINR}_{ind,i} &= \frac{\mathbf{c}_i^T\mathbf{\Phi}_i^{-1}\mathbf{c}_i}{1-\mathbf{c}_i^T\mathbf{\Phi}_i^{-1}\mathbf{c}_i} = \frac{1}{\mathrm{MSE}_{ind,i}}-1 && (22) \end{aligned}$$

and it is straightforward to verify that $\text{SINR}_{ind,i}$ is also the maximum, MSINR_i.

For the jointly LMMSE detector in $\chi_L(\mathbf{SA})$ the performance is given by:

$$\text{MSE}_{jnt,i} = 1 - 2\mathbf{c}_i^T \mathbf{\Phi}^{-1}\mathbf{c} + \mathbf{c}^T \mathbf{\Phi}^{-1}\mathbf{\Phi}_i \mathbf{\Phi}^{-1}\mathbf{c}^T \quad (23)$$

$$\text{SINR}_{jnt,i} = \frac{\left(\mathbf{c}_i^T \mathbf{\Phi}^{-1}\mathbf{c}\right)^2}{\mathbf{c}^T \mathbf{\Phi}^{-1}\mathbf{\Phi}_i \mathbf{\Phi}^{-1}\mathbf{c} - \left(\mathbf{c}_i^T \mathbf{\Phi}^{-1}\mathbf{c}\right)^2} = \frac{\left(\mathbf{c}_i^T \mathbf{\Phi}^{-1}\mathbf{c}\right)^2}{\text{MSE}_{jnt,i} - \left(\mathbf{c}_i^T \mathbf{\Phi}^{-1}\mathbf{c} - 1\right)^2} \quad (24)$$

and in general

$$\text{SINR}_{jnt,i} = \begin{cases} < \text{MSINR}_i & \text{for} \quad L < K \\ = \text{MSINR}_i & \text{for} \quad L \geq K \end{cases}. \quad (25)$$

For $L \geq K$ the equality is due to the fact that the jointly LMMSE detector and then, obviously, also the individually LMMSE detector in $\chi_L(\mathbf{SA})$ coincide with the true LMMSE [4] and, as well known, the latter maximizes jointly the MSINR_i for all the users.

In the asymptotic case, the performance of both the individually and the jointly LMMSE detectors in $\chi_L(\mathbf{SA})$ is obtained respectively from (21)-(22) and (23)-(24) substituting $\mathbf{c}$ with $\mathbf{c}^\infty$, $\mathbf{\Phi}$ with $\mathbf{\Phi}^\infty$, $\mathbf{c}_i$ with $\mathbf{c}_i^\infty$, a vector whose elements are defined by $(\mathbf{c}_i^\infty)_j = R_{ii,\infty}^j$ and $\mathbf{\Phi}_i$ with $\mathbf{\Phi}_i^\infty$, a matrix whose elements are given by $(\mathbf{\Phi}_i^\infty)_{lm} = R_{ii,\infty}^{l+m} + \sigma^2 R_{ii,\infty}^{l+m-1}$. Then, in the asymptotic case the performance depends only on the power $|a_{ii}|^2$ of the i-th symbol, the moments of power distribution, $m_{|\mathbf{A}|^2}^l$, β and σ^2. The relative SINR degradation for the jointly MMSE detector compared to the individually LMMSE detector in $\chi_M(\mathbf{SA})$ is obtained substituting $\overline{\mathbf{w}}_i$ with $(\mathbf{\Phi}^\infty)^{-1}\mathbf{c}^\infty$, $\mathbf{\Phi}_i$ with $\mathbf{\Phi}_i^\infty$ and $\mathbf{c}_i$ with $\mathbf{c}_i^\infty$ in the general equation:

$$\frac{\text{SINR}_{ind,i} - \text{SINR}_{\overline{\mathbf{w}}_i}}{\text{SINR}_{ind,i}} = \frac{\overline{\mathbf{w}}_i^H \left(\mathbf{\Phi}_i \overline{\mathbf{w}}_i \mathbf{c}_i^T - \mathbf{c}_i \overline{\mathbf{w}}_i^H \mathbf{\Phi}_i\right) \mathbf{\Phi}_i^{-1}\mathbf{c}_i}{\overline{\mathbf{w}}_i^H \left(\mathbf{\Phi}_i - \mathbf{c}_i \mathbf{c}_i^T\right) \overline{\mathbf{w}}_i \mathbf{c}_i^T \mathbf{\Phi}_i^{-1}\mathbf{c}_i}. \quad (26)$$

Corollary 1 ensures that $\mathbf{\Phi}_i^\infty = \mathbf{\Phi}^\infty$ and $\mathbf{c}_i^\infty = \mathbf{c}^\infty$ as $\mathbf{A}\mathbf{A}^H = \mathcal{P}\mathbf{I}$. Thus, the jointly and the individually LMMSE detectors in $\chi_L(\mathbf{SA})$ coincide asymptotically in the equal power case and (26) is equal to zero.

Additionally, equations (19)-(22) allow the performance evaluation of both the asymptotic LMMSE detectors when they are used in real scenarios with finite system size. For the asymptotic jointly LMMSE detector in $\chi_M(\mathbf{SA})$ it holds:

$$\text{MSE}_{jt,i}^\infty = 1 - 2\mathbf{c}_i^T(\Phi^\infty)^{-1}\mathbf{c}^\infty + (\mathbf{c}^\infty)^T(\Phi^\infty)^{-1}\mathbf{\Phi}_i(\Phi^\infty)^{-1}\mathbf{c}^\infty \quad (27)$$

and

$$\text{SINR}_{jnt,i}^\infty = \frac{\left((\mathbf{c}^\infty)^T(\Phi^\infty)^{-1}\mathbf{c}_i\right)^2}{(\mathbf{c}^\infty)^T(\Phi^\infty)^{-1}\left(\mathbf{\Phi}_i - \mathbf{c}_i\mathbf{c}_i^T\right)(\Phi^\infty)^{-1}\mathbf{c}^\infty}. \quad (28)$$

Analogous relations can be written for the individually LMMSE detector in $\chi_M(\mathbf{SA})$ and the relative SINR degradation can be derived. However all these equations depend also on the specific realizations of $\mathbf{A}$ and $\mathbf{S}$ and not only on $|\mathbf{a}_{ii}|^2$, $m_{|\mathbf{A}|^2}^l$, β and σ^2 as it was the case in asymptotic conditions.

III. Prospect

Investigation of the asymptotic performance in presence of multipath fading will be presented in subsequent work.

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