

# On the Transport Capacity of a Multiple Access Gaussian Channel

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**Abstract**—We address the problem of maximizing the transport capacity of a wireless network, defined as the sum, over all transmitters, of the products of the transmission rate with a reward  $r(x)$ , which is a function of the distance  $x$  separating the transmitter and its receiver. When  $r(x) = x$ , this product is measured in *bps × meters*, and is the natural measure of the usefulness of a transmission in a multihop wireless ad hoc network.

We first consider a single transmitter-receiver pair, and determine the optimal distance between the two that maximizes the rate-reward product, for reward functions of the form  $r(x) = x^\rho$  and when the signal power decays with distance according to a power law. We then calculate the scheme that maximizes the transport capacity in a multiple access network consisting of a single receiver and a number of transmitters, each placed at a fixed distance from the receiver, and each with a fixed power constraint. We conclude by showing that when the per-transmitter power constraints are substituted with a single constraint on the sum of the powers, the maximum transport capacity and the power allocation scheme that achieves it can be found by solving a convex optimization problem.

## I. INTRODUCTION

In multihop wireless ad hoc networks it is important not only that nodes transmit data packets with high bit rates, but also that the packets are transmitted over long distances. Indeed, assuming that the transmission rates of the links remain fixed, the larger the distances between the transmitter and the receiver, the fewer are the transmissions that will be needed until the packets reach their final destination.

In fact, in multihop wireless network, the appropriate metric for the usefulness of a transmission is not its data rate, but rather, the *product* of the data rate multiplied by the distance separating the transmitter and receiver, measured in *bps × meters*. The higher this product is for a transmission, the fewer transmissions of the same kind are needed for a specific amount of data to be delivered to their destinations. Two transmissions that achieve the same rate-distance product, while consuming the same bandwidth and power, are equivalent from a network capacity perspective, even if the two data rates are very different.

This idea was first explicitly stated in a seminal paper by Gupta and Kumar [1]. There, the summation of the distance-rate products over all links that are active at a given moment in time is defined as the **transport capacity**. The authors establish lower and upper bounds on the transport capacity that a large wireless network with an arbitrary topology can support at a given time instant.

More recently, Reznik and Verdú [2] considered a broadcast network consisting of a single transmitter and a number of receivers  $A_1, \dots, A_n$ , placed in strictly increasing distances  $0 < d_1 < d_2 < \dots < d_n$  from the transmitter, and

each subject to additive white Gaussian noise. The authors define the transport capacity of the network as the summation, over all receivers  $A_i$ , of the products of the received rate  $R_i$  with a reward  $r(d_i)$ , where the **reward function**  $r(\cdot)$  is non-decreasing, but otherwise arbitrary. This definition of the transport capacity contains the definition of [1] as the special case  $r(x) = x$ . The capacity region in this setting, i.e., the set of all combinations of communication rates between the transmitter and each of the receivers that are simultaneously achievable, is known [3]. The main contribution of [2] is the calculation of the particular point in the capacity region that maximizes the transport capacity.

In this work, we continue the investigation of transport capacity, along the lines of [2]. Instead of considering the broadcast channel, however, we study the multiple access channel, which consists of a single receiver and an arbitrary number of transmitters.

In Section II we start by identifying the optimal distance between a single transmitter and a single receiver, that maximizes the rate-reward product, in the case of the monomial reward function  $r(x) = x^\rho$ , and when the transmitted power decays according to a power law.

In Section III we consider a multiple access network consisting of a single receiver and a number of transmitters, placed at increasing distances from the receiver, and each with a fixed upper bound on its power. In this setting, we establish the point in the capacity region that maximizes the transport capacity. We show that, irrespective of how the signal powers decay with distance, and as long as the reward function is monotonically non-decreasing, the optimal strategy for the receiver is to successively decode the signals of all transmitters with the order of decoding being exactly the same as the order of the transmitter placements. In other words, the further away a transmitter lies, the later its signal will be decoded.

