

**Multiuser Detection of CDMA by Iterated
Soft-Cancellation
(Turbo Multiuser Detection)**

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Zusammenfassung. Mehrteilnehmerdetektion durch iterierte Elimination der Interferenz mittels weicher Entscheidungen ist als Verfahren bekannt, das nahezu die Qualität eines Einteilnehmersystems erreicht, wenn die Interferenz schwach bis mäßig ist. In diesem Aufsatz wird gezeigt, wie dieses Detektionsverfahren durch eine adaptive Anpassung der weichen Entscheidungsregel weiter verbessert werden kann. Dies ermöglicht die Detektion von wesentlich mehr Benutzern, die gleichzeitig im selben Frequenzband senden. Daraus resultiert eine Verbesserung der Bandbreiteneffizienz.

Abstract. Multiuser detection by iterated soft–decision interference cancellation is known to achieve almost single user performance in low and moderate interference scenarios. In this paper, it is shown how this detection scheme can be further improved by adaptive calculation of the soft–decision rule. This enables a significantly higher number of active users signaling within the same bandwidth to be detected, implying that spectral efficiency is increased.

1 Introduction

Code–division multiple–access (CDMA) is a promising multiple–access technique for future mobile communication systems. Its main advantages are firstly a wideband transmitter signal reducing the negative effects of frequency–selective fading, and secondly the fact, that in cellular environments the same carrier frequency can be used in every cell. An important drawback of CDMA systems is the distortion caused by multiple–access interference (MAI). Since the pioneering work of Verdú (1986), who has shown that single user performance can be approached by the cost of complexity increasing exponentially with the number of users, there has been spent much effort in finding less complex demodulator structures that combat degradation due to MAI.

Linear demodulators, e.g. (Lupas and Verdú, 1990; Xie et al., 1990; Rupf et al., 1994; Madhow and Honig, 1994), show significant performance gains in comparison to conventional demodulation where MAI is treated as additive white Gaussian noise (AWGN). Further performance improvement is possible by the use of interference cancellation, cf. Duel–Hallen (1993). The basic idea of this concept is to detect for a certain user, remodulate its signal, and then to subtract it from the received signal. This implies, that the cancelled user’s signal does not disturb the subsequently detected users, if it has been detected correctly. Hereby, starting with detection of the stronger users prior to the weaker ones (channel sorting) is reasonable. Obviously, this procedure can be iterated after the detection for all users by detecting for the first one again, cf. Varanasi and Aazhang (1991). This is reasonable, as the first user’s signal can now be detected under much less interference than for its first detection. An excellent overview concerning this types of suboptimum multiuser detectors has been given by Duel–Hallen et al. (1995).

Significant performance improvement of iterated interference cancellation can be achieved, if hard detection is replaced by soft–decision. This was firstly introduced in an indirect way by considering Hopfield neural networks for multiuser detection (Kechriotis and Manolakos, 1996; Nagaosa et al., 1996; Teich and Seidl, 1996), which in this case do exactly the same as iterated soft–decision interference cancellation (ISDIC). Their good performance may be understood taking into account, that they work in a similar way as the decoder for turbo codes, introduced by Berrou et al. (1993). Therefore, they can be considered as a kind of *turbo multiuser detectors*.

In all publications known to the author, the soft–decision rule is not adapted to the decreasing interference power during the iterations. Therefore, the real possibilities of ISDIC have not been fully highlighted until now. In Section 2 the MMSE optimized soft–cancellation rule is derived. Section 3 will show, that it is possible

¹This work was presented in part at the 9th Tyrrhenian International Workshop on Digital Communications, Lerici, Italy, 1997.

