ON THE ABILITY OF CDMA TO COMBAT LOG–NORMAL FADING

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Abstract

In contrast to the common opinion, that CDMA offers only advantages to overcome frequency–selective fading, in this paper, it is shown that CDMA can combat log–normal fading (long–term fading), too. This statement holds under the following two conditions: First, equal transmission rates for all users are presupposed. Second, interference cancellation is applied to combat multiple access interference. Including channel coding close to capacity limit, CDMA may even outperform orthogonal multiple–access in this case.

1 Introduction

In time–division multiple–access (MA) or frequency–division MA, the signals of different users are orthogonal ($\perp$). This implies that minimization of required signal energy can be performed independently among the users, if they are presupposed to transmit equal amount of information. If every user transmits with minimum possible energy, the overall transmit energy is minimized and the system design is called to be optimum. Obviously, there is no need to distinguish receive and transmit energy because of their proportionality. Thus, for simplicity, receive energy is used to be considered in literature, even though the actual goal is minimization of transmit energy.

In code–division (CD) MA the conditions mentioned above usually are not fulfilled, and the optimization problem is no longer separable among the users. Therefore, a more detailed analysis is given in Section 2 and 2.2 for receive and transmit energy, respectively.

It is well–known that orthogonality is not essential to achieve total capacity of the Gaussian MA channel for a fixed ratio of *receive* energy to noise–power–density [1]. For CDMA with random signature waveforms[1], a possible way to achieve capacity is application of interference cancellation (IC) [2]. Although this is an important result, it cannot be concluded that CDMA and $\perp$MA are also equivalent if the ratio *transmit* energy to noise–power–density is considered. In particular, this equivalence is lost if the signal attenuations for at least two users differ. This result may be somewhat surprising, as there is the same relationship between receive and transmit energy for each user in both, $\perp$MA and $\not\perp$MA. This paradox is resolved by considering the different signal attenuations for all users due to long–term fading. On the one hand, for $\perp$MA the required total transmit energy is not affected by the specific distribution of signal attenuations to the different users. One the other hand, IC for $\not\perp$MA needs the receive energies to be ordered properly. Thus, both the distribution of signal attenuation and the ordering of the users strongly influence the overall transmit energy in a $\not\perp$MA–system.

The problem of minimizing total transmit energy for CDMA with randomly chosen signature waveforms (as an example of $\not\perp$MA) and path loss within a circular cell (as an example of different signal attenuations among the users) has been addressed in [3]. In contrast to this, we neglect the effects of path loss in this paper and exclusively focus on signal attenuation by long–term fading. In addition to the considerations in Section 2, Section 3 includes channel coding and gives insight into the influences of several system paprameters. Hereby, the optimum system design turns out to be very disadvantageous for practical implementation. Therefore, in Section 4 we investigate the drawback which results if system parameters are chosen suboptimum in order to get system design more suitable with respect to the effort required for signal processing. Finally, Section 5 points out some conclusions.

2 Energy Distributions

For simplicity, our considerations are restricted to transmission over the additive white Gaussian noise (AWGN) channel, but they can easily be generalized to other channel models by replacing Eq. (14) by their corresponding counterparts. Without loss of generality it is assumed that K users are numbered in the inverse order of being demodulated and cancelled. IC is assumed to be imperfect which is represented by a factor β denoting the quotient of interference power after and before cancellation. In an implemented system there are various reasons, why IC is not perfect, e.g. imperfect estimation of fading

[1]In this paper CDMA is used as synonym for CDMA with random signature waveforms.

amplitudes. Therefore, one user's part of the received signal cannot be reconstructed exactly, even if it has been decoded correctly.