Finally, in Section IV we consider a multiple access network in which there is no per-transmitter power bound, but rather the sum of all powers is bounded. In this case, the power allocation that maximizes the transport capacity can be found by solving a convex optimization problem. We present a few examples, which show that the optimal power allocation has a remarkably simple structure. We conclude in Section V.

## II. THE SINGLE TRANSMITTER CASE

We study a communication environment in which the successful transmission of a bit of information over a distance  $x$  is associated with a reward  $r(x)$ , where  $r(\cdot)$  is the **reward function**. We assume that  $r(0) = 0$  and that  $r(x) \geq r(y)$  when  $x > y$ . As a special case, we consider the **monomial reward function**

$$r(x) = x^\rho, \quad (1)$$

where  $\rho > 0$  is the **reward exponent**.

Regarding the propagation of signals, we assume that when a transmitter transmits with power  $P$ , a receiver lying at some

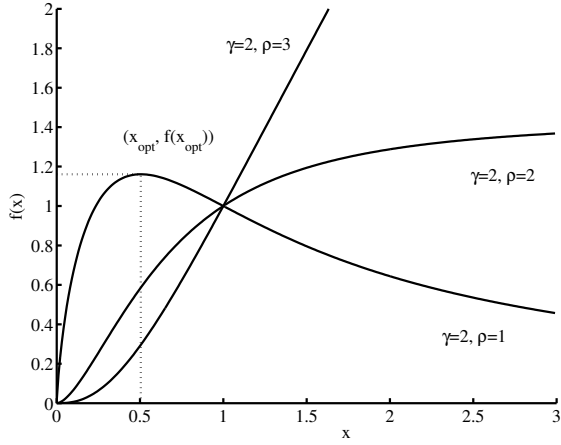


Fig. 1. The function  $f(x)$ , defined in (5), for various combinations for the values of  $\rho$  and  $\gamma$ , and for  $A = B = 1$ .

distance  $x$  away will receive an attenuated version of the signal of power  $Pd(x)$ , where  $d(\cdot)$  is the **decay function**. As a special case, we consider the **monomial decay function**

$$d(x) = Kx^{-\gamma}, \quad (2)$$

where  $\gamma > 0$  is the **decay exponent**, and  $K$  is a normalization constant.

Let us consider a single pair of nodes, a transmitter  $T$  and a receiver  $R$ , separated by a distance  $x$  which is allowed to vary. The transmitter power is  $P$ , the bandwidth available for the communication is  $B$ , and the receiver is susceptible to additive white Gaussian noise of density  $\eta$ . We assume that the two nodes achieve the Shannon capacity  $C = B \log_2(1 + \frac{Pd(x)}{\eta B})$ . We define the **transport capacity**  $C_T(x)$  with the equation

$$C_T(x) \triangleq r(x)B \log_2(1 + \frac{Pd(x)}{\eta B}). \quad (3)$$

The case of the monomial reward function with  $\rho = 1$  is the most interesting. As discussed in Section I, in this case the transport capacity is measured in bps · meters and is the natural figure of merit for the usefulness of a link from a networking perspective, in a wireless ad hoc network. However, as was pointed out in [2], there are other applications in which a different functional form for  $r(\cdot)$  would be more appropriate. As an example, consider a scenario in which the transmitter is associated with a sensor, and the receiver is associated with a base station. If the base station would like to receive information about the surroundings of the sensor, and the importance that the base station places on information coming from location  $x$  is given by an arbitrary function  $r(\cdot)$ , the transport capacity of (3) is the appropriate figure of merit about the usefulness of the transmission. Other examples are offered in [2].

We are interested in determining the maximum possible value for  $C_T(x)$ ,  $C_T^{\text{opt}} \triangleq \sup_{0 < x < \infty} C_T(x)$ . Clearly, unless specific cases for  $r(\cdot)$  and  $d(\cdot)$  are considered, we can not go much further. So let us limit the discussion to the monomial reward and decay functions. In this case,

$$C_T^{\text{opt}} = \sup_{0 < x < \infty} Bx^\rho \log_2(1 + \frac{KP}{\eta Bx^\gamma}). \quad (4)$$

This maximization was first considered in [2], however no expression for  $C_T^{\text{opt}}$  was calculated there, and only an asymptotic expression as  $\gamma \rightarrow \infty$  was offered.

