Multiuser Diversity in Delay-Limited Cellular Wideband Systems

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***Abstract*— We consider the uplink and the downlink of a multiuser wireless system with one base station and K user terminals. We model wideband transmission by considering M parallel subchannels, each of which is affected by fading. The fading processes in each subchannel are slowly time-varying with respect to the coding block length. Hence, in order to maintain given rate requirements for each user and each channel state, power control is used. We study the delay-limited achievable sum rate (throughput) versus the system E_b/N_0, under orthogonal and optimal signaling. We show that for both the orthogonal and the optimal schemes, in the limit of large K and finite M, the optimal allocation strategy consists of allocating each user to its best subchannel only. Hence, we are able to quantify the multiuser diversity gain by comparing the case $K \to \infty$ with the single-user delay-limited case. Finally, we show that the limits of optimal signaling can be approached by relatively simple convolutional codes and iterative joint multiuser decoding with appropriate power control. The proposed scheme can be regarded as a practical version of the optimal successive decoding approach, that mitigates the error propagation due to the suboptimality of the user channel codes.**[1]

I. Introduction

Tse and Hanly put forward a comprehensive treatment of multiple-access channels with fading [1], [2]. They distinguished between ergodic fading and non-ergodic, or delay-limited, fading. In this context, ergodic means that the channel shows its statistics during the time that a single codeword needs to be transmitted.

The multiple-access channel with arbitrarily slow fading was addressed by Hanly and Tse who characterized the so-called delay limited capacity region [2]. This is the case where the fading is so slow that it can be considered random but constant over a coding interval, while the codeword block length is large enough such that the theoretical Shannon limit is practically approached. The boundary of the delay-limited capacity is explicitly characterized in [2] in the flat-fading (narrowband) case, exploiting the contra-polymatroid structure of the region of received signal powers supporting a given rate K-tuple. In the case of frequency-selective fading (wideband case), this explicit characterization is not possible and only an implict Lagrangian characterization is given. The reason is that in the frequency-selective case the contra-polymatroid structure is lost.

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In this paper we consider a wideband system with M parallel subchannels (modeling the fact that the system bandwidth is M times larger than the channel coherence bandwidth) and K users. The users have to maintain fixed rates irrespectively of the fading, which is assumed to be random but constant over the codeword block length, as in the setting of [2]. We characterize the system throughput versus the system E_b/N_0 (to be defined later on) under the optimal (i.e., delay-limited capacity achieving) and an orthogonal signaling scheme in the limit of K large and M fixed, under certain symmetry conditions of the fading distribution. We show that in this limit the optimal strategy for both signaling schemes consists of letting each user transmit on its own best subchannel only, irrespectively of the other users. We quantify the multiuser diversity gain of the optimal and the orthogonal schemes over a single-user delay-limited system. Thanks to uplink-downlink duality [3], it is immediate to conclude that exactly the same performance and relative gains are achieved in the downlink (delay-limited parallel Gaussian broadcast channel).

II. Background

A. *Capacity region, power region and* $(E_b/N_0)_{\rm sys}$

The capacity region of a K-user multiple-access channel in additive white Gaussian noise N

$$Y = N + \sum_{k=1}^{K} X_k \tag{1}$$

is given by [4]

$$\sum_{k\in\mathcal{S}} R_k \le \log\left(1 + \frac{\sum\limits_{k\in\mathcal{S}} E_k^{(r)}}{N_0}\right) \quad \forall \mathcal{S} \subseteq \{1,\ldots,K\}. \tag{2}$$

with $E_k^{(r)}$ denoting the received energy per symbol [5] of users k and N_0 denoting the noise spectral density.