2.1 Receive Energy

If all users apply randomly chosen signature sequences, the average signal–to–interference ratio of user κ including imperfect IC and additive noise is given by, see e.g. [2, 4],

$$\mathrm{SIR}_\kappa = \frac{E_\kappa}{\mathcal{N}_0 + \sum_{i=1}^{\kappa-1} \frac{E_i}{N} + \sum_{i=\kappa+1}^{K} \frac{\beta E_i}{N}}. \quad (1)$$

Here $\mathcal{N}_0$, E_κ, and N are denoting one–sided noise power density, receive energy per symbol of user κ, and the spreading factor, respectively. The average signal–to–interference ratio is a reasonable measure, as for large N the variance of the signal–to–interference ratio vanishes. Now we assume, that all users (want to) transmit at the same information rate R. Using the Gaussian interference approximation[2] this implies that E_κ has to be controlled in such a way that

$$\mathrm{SIR}_\kappa = \mathrm{SIR} \quad \forall\, \kappa \quad (2)$$

is valid for all users. This enables us to rewrite Eq. (1) into

$$E_\kappa - E_{\kappa-1} = \mathrm{SIR}\left(\frac{E_{\kappa-1}}{N} - \frac{\beta E_\kappa}{N}\right). \quad (3)$$

The solution of the difference equation (3) can be simplified for large N via Taylor series to

$$\begin{aligned} E_\kappa &= E_1 \exp\left((\kappa-1)\log\frac{1+\mathrm{SIR}/N}{1+\beta\mathrm{SIR}/N}\right) \quad (4) \\ &\xrightarrow{N \gg \mathrm{SIR}} E_1 \exp\left(\frac{\kappa-1}{N}\mathrm{SIR}\,(1-\beta)\right). \end{aligned}$$

Eq. (4) describes the optimum distribution of the receive energy per symbol among the users in a CDMA system applying IC, cf. [2, 3]. For $\beta < 1$, i.e. IC operating, this is not a uniform but an exponentional distribution. In order to achieve the complete performance gain of IC, it is very important to provide the energy distribution established by Eq. (4) by power control.

2.2 Transmit Energy

In Section 2 the optimum *receive* energy distribution for CDMA has been given. Now, we will use this result to calculate the optimum *transmit* energy distribution. For simplicity, we assume that there are so many users, that the law of large numbers can be applied to the users' fading amplitudes, i.e. the relative frequency tends to the probability density function. This implies, that the ratio κ/K is no longer a quantized variable, but a continuous one. Moreover, we assume the signals to fade slowly enough, so that power control can perfectly provide the optimum nonuniform receive energy distribution, see Eq.(4). As long–term fading provides a strongly nonuniform distribution of signal attenuations among the users, only small additional adjustment by power control is actually needed to provide the optimum receive energy distribution. For this purpose, the user indices have to be properly mapped to the indices of the fading processes.

Considering only long–term fading the users' logarithmic attenuation $\log D$ is Gaussian (normal) distributed with mean $\mu_{\log D}$ and variance $\sigma^2_{\log D}$ [5]. Therefore, the probability of the logarithmic attenuation $\log D$ being larger than user κ's minimal logarithmic attenuation $\log d_\kappa$ is

$$\begin{aligned} &\Pr\{\log D > \log d_\kappa\} = \\ &= \int_{\log d_\kappa}^{\infty} \exp\left(-\frac{(\log D - \mu_{\log D})^2}{2\,\sigma^2_{\log D}}\right)\frac{d(\log D)}{\sqrt{2\pi}\sigma_{\log D}} \quad (5) \\ &= \mathrm{Q}\left(\frac{\log d_\kappa - \mu_{\log D}}{\sigma_{\log D}}\right) \quad (6) \end{aligned}$$

with $\mathrm{Q}(x) \overset{\mathrm{def}}{=} \int_x^\infty \exp(-t^2/2)\,dt/\sqrt{2\pi}$ denoting the Gaussian error probability function. Proper mapping of user indices to signal attenuations forces

$$\Pr\{\log D > \log d_\kappa\} = \frac{\kappa}{K}, \quad (7)$$

implying

$$\log d_\kappa = \sigma_{\log D}\,\mathrm{Q}^{-1}\left(\frac{\kappa}{K}\right) + \mu_{\log D} \quad (8)$$

with $\mathrm{Q}^{-1}(x)$ denoting the inverse Gaussian error probability function.