By defining  $A \triangleq \frac{KP}{\eta B}$  and

$$f(x) \triangleq Bx^\rho \log_2(1 + \frac{A}{x^\gamma}), \quad (5)$$

we bring (4) to the form

$$C_T^{\text{opt}} = \sup_{0 < x < \infty} f(x) = \sup_{0 < x < \infty} Bx^\rho \log_2(1 + \frac{A}{x^\gamma}). \quad (6)$$

When  $\gamma < \rho$ , clearly  $\lim_{x \rightarrow \infty} f(x) = \infty$ , so that  $C_T^{\text{opt}} = \infty$ . When  $\gamma = \rho$ ,  $f(x)$  is monotonically increasing so that its supremum is approached as  $x \rightarrow \infty$ , and  $C_T^{\text{opt}} = AB \log_2(e)$ . However, as in [2], we are mostly interested in the case  $\gamma > \rho$ . (Indeed, in most environments  $\gamma > 2$ , and the case  $\rho = 1$  is the most relevant in wireless ad hoc networks.) In this case, as shown in Fig. 1,  $f(x)$  achieves a single maximum for an optimum value of  $x$ ,  $x_{\text{opt}}$ . We will develop expressions for both  $x_{\text{opt}}$  and  $C_T^{\text{opt}} = f(x_{\text{opt}})$ .

To find  $x_{\text{opt}}$ , we set the derivative of  $f(x)$  equal to 0:

$$\frac{\rho}{\gamma} \log(1 + \frac{A}{x^\gamma}) - \frac{A}{A + x^\gamma} = 0,$$

where by  $\log(x)$  we denote the natural logarithm of  $x$ . We make the substitution

$$y = \log(1 + \frac{A}{x^\gamma}) - \frac{\gamma}{\rho} \quad (7)$$

and after simplifying we arrive at:

$$ye^y = (-\frac{\gamma}{\rho})e^{(-\frac{\gamma}{\rho})} \triangleq z_0. \quad (8)$$

This equation is of the form  $ye^y = z_0$  where  $z_0$  is given and we must find  $y$ . In other words, solving (8) is equivalent to calculating the inverse of the function  $y \mapsto z = ye^y$ . This is actually a very old problem, predating Euler, who has himself worked on it [4]. The inverse is known in the literature as Lambert's  $W$  function, and it appears often in a variety of situations from the enumeration of trees in computer theory to the calculation of wave heights in physics [5].

In general  $W(z)$  is defined for complex  $z$  and is complex and multivalued. In our context, however, both  $z$  and  $W(z)$  are real, and the situation is relatively simple. In Fig. 2 we plot  $z = ye^y$  for real  $y$ . From the figure it is clear that for  $z \geq 0$   $W(\cdot)$  has a single branch, the curve on the right of point  $P_1$ . For  $z < -e^{-1}$  it has no branches, and for  $-e^{-1} \leq z < 0$  it has two branches,  $W_1(\cdot)$  and  $W_2(\cdot)$ . The branch  $W_1(\cdot)$  is the curve that lies on the left of point  $P_2$ , and the branch  $W_2(\cdot)$  is the curve that lies between the points  $P_1$  and  $P_2$ . Unfortunately, no closed-form expressions are known for the two branches. We define

$$g(z) \triangleq W_2(z) - W_1(z) \quad (\text{for } z \in [-e^{-1}, 0]) \quad (9)$$

The value  $z_0$  of (8) lies in the interval  $(-e^{-1}, 0)$ , so (8) has two solutions, one for each branch.  $W_1(z_0)$  is clearly equal to  $-\frac{\gamma}{\rho}$  and is not acceptable, since plugging this to (7) implies that  $x = +\infty$ . However  $W_2(z_0)$  is acceptable, as it leads to the following solution:

$$x_{\text{opt}} = \left[ \frac{A}{e^{W_2(z_0) + \frac{\gamma}{\rho}} - 1} \right]^{\frac{1}{\gamma}}.$$