If the users are separated by orthogonal signaling, e.g. by TDMA or FDMA, the achievable rates are

$$R_k \le \Theta_k \log_2\left(1 + \frac{E_k^{(r)}}{\Theta_k N_0}\right) \quad \forall k \tag{3}$$

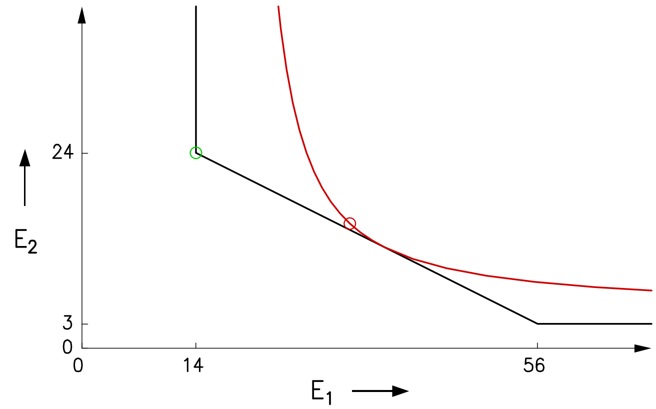

Fig. 1. Transmit power region for two users with $R_1 = 3$, $R_2 = 2$ bits, $N_0 = 1$, $d_1 = \frac{1}{2}$, and $d_2 = 1$. The curve refers to orthogonal signaling.

subject to

$$\sum_{k=1}^{K} \Theta_k \leq 1 \tag{4}$$

where Θ_k denotes the resource-sharing fraction (proportion of channel dimensions) given to user k. If these fractions are chosen appropriately, the optimal sum rate can be achieved.

The received power region supporting a given set of user rates is obtained solving the $2^K - 1$ equations in (2) and the K equation in (3) for the symbol energies. This yields

$$\sum_{k\in\mathcal{S}} E_k^{(r)} \geq N_0 \left[\exp\left(\sum_{k\in\mathcal{S}} R_k\right) - 1\right] \quad \forall \mathcal{S} \subseteq \{1,\ldots,K\} \tag{5}$$

for optimal signaling and

$$E_k^{(r)} \geq \Theta_k N_0 \left[\exp\left(R_k/\Theta_k\right) - 1\right] \quad \forall k, \tag{6}$$

for orthogonal signaling, respectively. Again, constraining to orthogonal signaling does not increase the required sum power if the fractions Θ_k are chosen appropriately.

Next, we introduce the *near-far* effect by formulating the problem in terms of *transmit* powers: namely, the received symbol $E_k^{(r)}$ energy is related to the transmit symbol energy E_k by

$$E_k^{(r)} = d_k E_k, \tag{7}$$

where d_k denotes the channel (power) gain of user k. The transmit power region is illustared in Fig. 1. Constraining to orthogonal signaling implies an increase in total transmit power unless all channel gains are identical. In addition, the optimal choice of the fractions is influenced by the channel gains. In order to help users with bad channels, their fractions are increased at the expense of users with good channels.

Fig. 1 shows that the minimum received power for given user rates is achieved by a unique vertex of the power region (assuming all user gains distinct). At this vertex, the receiver can make use of successive decoding (stripping) without loss of performance. Clearly, users are decoded in decreasing order of the strength of their channels [6], [2].

The minimum total energy supporting a given rate K-tuple is obtained by finding the symbol energies solution of

$$\begin{array}{c} \min_{\mathbf{E}\in\mathbb{R}_+^K} \;\; \sum_{k=1}^K E_k \\ \text{subject to} \;\; \mathbf{R} \in \mathcal{C}_{\rm MAC}(\mathbf{d};\mathbf{E}), \end{array} \tag{8}$$

where $\mathcal{C}_{\rm MAC}(\mathbf{d};\mathbf{E})$ is the region defined in (2) letting $E_k^{(r)} = d_k E_k$. Thanks to the fact that the received energy region is a contra-polymatroid [2], the solution of (8) is found explicitly as

$$E_{\pi_k} = \frac{N_0}{d_{\pi_k}} \exp\left(\sum_{i<k} R_{\pi_i}\right)\left(\exp(R_{\pi_k}) - 1\right) \tag{9}$$

where π is the permutation of $\{1,\ldots,K\}$ that sorts the channel gains in increasing order, i.e., $d_{\pi_1} \leq \cdots \leq d_{\pi_K}$ and the associated decoding order is given by $\pi_K, \pi_{K-1}, \ldots, \pi_1$.