Using Eqs. (4), (8), and the K/N–ratio $\zeta \overset{\mathrm{def}}{=} K/N$, the optimum *transmit* energy distribution

$$\tilde{E}_\kappa = E_\kappa d_\kappa \quad (9)$$

for large N and K is given by

$$\begin{aligned} \tilde{E}_\kappa &= E_1 \exp\left(\frac{\kappa}{K}\zeta\,\mathrm{SIR}\,(1-\beta)\right. \quad (10) \\ &\left. + \sigma_{\log D}\,\mathrm{Q}^{-1}\left(\frac{\kappa}{K}\right) + \mu_{\log D}\right). \end{aligned}$$

This distribution is plotted in Fig. 1 for various standard deviations of the log–normal fading. In the unfaded case ($\sigma_{\log D} = 0$) the exponential distribution occurs. Involving weak log–normal fading the transmit energy distribution becomes flatter[3],

[2]For an exponential energy distribution, as derived to be optimum in the following, the central limit theorem is not valid. Therefore, Gaussian interference implies Gaussian distributed signals of all users.

[3]Obviously, in the presence of log–normal fading the transmit energy distribution shows a pole for $\kappa/K \to 0$. This is no problem for the following theory, as the integral over the pole does exist, cf. Section 3. In implemented systems, however, this leads to nonzero outage probability.

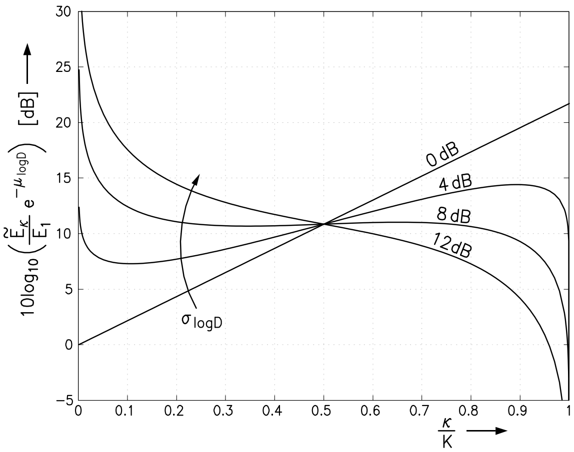

Figure 1: Normalized transmit energy distribution for $\zeta\,\mathrm{SIR}\,(1-\beta)=5$ and different values of the fading standard deviation $\sigma_{\log D}$ =0, 0.92, 1.84, and 2.76, respectively, which is equivalent to 0 dB, 4 dB, 8 dB, and 12 dB, respectively. Typical for mobile radio transmission is 6 to 8 dB [5].

which implies both less total transmit energy thus increasing system capacity and less peak transmit energy thus simplifying power amplifier design. But this effect turns into the opposite, if fading is too strong. Then, fading dominates the exponential receive energy distribution, and the transmit energy distribution is no longer flat. Now the question is what determines whether log–normal fading being too strong or not. The receive energy distribution and signal attenuation have to balance each other. In first order approximation this means

$$\zeta\,\mathrm{SIR}\,(1-\beta) \approx -\sigma_{\log D}\,\frac{d}{dx}\mathrm{Q}^{-1}(x)\Big|_{x=1/2} \qquad (11)$$

$$\zeta \approx \frac{\sqrt{2\pi}\sigma_{\log D}}{(1-\beta)\,\mathrm{SIR}}. \qquad (12)$$

Now it becomes obvious, why CDMA with IC may offer nice advantages over $\perp$MA in the presence of log–normal fading: For $\perp$MA the receive energy distribution has to be uniform, implying that the transmit energy distribution is not flat. But by applying CDMA, the receive energy distribution can be matched to the distribution of the signal attenuation via chosing $\zeta = K/N$ properly, i.e. according to approximation (12).