Noting that  $W_2(z_0) + \frac{\gamma}{\rho} = g(z_0)$  and that  $A \triangleq \frac{KP}{\eta B}$ , we have that:

$$x_{\text{opt}} = \left[ \frac{KP}{\eta B} \right]^{\frac{1}{\gamma}} \left[ \frac{1}{e^{g(z_0)} - 1} \right]^{\frac{1}{\gamma}}. \quad (10)$$

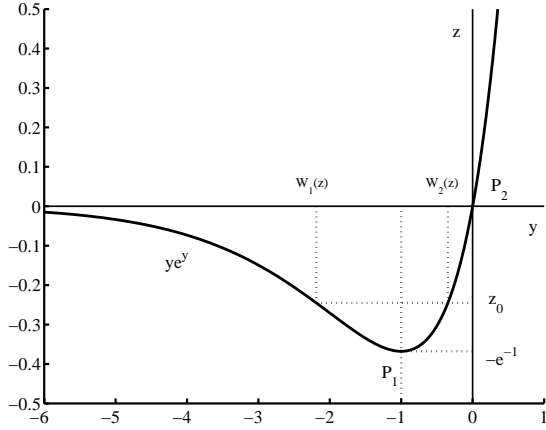


Fig. 2. The plot of the function  $ye^y$ .

The optimal transport capacity becomes

$$C_T^{\text{opt}} = B \left[ \frac{KP}{\eta B} \right]^{\frac{\rho}{\gamma}} G\left(\frac{\rho}{\gamma}\right), \quad (11)$$

where

$$G\left(\frac{\rho}{\gamma}\right) \triangleq \left[ e^{g(z_0)} - 1 \right]^{-\frac{\rho}{\gamma}} g(z_0) \log_2(e). \quad (12)$$

A few interesting observations can be made from (11). For example,  $\rho$  and  $\gamma$  affect the transport capacity only through their quotient  $\frac{\rho}{\gamma}$ . Therefore, changing both their values while leaving their quotient fixed does not change the value of the transport capacity. In addition, in contrast to Shannon capacity, the dependence of the transport capacity on the available bandwidth and transmitter power is monomial.

### III. THE MULTIPLE TRANSMITTER CASE

We now consider a multiple access network that consists of a single receiver  $R$ , placed at the origin, and  $n$  transmitters  $T_1, T_2, \dots, T_n$  placed along the  $x$ -axis, at points  $x_1 < x_2 < \dots < x_n$ . The receiver is subject to additive white Gaussian noise with spectral density  $\eta$ . Transmitter  $T_i$  can transmit with a maximum power  $P_i^1$ , and the total bandwidth available for communication is equal to  $B$ . For simplicity, we use the notation  $d_i \triangleq d(x_i)$  and  $r_i \triangleq r(x_i)$ .

The multiple access channel has attracted significant research interest in the past, and its capacity region  $\mathcal{C}$ , i.e., the set of all the combinations of communication rates that are simultaneously achievable, has been discovered [3]. In particular, each of the transmitters  $T_i$  can send data to the receiver with rate  $R_i$  as long as

$$\sum_{i \in \mathcal{I}} R_i \leq B \log_2 \left( 1 + \frac{\sum_{i \in \mathcal{I}} d_i P_i}{\eta B} \right) \quad \forall \mathcal{I} \subseteq \{1, 2, \dots, n\}. \quad (13)$$

Equations (13) show that the capacity region  $\mathcal{C}$  is a closed convex polyhedron. The number of vertices whose coordinates are *all* positive is exactly  $n!$ . Each of these vertices can be achieved by a successive interference cancellation scheme, in

<sup>1</sup>More formally, each codeword consisting of  $n$  channel uses is restricted to have an expected power equal to at most  $nP_i$ . Alternative constraints in a similar setting have been considered in [6].