With orthogonal signaling, the minimum total energy supporting a given rate K-tuple $\mathbf{R}$ with gains $\mathbf{d}$ is obtained by solving

$$\begin{array}{c} \min_{\mathbf{\Theta},\mathbf{E}\in\mathbb{R}_+^K} \;\; \sum_{k=1}^K E_k \\ \text{subject to} \;\; \mathbf{R} \in \mathcal{C}_{\rm orth}(\mathbf{d};\mathbf{E};\mathbf{\Theta}), \end{array} \tag{10}$$

where $\mathcal{C}_{\rm orth}(\mathbf{d};\mathbf{E};\mathbf{\Theta})$ is the region defined by (3) and (4), letting $E_k^{(r)} = d_k E_k$.

Assume that the channel gain vector is constant over the duration of a codeword and it is randomly distributed according to some joint probability law $\mathbf{d} \sim F(\mathbf{d})$. The delay-limited capacity introduced in [2] is the set of all rate K-tuples $\mathbf{R}$ that can be attained for all $\mathbf{d} \in \mathbb{R}_+^K$, subject to *average power constraints* $\mathbb{E}[E_k] \leq \overline{E}_k$. From the operational point of view, this setting is relevant in systems with quality-of-service guarantee: users sign-up for a given desired rate that must be maintained in any channel conditions. The system makes use of *power control* in order to compensate for the channel gains' instantaneous realizations.

In this work we are interested in the total system throughput (sum rate) versus the total average transmit energy. Following [7], [8], [9], we define the system E_b/N_0 under a coding strategy that supports user rates $(R_1,\ldots,R_K)$ with sum $\Gamma = \sum_{k=1}^K R_k$ subject to average transmit energy per symbol constraints $(\overline{E}_1,\ldots,\overline{E}_K)$ as

$$\left(\frac{E_b}{N_0}\right)_{\rm sys} = \frac{\sum_{k=1}^K \overline{E}_k}{N_0 \Gamma} \tag{11}$$

where Γ is expressed in bits. In the case of equal individual rates $R_k = \Gamma/K$, this coincides with the individual user transmit E_b/N_0. In general, for finite K, an operating point $((E_b/N_0)_{\rm sys}, \Gamma)$ on the spectral-power efficiency plane is a function of both the signaling strategy and of the individual user rates $\mathbf{R}$, as well as of the channel gain joint distribution $F(\mathbf{d})$. In the next sections we investigate a regime of large K for which, under mild assumptions on the user individual rates, the dependency on $\mathbf{R}$ disappears and the total instantaneous transmit energy converges to its average value.

B. Downlink

So far we have treated the multiple-access case, that models the uplink of a wireless system with a single base station and many users. Fortunately, exploiting the recent result on the duality of the Gaussian multiple-access and broadcast channel, it is immediate to see that for a given set of user rates $\mathbf{R}$ and channel gain statistics, the downlink channel supports $\mathbf{R}$ with exactly the same $(E_b/N_0)_{\rm sys}$.

For an orthogonal system this follows trivially from the fact that the uplink and downlink channel gains are identically distributed. In the case of optimal signaling, letting $\mathcal{C}_{\rm BC}(\mathbf{d}; E_{\rm tot})$ denote the Gaussian broadcast channel capacity region with gains $\mathbf{d}$ and transmit energy per symbol $E_{\rm tot}$, [3] showed that

$$\mathcal{C}_{\rm BC}(\mathbf{d}; E_{\rm tot}) = \bigcup_{\sum_{k=1}^K E_k = E_{\rm tot}} \mathcal{C}_{\rm MAC}(\mathbf{d}; \mathbf{E}) \tag{12}$$

Any point on the boundary of $\mathcal{C}_{\rm BC}(\mathbf{d}; E_{\rm tot})$ corresponds to the vertex of $\mathcal{C}_{\rm MAC}(\mathbf{d}; \mathbf{E})$ (for some choice of the individual transmit energies $(E_1, \ldots, E_K)$) associated to successive decoding in the order $\pi_K, \pi_{K-1}, \ldots, \pi_1$. For what said before, this vertex is precisely the one that minimizes the total (MAC) transmit energy for given user rates $\mathbf{R}$ and channel gains $\mathbf{d}$. Therefore, for any realization of $\mathbf{d}$, the downlink achieves the rate K-tuple $\mathbf{R}$ with the same (minimal) total transmit energy of the uplink. It follows that the optimal operating point $((E_b/N_0)_{\rm sys}, \Gamma)$ is the same for both uplink and downlink. Hence, from now on we shall focus on the uplink, taking into account that all the results and conclusions are immediately applicable to the downlink.