3 Total Multiuser Capacity

For simplicity, we restrict our subsequent considerations to perfect IC ($\beta=0$), implying, cf. Eq. (1),

$$E_1 = \mathrm{SIR}\,\mathcal{N}_0. \qquad (13)$$

Furthermore, we assume that all users apply single user channel coding close to capacity of the (complex) AWGN channel [6]

$$R = \log_2(1+\mathrm{SIR}). \qquad (14)$$

This assumption is reasonable, since in single user systems transmission close to capacity limit is possible by multilevel coding and Turbo Codes [7]. Here, the assumption of Gaussian noise is also applied to the multiple–access interference (MAI). Furthermore, in CDMA systems total multiuser capacity is given by $\mathit{\Gamma}=R\zeta = R\cdot K/N$ [4]. Therefore, applying Eqs. (10), (13), and (14), we obtain

$$\tilde{E}_\kappa = (2^{\mathit{\Gamma}/\zeta}-1)\mathcal{N}_0 \exp\left(\frac{\kappa}{K}\zeta\,(2^{\mathit{\Gamma}/\zeta}-1) + \sigma_{\log D}\,\mathrm{Q}^{-1}\left(\frac{\kappa}{K}\right) + \mu_{\log D}\right). \qquad (15)$$

Eq. (15) contains the K/N–ratio ζ as a free parameter. As the overall goal is to minimize the average transmit energy per information bit

$$\tilde{E}_\mathrm{b} = \frac{1}{K}\sum_{\kappa=1}^{K}\frac{\tilde{E}_\kappa}{R} = \frac{1}{K\mathit{\Gamma}}\sum_{\kappa=1}^{K}\zeta\tilde{E}_\kappa, \qquad (16)$$

the parameter ζ has to be chosen to minimize the expression $\zeta(2^{\mathit{\Gamma}/\zeta}-1)$, as $\tilde{E}_\mathrm{b}$ is a monotonously increasing function of $\zeta(2^{\mathit{\Gamma}/\zeta}-1)$. From channel capacity of the AWGN channel it is well known, see e.g. [1], that

$$\min_{1/\zeta}\left\{\frac{2^{\mathit{\Gamma}/\zeta}-1}{\mathit{\Gamma}/\zeta}\right\} = \log 2, \qquad (17)$$

with infinite ζ as optimum argument. Now, this result is used to obtain the required minimum average transmit energy per information bit to achieve total multiuser capacity $\mathit{\Gamma}$

$$\inf_\zeta\left\{\tilde{E}_\mathrm{b}\right\} = \qquad (18)$$

$$\mathcal{N}_0\int_0^{\log 2}\exp\left(\mathit{\Gamma}t + \sigma_{\log D}\,\mathrm{Q}^{-1}\left(t/\log 2\right) + \mu_{\log D}\right)dt.$$

Therefore, the minimum required transmit energy to noise power density ratio of CDMA is given by the integral in Eq. (18) using the substitution $x = \mathrm{Q}^{-1}(t/\log 2)$

$$\left(\frac{\tilde{E}_\mathrm{b}}{\mathcal{N}_0}\right)_\mathrm{CDMA} = \log 2\,\frac{\mathrm{e}^{\mu_{\log D}}}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}\exp\left(\mathit{\Gamma}\mathrm{Q}(x)\log 2 + \sigma_{\log D}x - x^2/2\right)dx. \qquad (19)$$

In contrast to $\not\perp$MA, for $\perp$MA the optimum receive energy has to be equal for all users. Using Eq. (3.323.2) in [8] we obtain in this case:

$$\left(\frac{\tilde{E}_\mathrm{b}}{\mathcal{N}_0}\right)_{\perp\mathrm{MA}} = \frac{2^{\mathit{\Gamma}}-1}{\mathit{\Gamma}}\cdot\frac{\mathrm{e}^{\mu_{\log D}}}{\sqrt{2\pi}}\cdot \qquad (20)$$

$$\cdot\int_{-\infty}^{+\infty}\exp\left(\sigma_{\log D}x - x^2/2\right)dx$$

$$= \frac{2^{\mathit{\Gamma}}-1}{\mathit{\Gamma}}\exp\left(\sigma_{\log D}^2/2 + \mu_{\log D}\right). \qquad (21)$$