which the signals from the  $n$  transmitters are decoded one by one. When decoding the signal of transmitter  $T_i$ , those signals that have already been decoded do not affect the decoding, however those signals that have not been decoded yet appear as additive white Gaussian noise to the receiver. In particular, consider a successive interference cancellation scheme in which the signal from  $T_{\pi(j)}$  is decoded  $j$ -th, and  $\pi(\cdot)$  is a permutation of the set  $\{1, 2, \dots, n\}$ . (Consequently,  $\pi^{-1}(i)$  is the rank with which the signal of  $T_i$  is decoded.) Then the components of the vertex  $V_\pi \triangleq (R_{1,\pi}, \dots, R_{n,\pi})$  are given by:

$$R_{i,\pi} = B \log_2 \left( 1 + \frac{d_i P_i}{\eta B + \sum_{k: \pi^{-1}(k) > \pi^{-1}(i)} d_k P_k} \right)$$

The **transport capacity** associated with the rate vector  $\{R_i\} \triangleq (R_1, R_2, \dots, R_n) \in \mathcal{C}$  is defined as

$$C_T(\{R_i\}) \triangleq \sum_{i=1}^n r_i R_i,$$

and the optimum transport capacity is

$$C_T^{\text{opt}} \triangleq \sup_{\{R_i\} \in \mathcal{C}} C_T(\{R_i\}) = \sup_{\{R_i\} \in \mathcal{C}} \sum_{i=1}^n r_i R_i. \quad (14)$$

The quantity that must be maximized is a linear function of the rates  $R_i$ , who in turn belong in the convex polyhedron  $\mathcal{C}$  defined by (13). Therefore, the maximization of (14) is a linear program, and the supremum is actually achieved in one of the  $n!$  vertices  $V_\pi$ . It would seem that the complexity of the problem increases factorially with the number of nodes, however because of the special structure of the problem, the solution is remarkable simple, as the next theorem shows:

*Theorem 1: The optimum transport capacity is achieved by a successive interference cancellation scheme under which the signal of transmitter  $T_j$  is decoded  $j$ -th. In other words, the optimal permutation  $\pi(\cdot)$  is the identity permutation  $\pi(i) = i$ . This result holds irrespective of the particular form of the functions  $r(\cdot)$  and  $d(\cdot)$ , as long as  $r(\cdot)$  is monotonically non-decreasing. Therefore, the optimum transport capacity is given by:*

$$C_T^{\text{opt}} = B \sum_{i=1}^n r_i \log_2 \left( 1 + \frac{d_i P_i}{\eta B + \sum_{j=i+1}^n d_j P_j} \right). \quad (15)$$

**Proof:** Let us assume that the transport capacity is achieved by using a permutation  $\pi_1(\cdot)$  other than the identity permutation. Therefore, there is a  $j_0$  such that  $k \triangleq \pi_1(j_0) > \pi_1(j_0 + 1) \triangleq l$ . We define a new decoding order, specified by the permutation  $\pi_2(\cdot)$ :

$$\pi_2(j) \triangleq \begin{cases} \pi_1(j+1) & \text{if } j = j_0, \\ \pi_1(j-1) & \text{if } j = j_0 + 1, \\ \pi_i(j) & \text{otherwise.} \end{cases}$$

Both orders of decoding achieve exactly the same transmission rate, for all transmitters other than  $T_k$  and  $T_l$ . Any difference in the transport capacities  $C_T(\{R_{i,\pi_2}\}) - C_T(\{R_{i,\pi_1}\})$  will be due to the different rates achieved by  $T_k$  and  $T_l$ . Let  $I$  be the combined noise and interference power that the receiver sees when decoding the signal coming from  $T_l$ , under the

original decoding order  $\pi_1$ . Also let  $p_k = d_k P_k$  and  $p_l = d_l P_l$ . Then:

$$\begin{aligned}
& \frac{1}{B} [C_T(\{R_{i,\pi_2}\}) - C_T(\{R_{i,\pi_1}\})] \\
&= r_k \log_2\left(1 + \frac{p_k}{I}\right) + r_l \log_2\left(1 + \frac{p_l}{I + p_k}\right) \\
&\quad - r_k \log_2\left(1 + \frac{p_k}{I + p_l}\right) - r_l \log_2\left(1 + \frac{p_l}{I}\right) \\
&= (r_k - r_l) [\log_2(I + p_k) + \log_2(I + p_l)] \\
&\quad + (r_l - r_k) [\log_2(I) + \log_2(I + p_l + p_k)] \\
&= (r_k - r_l) \log_2 \left[ \frac{I^2 + Ip_k + Ip_l + p_k p_l}{I^2 + Ip_k + Ip_l} \right] \\
&\geq 0.
\end{aligned}$$

Therefore, the transport capacity increases if we exchange the decoding orders of nodes  $k$  and  $l$ . Repeating the process, we can create a finite sequence of permutations  $\pi_1, \pi_2, \dots, \pi_m$  of increasing transport sum rate, with  $\pi_m$  being the identity permutation. The result follows.  $\square$

#### IV. MULTIPLE TRANSMITTERS WITH A SUM-POWER CONSTRAINT

Until now we have assumed that the power  $P_i$  available to each transmitter  $T_i$  is fixed, and is independent of the powers of other transmitters. We now assume that the transmitter powers are allowed to vary, but are subject to a global constraint. In particular, they must satisfy the inequality  $\sum_{i=1}^n P_i \leq P_0$ .

We are interested in discovering the distribution of powers that maximizes the transport capacity. In light of Theorem 1, our optimization problem is the following:

$$\begin{aligned}
& \text{maximize:} && B \sum_{i=1}^n r_i \log_2\left(1 + \frac{d_i P_i}{\eta B + \sum_{j=i+1}^n d_j P_j}\right) \\
& \text{subject to:} && \begin{cases} \sum_{i=1}^n P_i \leq P_0, \\ P_i \geq 0. \end{cases} \quad (16)
\end{aligned}$$

Such an optimization is relevant in a number of scenarios. For example, during the planning phase for a sensor network, the network planners might have a fixed upper bound on the total number of batteries, and they will need to know beforehand what is their optimum distribution. Alternatively, the network may have to comply with regulations that limit the total transmitted power (as is the case with networks operating in the ISM band).

The optimization problem (16) may easily be shown to be convex. In particular, define  $a_i \triangleq \eta B + \sum_{j=i}^n d_j P_j$  for  $i = 1, \dots, n+1$  (so that  $a_{n+1} = \eta B$ ), and  $r_0 \triangleq 0$ . Then:

$$\begin{aligned}
C_T &= B \sum_{i=1}^n r_i \log_2\left(\frac{a_i}{a_{i+1}}\right) \\
&= \sum_{i=1}^n [B(r_i - r_{i-1})] \log_2 a_i - B r_n \log_2(\eta B),
\end{aligned}$$

and the optimization problem (16) can be written as:

$$\begin{aligned}
& \text{maximize:} && \sum_{i=1}^n [B(r_i - r_{i-1})] \log_2 a_i - B r_n \log_2(\eta B) \\
& \text{subject to:} && \begin{cases} \sum_{i=1}^n P_i = P_0, \\ P_i \geq 0, \\ a_i = \eta B + \sum_{j=i}^n d_j P_j. \end{cases} \quad (17)
\end{aligned}$$

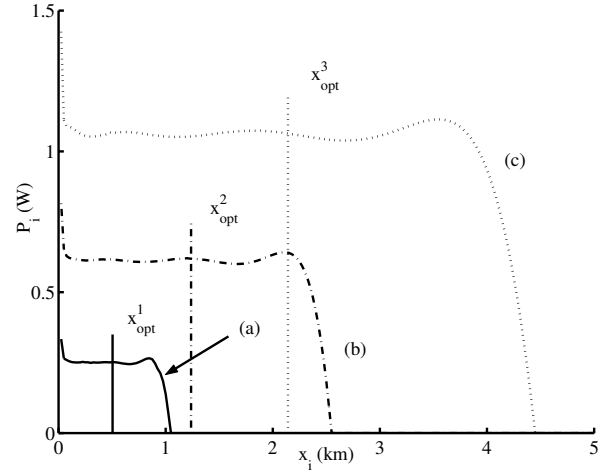


Fig. 3. The optimal power distribution for three different values of the total available power: (a)  $P_0^1 = 10$  W. (b)  $P_0^2 = 60$  W. (c)  $P_0^3 = 180$  W.