III. Parallel Channels

Consider now the more general case that there are M parallel multiple-access channels where the channel gain of each user may differ from channel to channel. Namely, we let

$$Y^m = \sum_{k=1}^K \sqrt{d_k^m} X_k^m + N^m, \qquad m = 1, \ldots, M \tag{13}$$

This is an accurate model for frequency-selectivity where m can be interpreted as the sub-band index. The theoretical foundations for this case were laid down in [10].

The capacity region of this channel can be achieved by letting each user split its information messages into M parallel streams, encode them independently, and send the resulting independent codewords over the parallel channels.The aggregate rate and aggregate energy per symbol of user k are given by

$$R_k = \sum_{m=1}^M R_k^m, \qquad k = 1, \ldots, K \tag{14}$$

$$E_k = \sum_{m=1}^M E_k^m, \qquad k = 1, \ldots, K \tag{15}$$

respectively, where R_k^m and E_k^m denote the rate and the energy per symbol allocated by user k on subchannel m. Let $\mathbf{E}^m = (E_1^m, \ldots, E_K^m)$, $\mathbf{R}^m = (R_1^m, \ldots, R_K^m)$ and $\mathbf{d}^m = (d_1^m, \ldots, d_K^m)$. The capacity region for given per-user energies $\mathbf{E} = (E_1, \ldots, E_K)$ and channel gains can be written as

$$\mathcal{C}_{\rm MAC}(\mathbf{d}^1, \ldots, \mathbf{d}^M; \mathbf{E}) = \bigcup_{\substack{\sum_m E_k^m \le E_k \\ k=1,\ldots,K}} \left\{ \mathbf{R} = \sum_m \mathbf{R}^m : \mathbf{R}^m \in \mathcal{C}_{\rm MAC}(\mathbf{d}^m; \mathbf{E}^m) \right\}$$

In other words, the partial rates R_k^m and energies E_k^m must obey the constraints (2) and (5) in each of the subchannels m.

When it comes to transmit energy, we make use of the fact that the successive decoding order depends only on the channel gains, but not on the rates. This implies the following:

- The decoding order is independent of the split of rates into partial rates.
- The decoding order differs, in general, from subchannel to subchannel.

Let π^m denote the permutation that sorts the gains $\mathbf{d}^m$ in increasing order. The required transmit energy per symbol of user π_k^m in subchannel m is given by

$$E_{\pi_k^m}^m = \frac{N_0}{d_{\pi_k^m}^m} \exp\left(\sum_{i<k} R_{\pi_i^m}^m \right) \left(\exp(R_{\pi_k^m}^m) - 1 \right). \tag{16}$$

Optimizing the partial rates R_k^m for minimizing

$$\left(\frac{E_b}{N_0} \right)_{\rm sys} = \frac{1}{\Gamma N_0} \sum_{k=1}^K \sum_{m=1}^M E_k^m \tag{17}$$

(recall that $\Gamma = \sum_k R_k$ is fixed by the user rates) subjects to the constraints (14), is a convex optimization problem.