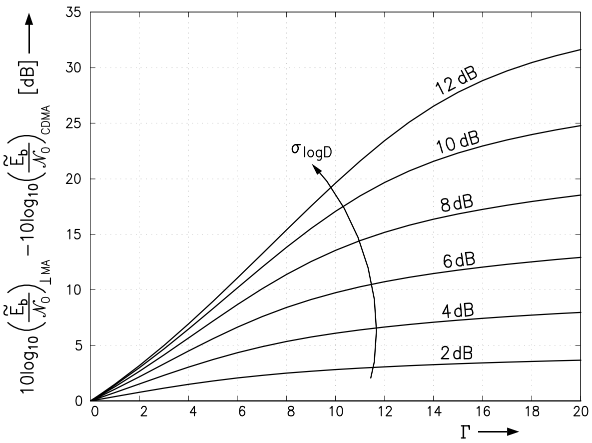

Figure 2: Gain in power efficiency of CDMA in comparison to $\perp$MA versus total multiuser capacity for different log–normal fading standard deviations $\sigma_{\log D} = 0.46, 0.92, 1.38, \ldots, 2.76$, respectively, which are equivalent to $2, 4, 6, \ldots, 12$ dB, respectively.

In Fig. 2, the gain in transmit power efficiency of CDMA in comparison to $\perp$MA is plotted versus total multiuser capacity Γ. Note that the gain increases with larger fading variance $\sigma^2_{\log D}$ and larger capacity Γ. In mobile communications typical values of the fading standard deviation are about 6 to 8 dB, cf. [5], implying about 10 to 20 dB gain for high multiuser capacity. Of course, this large gain is only achievable by an enormous increase in receiver complexity, as high spreading factors and a huge number of users are required. Nevertheless, we suggest this gain should be used in future mobile radio systems, as bandwidth is becoming a more and more expensive resource and complexity less and less important.

3.1 Why to Spread at All?

Previously, we discussed the optimum trade–off between spreading and coding. Obviously, the optimum choice is to perform all bandwidth extension by coding and not by spreading, as one can conclude by the fact that the optimum value for ζ is infinity. This is not surprising, as spreading is simply a repetition code which does not get any coding gain. So only coding can provide reaching the total capacity.

Our calculations are only valid for $N \gg$ SIR, cf. Eq. (4). Is this a restriction implying large spreading factor? Code rate $R \to 0$ which has been shown to be optimum implies SIR $\to 0$. Therefore, spreading factor $N = 1$, i.e. no spreading, is sufficient to establish validity of Eq. (4) in coded systems.

On the other hand there is no drawback by applying spreading, if the number of users grows over all bounds [3]. But is it necessary? In Section 2.2, we assumed a smooth continuous distribution of the fading amplitudes justified by a large number of users which has to be mapped to the energy distribution of the users. Therefore, the energy distribution has to be smooth and continuous, too. Recovering Eq. (4) you can see, that both the spreading factor and the SIR can provide this. Spreading involves additional complexity. Therefore, one would like to avoid spreading in coded systems as much as possible, implying not to spread on the AWGN channel at all. However, considering other channel models like Rayleigh or Ricean fading channels spreading can be used to exploit diversity gains.

3.2 Heuristic Interpretation

In Section 3.1 spreading has been shown to be unnecessary. This allows an interpretation of the results of Section 3 which is rather heuristical, but easy to understand. In an $\perp$MA scheme all users require the same signal–to–noise ratio depending only on their information rate. In $\not\perp$MA there is much more bandwidth available, implying that at least the user demodulated last (who is not affected by MAI, because all interference is cancelled) needs less signal–to–noise ratio than the users in an orthogonal system. Of course, the user demodulated first requires higher signal–to–noise ratio than an orthogonal user. Although he can use more bandwidth, he is disturbed by severe MAI. Calculating the average required energy over all users the equivalence of $\perp$MA and $\not\perp$MA due to receive energy becomes obvious. This is a consequence of the fundamental property of the capacity function of the AWGN channel [1]

$$\mathrm{C} : x \quad \mapsto \quad \log(1 + x) \tag{22}$$

$$\Downarrow$$

$$\mathrm{C}\left(\frac{E_1 + E_2}{\mathcal{N}_0}\right) = \mathrm{C}\left(\frac{E_1}{\mathcal{N}_0}\right) + \mathrm{C}\left(\frac{E_2}{\mathcal{N}_0 + E_1}\right). \tag{23}$$