The vector  $(P_1, \dots, P_n, a_1, \dots, a_n)$  is the optimization variable. The objective function is easily shown to be concave by considering its Hessian (note that we have assumed that  $r(\cdot)$  is non-decreasing). Since all the constraints are linear, (17) is a convex problem. Therefore, there is no local maximum other than the global maximum, which in addition may be found very quickly, using a variety of methods [7].

As an example, we consider a network that consists of a single receiver, placed at the origin, and 200 transmitters, placed uniformly every 25 m along the  $x$ -axis, the closest of them being 25 m from the receiver and the furthest of them being 5 km from the receiver. The bandwidth available for communication is  $B = 10$  MHz, and the receiver is susceptible to additive white Gaussian noise with power spectral power density  $\eta = 10^{-16} \frac{\text{W}}{\text{Hz}}$ . The monomial reward and decay functions are used, with  $\rho = 1$ ,  $\gamma = 2$ , and  $K = 10^{-4} \text{ m}^2$ .

In Fig. 3 we plot the optimal power allocation that maximizes the transport capacity, for the three cases  $P_0^1 = 10$  W,  $P_0^2 = 60$  W,  $P_0^3 = 180$  W. In Fig. 4 we plot the contributions to the transport capacity of each of the transmitters. In the figures, we have also denoted the optimal positions  $x_{\text{opt}}^1, x_{\text{opt}}^2, x_{\text{opt}}^3$ , for the case of a single transmitter with power  $P_0^1, P_0^2$ , and  $P_0^3$  respectively. These values are given by (10).

The form of the solutions is remarkably simple: the optimal power allocation is very close to a uniform distribution of power among all the nodes that are between the origin and a distance equal to twice the distance which is the optimum for the single transmitter case. With this power allocation, all nodes in this range are ensured the same bps  $\times$  meters to the transmitter, as shown in Fig. 4.

In all three cases, the gains, in terms of transport capacity, of moving from a single transmitter to multiple transmitters are relatively modest: The transport capacity goes from 11.6 kbps  $\times$  km to 14.3 kbps  $\times$  km in the first case, from 28.4 kbps  $\times$  km to 35.2 kbps  $\times$  km in the second case, and from 49.2 kbps  $\times$  km to 61.1 kbps  $\times$  km in the third case, a gain of approximately 24% in all three cases. However, note that the multiple transmitter scheme offers an improvement even though most of the active transmitters lie in positions that would not be at all favorable, if these transmitters were by themselves in the network.

In Fig. 5 we redraw the power distribution for the case

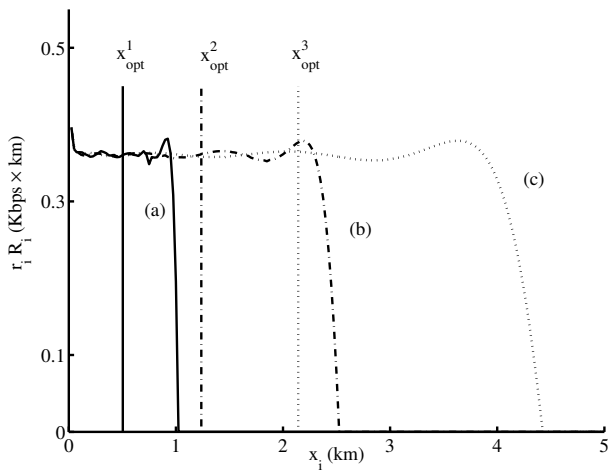


Fig. 4. The contributions  $r_i R_i$  of the transmitters to the transport capacity that are induced by the power distributions of Fig. 3.