For orthogonal multiple-access, we let $\mathbf{\Theta}^m = (\Theta_1^m, \ldots, \Theta_K^m)$, where Θ_k^m denotes the resource-sharing fraction of user k over subchannel m. The achievable rate region under orthogonal signaling can be written as

$$\mathcal{C}_{\rm orth}(\mathbf{d}^1, \ldots, \mathbf{d}^M; \mathbf{E}) = \bigcup_{\sum_k \Theta_k^m \le 1 \ \forall\ m} \bigcup_{\substack{\sum_m E_k^m \le E_k \\ k=1,\ldots,K}} \left\{ \mathbf{R} = \sum_m \mathbf{R}^m : \mathbf{R}^m \in \mathcal{C}_{\rm orth}(\mathbf{d}^m; \mathbf{\Theta}^m; \mathbf{E}^m) \right\}$$

For orthogonal signaling, the transmit energy per symbol of user k in subchannel m is given by

$$E_k^m = \Theta_k^m \frac{N_0}{|d_k|^2} \left(2^{R_k^m/\Theta_k^m} - 1 \right) \tag{18}$$

The minimization of $(E_b/N_0)_{\rm sys}$ with respect to $\{\mathbf{\Theta}^m, \mathbf{R}^m\}$, $\forall m$, subject to (14) can be stated as a convex optimization problem by letting $r_k^m = R_k^m/\Theta_k^m$. This results in

$$\min_{\mathbf{\Theta}^m, \mathbf{r}^m,\, m=1,\ldots,M} \frac{1}{\Gamma} \sum_{k=1}^K \sum_{m=1}^M \frac{\Theta_k^m}{d_k^m} \left(\exp(r_k^m) - 1 \right) \tag{19}$$

subject to

$$\sum_{m=1}^M \Theta_k^m r_k^m = R_k, \quad k = 1, \ldots, K$$

and to

$$\sum_{k=1}^{K} \Theta_k^m \le 1, \quad m = 1, \ldots, M$$

IV. Asymptotically optimal subchannel allocation

In order to quantify the near-far gain for various distributions of (quasi) static channel gains in the case of M parallel channels discussed before, we address the problem in the large-user limit. In particular, we make the following assumptions:

A1 M is fixed while K gets arbitrarily large.

A2 As $K \to \infty$ the sequence of the channel user gains $\{(d_k^1, \ldots, d_k^M) : k = 1, \ldots, K\}$ converges to an ergodic vector process over the user population. In particular, the empirical joint channel gain distribution, defined by

$$F^{m,K}(x^1, \ldots, x^M) = \frac{1}{K} \sum_{k=1}^{K} \prod_{m=1}^{M} 1\{d_k^m \le x^m\} \quad (20)$$

converges almost surely to a given deterministic cumulative distribution function (cdf) $F_{\mathbf{d}}(x^1, \ldots, x^M)$ as $K \to \infty$. Moreover, $F_{\mathbf{d}}$ is assumed to be symmetric, i.e., for any $\mathbf{x}$ and any permutation π of M elements, $F_{\mathbf{d}}(x^1, \ldots, x^M) = F_{\mathbf{d}}(x^{\pi_1}, \ldots, x^{\pi_M})$. In particular, the marginal cdfs of $F_{\mathbf{d}}$ are identical (no subchannel is better of worse than another one).

A3 For a given system throughput Γ, the user individual rates are given by $R_k = \frac{\Gamma}{K}\nu_k$, where ν_k is the rate allocation factor for user k. As $K \to \infty$ the sequence of rate allocation factors $\{\nu_k : k = 1, \ldots, K\}$ converges to an ergodic process over the user population. In particular, the empirical rate allocation distribution, defined by

$$G^K(x) = \frac{1}{K} \sum_{k=1}^{K} 1\{\nu_k \le x\} \quad (21)$$

converges almost surely to a given deterministic cdf $G_\nu(x)$ with mean 1 and support in $[a, b]$ as $K \to \infty$, where $0 \le a \le b < \infty$ are constants independent of K.

A4 The rate allocation factors are fixed independently of the realization of the channel gains. Therefore, the empirical joint distribution of $(d_k^1, \ldots, d_k^M)$ and ν_k converges to the product cdf $F_{\mathbf{d}}(x^1, \ldots, x^M) G_\nu(z)$.

We have proofs (omitted here) for the following results.