The different signal attenuations affecting the different users can be mapped in an intelligent manner to the different required signal–to–noise ratios in a non–orthogonal system. In orthogonal systems however no such optimization by mapping is possible. Therefore, the performance of a properly designed $\not\perp$MA scheme cannot be reached.

4 Implementational Aspects

In Section 3 the optimum system design has been proposed. Hereby, no respect to implementational aspects has been taken. Unfortunately, the optimum system design is not suitable in practice. Therefore, in the following we investigate the performance loss due to realistic system parameters.

4.1 Bounded Rate

It has been shown that the required transmit energy is minimized if the K/N–ratio grows over all bounds, i.e. transmission rate R converges to zero. Obviously, this is impractical for implementation. Therefore, we have to look at the penalty to be paid for lower bounded rate ($R > 0$). This can be done by a straightforward specialization of Eq. (15). Therefore, no detailed analysis is presented here, but the results are

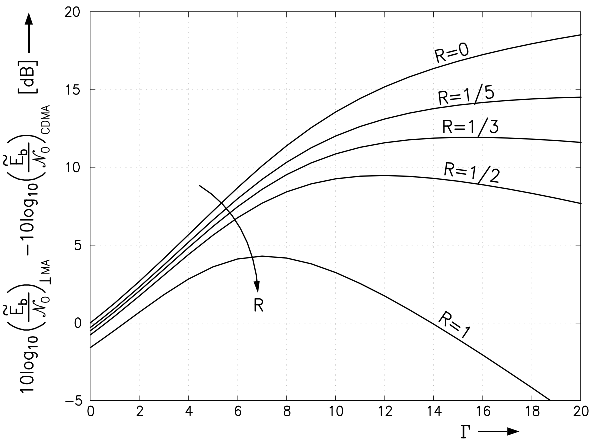

Figure 3: Gain in power efficiency of CDMA in comparison to ⊥MA versus total multiuser capacity for fixed $\sigma_{\log D} = 1.84$ (8 dB) with transmission rate $R = 0, 1/5, 1/3, 1/2, 1$, respectively.

summarized in Fig. 3. It can be observed, that with rate 1/5 or 1/3 bit/(s·Hz) which may easily be achieved by Turbo Codes [9], there still remains a large gain of CDMA in comparison to ⊥MA if total capacity Γ is not very small.

Nevertheless, we should note that another problem may occur. Bounded rate requires some bandwidth extension by spreading to achieve validity of Eq. (4), as the considerations of Section 3.1 are no longer valid. However, the question whether bounded rate indeed needs additional spreading can only be answered by a more sophisticated analysis which avoids the approximation $N \gg \mathrm{SIR}$ and has not been performed, yet.

4.2 Imperfect Coding

In all the previous analyses perfect channel coding arbitrarily close to capacity limit of the AWGN channel has been considered. This has been justified by the fact, that transmission close to less than 1 dB to channel capacity is possible using Turbo Codes [9]. This means systems employing ⊥MA are less than 1 dB behind *their* theoretical limit. If properly designed coded modulation is used this statement even holds for arbitrary trade–off between power and bandwidth efficiency, cf. [7].