$P_0^3 = 180$  W, and the power distribution under the same conditions, but with a different distribution of nodes. In particular, transmitters are placed every 25 m within the ranges (200 m, 1800 m) and (2500 m, 3800 m). As is evident from Fig. 5, the optimal strategy is that the nodes now lying in the extremes of the two ranges use all the power that was allocated to the nodes that were removed from the system. A very similar phenomenon was observed in [2] for the case of the broadcast channel. Despite the removal of a significant amount of nodes, the transport capacity is not reduced significantly; it changes from 61.1 Kbits  $\times$  km to 60.3 Kbits  $\times$  km, showing that the system is very robust and adapts very well to the removal of transmitters.

## V. CONCLUSIONS

In this paper, we focus on the maximization of the transport capacity, first in the case of a single transmitter-receiver pair, and then in the case of a multiple access network consisting of a single receiver and many transmitters.

In the case of a single transmitter-receiver pair, we calculate the distance between the two that maximizes the transport capacity, assuming a monomial reward function of the form  $r(x) = x^\rho$ , and a monomial power decay function of the form  $d(x) = \frac{K}{x^\gamma}$ . The solution is given in closed form, using Lambert's  $W$  function.

In the case of the multiple access network, we show that, in order to maximize the transport capacity, the received signals should be decoded successively, with the order of the decoding being the same as the order of the transmitter placements. In other words, the signal of the transmitter who is closest to the receiver should be decoded first, and so on. In the case where the transmitter powers are allowed to vary, but are subject to a constraint on their sum, the optimal power allocation can be determined by solving a convex optimization problem. Our preliminary investigation suggests that the solutions have a remarkably simple structure.

This work only scratches the surface of a very important topic that, with a few notable exceptions [1], [2], [8] has remained largely unexplored. Therefore, there are many possible lines of future research. A natural next step would be the embedding of our line of research into a more challenging interference environment with many transmitters and many

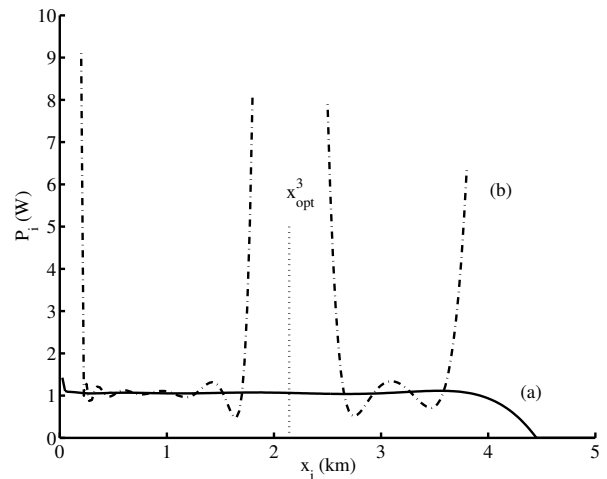


Fig. 5. The optimal power distributions with a total available power equal to  $P_0^3 = 180$  W. (a) 200 transmitters are placed uniformly, every 25 m, in the interval [25 m, 5000 m]. (b) 118 transmitters are placed uniformly, every 25 m, in the intervals [200 m, 1800 m] and [2500 m, 3800 m].

receivers. This area is the subject of [8], however because the capacity regions of more challenging topologies are not yet known, the focus there is toward the establishment of order laws that work in the limit of a large number of nodes.

An alternative future line of research would be toward the establishment of the optimal transport capacity when the locations of the transmitters are allowed to vary. Such an investigation would shed light on the spatial configurations that promote the high utilization of network resources.

## ACKNOWLEDGMENT

The authors would like to thank Prof. Sriram Vishwanath, of UT Austin, for pointing us to [9], which contains the idea of Theorem 1 in a different context, and for many useful discussions, while he was a visitor at the Telecommunications Research Center Vienna.

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