Transmit Power Allocation for Gaussian Multiple Access Channels with Diversity

Alexander Lampe, Ralf R. Müller, and Johannes B. Huber

Lehrstuhl für Nachrichtentechnik II, Universität Erlangen-Nürnberg
Cauerstr. 7/NT, 91058 Erlangen, Germany
Email: alampe@nt.e-technik.uni-erlangen.de

I. Introduction

Recently, Hanly and Tse [3, 6], see also Cheng [2], have generalized the results on the Gaussian multiple–access channel to the case where flat fading occurs which can be tracked by both receiver and transmitter. They distinguish two cases which they term Shannon and Delay–Limited capacity of that channel. With Shannon capacity they refer to the case where the capacity region expected over the fading statistics is maximized for constrained averaged sum power of all users. The practically more important so–called Delay–Limited capacity results from minimizing the sum energy of all users for fixed rates. In the following we focus on the second case. Instead of assuming a single receiver only we study the case of multiple receivers since in implemented systems diversity reception is used often. This can be caused e.g. by multiple receive antennas (receiver antenna diversity) for signal separation, or by multiple cell–site processing in cellular systems. Surprisingly, we show that some results which are valid for the case without diversity [4] do not generalize to the diversity model.

II. Uniqueness of Optimum Transmit Energy

Considering transmission of K users to M receivers over faded single path channels the received signal Y_m at the mth receiver is given as $Y_m = \sum_{k=1}^{K} d_{km}\tilde{X}_k + N_m$. Here $\tilde{X}_k$, d_{km} and N_m denote the i.i.d. data symbols of the kth user, the kth user's path gain to the mth receiver and the additive white Gaussian channel noise with power σ_n^2, respectively. The additive channel noise is supposed to be independent at all receiver sites.

Assuming perfect knowledge of the instantaneous path gains as well as joint processing of all receivers we are interested in those users' transmit energies $\tilde{E}_k = \mathcal{E}\{|\tilde{X}_k|^2\}, \forall k$, minimizing total transmit energy while allowing the users to transmit reliable at required rates $R_k, \forall k$. Mathematically, solve $\tilde{E}_{\text{opt}} = \min \sum_{k=1}^{K} \tilde{E}_k$ which fulfill the conditions $\sum_{i\in\Omega} R_i \leq I(Y_1, \ldots, Y_M; \tilde{X}_i \in \Omega), \forall \Omega \subseteq \{1, \ldots, K\}$, (see [1]). Investigating this problem, we find for the multiple receiver case with the definition of the kth user's path gain vector as $\boldsymbol{d}_k \triangleq (d_{k1}, \ldots, d_{kM})^T$

Theorem: The transmit energy tuple $\left(\tilde{E}_1, \ldots, \tilde{E}_K\right)$ achieving minimum total transmit energy for a given rate tuple $(R_1, \ldots, R_K), R_k > 0, \forall k$, and given channel state is unique if and only iff each pair of vectors $\boldsymbol{d}_k, \forall k$, is not collinear, i.e. $\boldsymbol{d}_k \neq \gamma \boldsymbol{d}_j, \forall j \neq k$, $\gamma \in \mathbb{C}, |\gamma| = 1$.[1]

Furthermore, it follows that the optimum transmit energy tuple relies not only on the users' path gains but also on the required rates if multiple receivers are employed. Next, in real environments the path gains d_{km} are samples of *continuous* random distributions in general. Therefore and because of the above results the optimum transmit energy tuple is unique with probability one in practical systems.

Having obtained the optimum transmit energy tuple we face the question whether overall joint multiuser decoding or merely single–user decoding is required.

It can be shown that like for the single receiver case rate–splitting [5] as well as time–sharing can be employed also for multiple receivers. So, reliable transmission at a required rate tuple $(R_1, \ldots, R_K), R_k > 0, \forall k$, using a multiple–user multiple–receiver system with given path gains and weighted energies can be reached by means of MMSE–Equalization and single user decoding combined with successive cancellation via splitting the optimum transmit energy tuple $\left(\tilde{E}_1, \ldots, \tilde{E}_K\right)$ into at most $2K-1$ new (virtual) transmit energies. In addition a closed analytical solution on the virtual users' optimum energies can be given.

III. Conclusions

Investigating the transmission of multiple–users employing diversity reception we see that in contrast to single–receiver applications the optimum transmit energies depend not only on the path gain vectors' magnitudes but on their correlations and the required rates, too. As a consequence, the decoding order of the users can no longer be determined knowing the path gains only. Finally, the optimization of transmission over bandlimited multipath–fading channels with perfect channel state information can be treated in a similar way.

[1] $\mathbb{C}$ denotes the set of complex numbers and $|x|$ the absolute magnitude of x.

References

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