Theorem 1. Under the assumptions A1, A2, A3 and A4, as $K \to \infty$ the minimum $(E_b/N_0)_{\text{sys}}$ for given system throughput Γ (in bits) is given by

$$\left(\frac{E_b}{N_0}\right)_{\text{sys}} = \log(2) \int_0^\infty \exp\left(\frac{\Gamma \log(2)}{M} F_{\max}(x)\right) \frac{dF_{\max}(x)}{x} \quad (22)$$

where $F_{\max}(x)$ is the limit cdf of the empirical distribution of $d_k^{\max} = \max\{d_k^1, \ldots, d_k^M\}$, as $K \to \infty$. This is achieved by letting each user transmit on its best subchannel only, and by using superposition coding and successive decoding on each subchannel.

Theorem 2. Under the assumptions A1, A2, A3 and A4, as $K \to \infty$ the minimum $(E_b/N_0)_{\text{sys}}$ for given system throughput Γ (in bits) achieved by orthogonal signaling is given by

$$\left(\frac{E_b}{N_0}\right)_{\text{sys}} = \log(2) \int_0^\infty \frac{\exp\left(1 + W\left(\frac{-\mu x - 1}{e}\right)\right) - 1}{1 + W\left(\frac{-\mu x - 1}{e}\right)} \frac{dF_{\max}(x)}{x} \quad (23)$$

where $F_{\max}(x)$ is defined as in Theorem 1, where $W : [-e^{-1}, +\infty) \to [-1, +\infty)$ is Lambert's W function, defined by $W(x)e^{W(x)} = x$ [11], and where μ is the solution of

$$\int_0^\infty \frac{dF_{\max}(x)}{1 + W\left(\frac{-\mu x - 1}{e}\right)} = \frac{M}{\Gamma \log(2)} \quad (24)$$

This is achieved by letting each user transmit on its own best subchannel only, and by using orthogonal signaling with optimized fractions on each subchannel.

Theorem 2 directly applies to orthogonal frequency division multiple access (OFDM) transmission and provides an ulimate limit for its potential performance on a frequency-selective fading channel with channel state information.

V. Iterative Multiuser Decoding

Iterative multiuser decoding is a practical means to benefit from the near-far situation. As shown in [8], iterative multiuser decoders require some kind of irregularity to aid the convergence properties of the iterative decoding process. The near-far situation provides irregularity among users without additional means. In order to shape the natural irregularity towards the desired one, we expect only little additional effort.

The problem of finding the power allocation policy that minimizes the total received power among the user population was solved in [8] by means of linear programming in the large-system limit for several detector front-ends in the iterative decoding process. The large-system results serve as good approximations for systems with only a few users. In the following, we develop a linear programm (LP) which finds that power allocation policy which minimizes the total transmitted power.

Define the received power distribution as

$$G(x) = \lim_{k \to \infty} \frac{1}{K} \sum_{k : E_k < x} 1 \quad (25)$$

and denote the multiuser efficiency by η, the LP solving the power allocation problem for minimum total received power is given by

$$\begin{aligned} &\text{minimize} && \int x \mathrm{d}G(x) \\ &\text{subject to} && \begin{cases} \Psi(G, \eta) \ge \eta & \forall \eta \\ \int \mathrm{d}G(x) = 1 \\ \frac{\mathrm{d}}{\mathrm{d}x} G(x) \ge 0 & \forall x \end{cases} \end{aligned} \quad (26)$$

where Ψ is a functional linear in G but not in η which depends on the building blocks of the iterative multiuser decoder, the code laws, the load and the noise power. More details on particular forms of Ψ are given in [8].

TABLE I

AVERAGE NORMALIZED TRANSMIT ENERGY PER BIT FOR ITERATIVE MULTIUSER DECODING WITH UNCOND. LINEAR MMSE DETECTION [8], PATH LOSS WITH $\alpha = 4$, RANDOM SPREADING, CONV. CODES WITH 64 STATES, RATE $\frac{1}{2}$, AND BIT ERROR RATE 10^{-5}.