Unfortunately, non–orthogonal systems using interference cancellation are more affected by imperfect channel coding. In order to proof this, we introduce an energy loss $V < 1$. With R^{Sub} denoting the maximum code rate of a suboptimal channel code Eq. (14) reads

$$R^{\mathrm{Sub}} = \log_2(1 + V\,\mathrm{SIR}). \qquad (24)$$

Following the argumentation from Eq. (15) to (21) it is straightforward to derive the transmit energy to noise power density ratios corresponding to imperfect

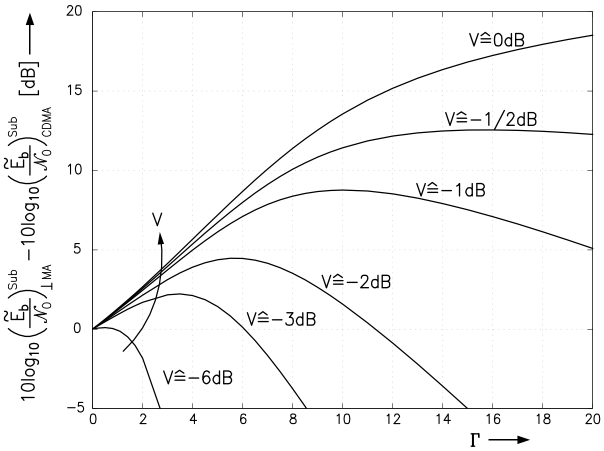

Figure 4: Gain in power efficiency of CDMA in comparison to ⊥MA versus total multiuser capacity for fixed $\sigma_{\log D} = 1.84$ (8 dB) with imperfect channel coding $10\log_{10} V = 0, -1/2, -1, -2, -3, -6$ dB, respectively.

channel coding for CDMA and ⊥MA, respectively:

$$\left(\frac{\tilde{E}_{\mathrm{b}}}{\mathcal{N}_0}\right)^{\mathrm{Sub}}_{\perp\mathrm{MA}} = \frac{2^{\Gamma}-1}{\Gamma V} \exp\left(\sigma^2_{\log D}/2 + \mu_{\log D}\right). \qquad (25)$$

$$\left(\frac{\tilde{E}_{\mathrm{b}}}{\mathcal{N}_0}\right)^{\mathrm{Sub}}_{\mathrm{CDMA}} = \frac{\log 2}{V} \cdot \frac{\mathrm{e}^{\mu_{\log D}}}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \exp\left(\frac{\Gamma \mathrm{Q}(x) \log 2}{V} + \sigma_{\log D}\, x - x^2/2\right) dx \qquad (26)$$

The effect of imperfect channel coding to ⊥MA is the same as in single user systems, see Eq. (25). For CDMA applying IC a much more serious degradation occurs, as the energy loss V takes place in the exponent, too. In the power–bandwidth plane, this means, that the total capacity curve is not only shifted to higher signal–to–noise ratios like in the orthogonal case, but that its slope is decreased, too.

The penalty to be paid for imperfect channel coding is depicted in Fig. 4. Considering Turbo Codes being 1 dB behind capacity, the gain due to ⊥̸MA occurs over a large range of spectral efficiency. But using convolutional codes with a loss of about 6 dB to capacity limit there is only a negligible gain for low spectral efficiency and a huge loss in all other cases. Therefore, searching for codes which are even closer to channel capacity than Turbo Codes is not only a kind of academic competition among scientists, but even of practical interest in systems which try to combat log–normal fading by ⊥̸MA.

5 Conclusions

Considering log–normal fading and users transmitting at equal information rate, CDMA applying perfect IC has been shown to achieve higher performance than orthogonal transmission schemes, although in this paper multipath fading, the widely known area for CDMA to achieve good performance in comparison to $\perp$MA, was not addressed. In order to achieve a significant fraction of this gain in practice, channel coding operating very close to channel capacity has turned out to be very important.

The key idea to performance improvement by $\not\perp$MA is flattening of the transmit energy distribution by correlations between the signature waveforms. Using randomly chosen signature waveforms, the transmit energies of most of the users are only approximately equal. This is an encouraging result for further investigations on design of signature waveforms for coded transmission over multiple–access channels similar to [10]: First, the traditional quality criterion, low cross correlation, is not the only reasonable in theory of coded mobile communications. Second, randomly chosen signature waveforms only approximately fulfill the requirements caused by long–term fading.

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