	$M \to \infty$		$M = 2$	
β	LP (26)	LP (30)	LP (26)	LP (30)
2	4.20 dB	4.20 dB	4.19 dB	4.19 dB
3	4.64 dB	4.63 dB	4.44 dB	4.43 dB
4	5.54 dB	5.53 dB	4.98 dB	4.98 dB
5	6.92 dB	6.92 dB	5.91 dB	5.90 dB
6	9.26 dB	9.10 dB	7.66 dB	7.55 dB
7	11.56 dB	11.41 dB	9.82 dB	9.47 dB

Define the joint distribution of received powers and attenuations as

$$J(x,y) = \lim_{k\to\infty} \frac{1}{K^2} \sum_{k:E_k<x} \sum_{i:|d_i|^2<y} 1 \tag{27}$$

and its marginal distributions

$$G(x) = \int_0^\infty J(x,y)\,\mathrm{d}y \tag{28}$$

$$F(y) = \int_0^\infty J(x,y)\,\mathrm{d}x. \tag{29}$$

The LP minimizing transmitted power among all power allocations allowing for convergence of the iterations is

$$\begin{aligned} &\text{minimize} && \int \frac{x}{y}\,\mathrm{d}J(x,y) \\ &\text{subject to} && \begin{cases} \Psi(G,\eta) \ge \eta & \forall \eta \\ \int_0^\infty J(x,y)\,\mathrm{d}x = F(y) & \forall y \\ \frac{\mathrm{d}^2}{\mathrm{d}x\,\mathrm{d}y} J(x,y) \ge 0 & \forall x,y \end{cases} \end{aligned} \tag{30}$$

with $G(x)$ being defined by (28). The constraint (29) ensures that the optimization of the joint distribution does not violate the distribution of attenuations given by the channel.

The LPs (26) and (30) are implemented numerically by sampling the continuous functions over a dense grid. This means approximating the continuous distributions by discrete ones. Details on numerical issues are discussed in [8]. The numerical complexity of (30) is significantly larger than the complexity of (26), as (30) requires two dimensional numerical integration for calculation of the objective function. However, the example in Table I shows that both LPs give very similar results unless the load is higher than $\beta = 5$. In practice, (26) can be used without noticeable loss in performance.

An example for the achievable spectral efficiency at a bit error rate of 10^{-5} with standard rate $\frac{1}{2}$ convolutional codes is given in Fig. 2. For high spectral efficiency, the near-far effect can be utilized very well. Even with convolutional codes, a higher spectral efficiency can be achieved than with capacity achieving codes when the near-far effect is not exploited. For $M = 2$, convolutional codes can even go beyond the

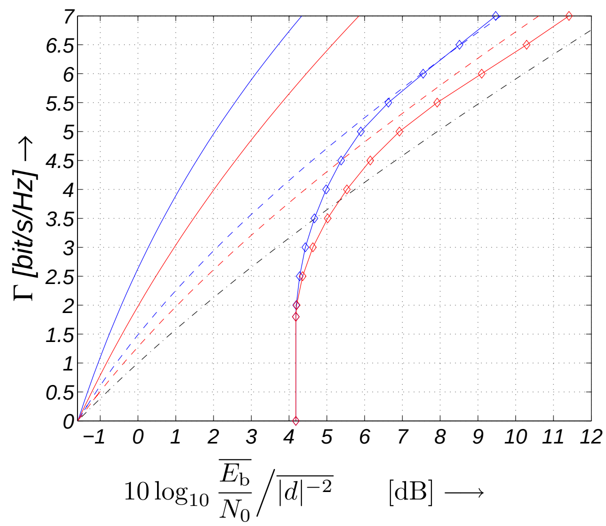

Fig. 2. Spectral efficiency vs. normalized energy per bit. The marked points refer to the iterative multiuser decoder of Table I with LP (30). The blue and red lines refer to $M = 2$ and $M \to \infty$, respectively. The solid (unmarked lines) refer to channel capacity, the dashed lines refer to the orthogonal protocol, the dash-dotted line refers to a system without fading.

performance of any orthogonal system even when capacity achieving codes are used and the bandwidth allocation is optimized with respect to the near-far situation.

VI. CONCLUSION

With respect to delay-limited capacity, the near-far effect is rather a blessing than a curse. It helps to greatly reduce the total amount of transmit power which is required to guarantee a ceratin quality of service. Iterative multiuser decoding is a powerful tool to achive the near-far gain in practice.

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