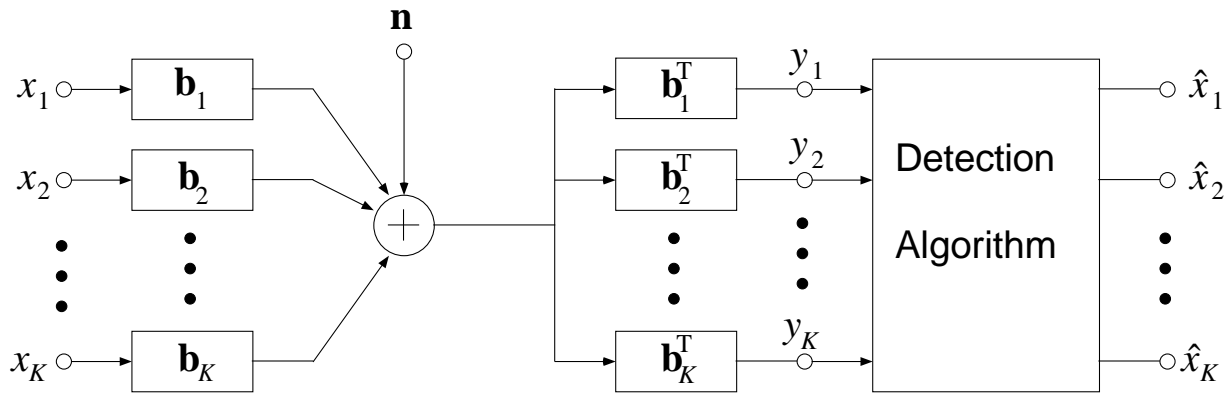


Figure 1. Modell of code-division multiple-access communications.

not only to achieve power efficiency of an orthogonal (interference free) system, but spectral efficiency, too. In Section 4 and 5 the influence of the processing gain to the performance of ISDIC is examined and fading channels are addressed, respectively. Finally, Section 6 points out the conclusions.

2 MMSE Optimized Soft-Cancellation

In the following we consider simultaneous transmission of K synchronous CDMA signals over the discrete time real valued AWGN channel as depicted in Fig. 1. As the complex valued AWGN channel can be split up into two independent real valued channels, this is no restriction of generality. The randomly chosen spreading sequences of length N are represented by the $N \times K$ matrix $\mathbf{B} = [\mathbf{b}_1, \mathbf{b}_2, \dots, \mathbf{b}_K]$. The transmission of one symbol is disturbed by N samples of AWGN with variance σ_n^2 , represented by the $N \times 1$ vector \mathbf{n} . Denoting the transmitted symbols of the K users by the $K \times 1$ vector² $\mathbf{x} = [x_1, x_2, \dots, x_K]^T \in \{\pm 1\}^K$, the received signal can be represented without loss of generality (Verdú, 1986; Rupf et al., 1994) by

$$\mathbf{y} = \mathbf{B}^T(\mathbf{B}\mathbf{x} + \mathbf{n}) \quad (1)$$

with \cdot^T denoting matrix transposition. Then ISDIC is defined by the algorithm for calculating an estimate $\hat{\mathbf{x}}$ for the transmitted vector \mathbf{x} shown in Fig. 2. State of the art is using the functions $g(\cdot) = \tanh(\cdot)$ and $f(\mathbf{R}, \cdot, \sigma_n^2, k) = c$ with the constant c being optimized empirically, cf. (Kechriotis and Manolakos, 1996; Nagaosa et al., 1996; Teich and Seidl, 1996). Now, a more reasonable choice of the function $f(\cdot)$ is shown: The best estimate according to minimum mean-squared error (MMSE) in the decision variable d_k is the conditional (a-posteriori) expectation for x_k (Tarköy, 1995; Papoulis, 1991), i.e.

$$d_k = \mathcal{E}\{x_k|y_k\}. \quad (2)$$

If it is assumed, that distortion is only caused by zero mean AWGN with variance σ^2 , it follows after some calculations that

$$d_k = \tanh\left(\frac{r_{kk}^2}{\sigma^2} y_k\right). \quad (3)$$

If the number of users is not too small, $K \gg 1$, the assumption of Gaussian distortion is a good approximation for our problem. The trouble occurring, if the Gaussian approximation of distortion is poor, e.g. the number of users is not large, are discussed in Section 4. After some derivations the expected (a-posteriori) residual interference power of user k after soft-cancellation due to Eq. (3) turns out to be

$$\mathcal{E}\{(x_k - d_k)^2|y_k\} = 1 - d_k^2. \quad (4)$$

Therefore, with the Gaussian approximation of MAI, it holds

$$f(\mathbf{R}, \cdot, \sigma_n^2, k) = \frac{r_{kk}^2}{\sigma_n^2 + \sum_{i=1}^{k-1} r_{ki}^2 (1 - d_i^2) + \sum_{i=k+1}^K r_{ki}^2 (1 - d_i^2)}. \quad (5)$$

²This notation for the components of a vector is used for all other vectors and matrices, too.

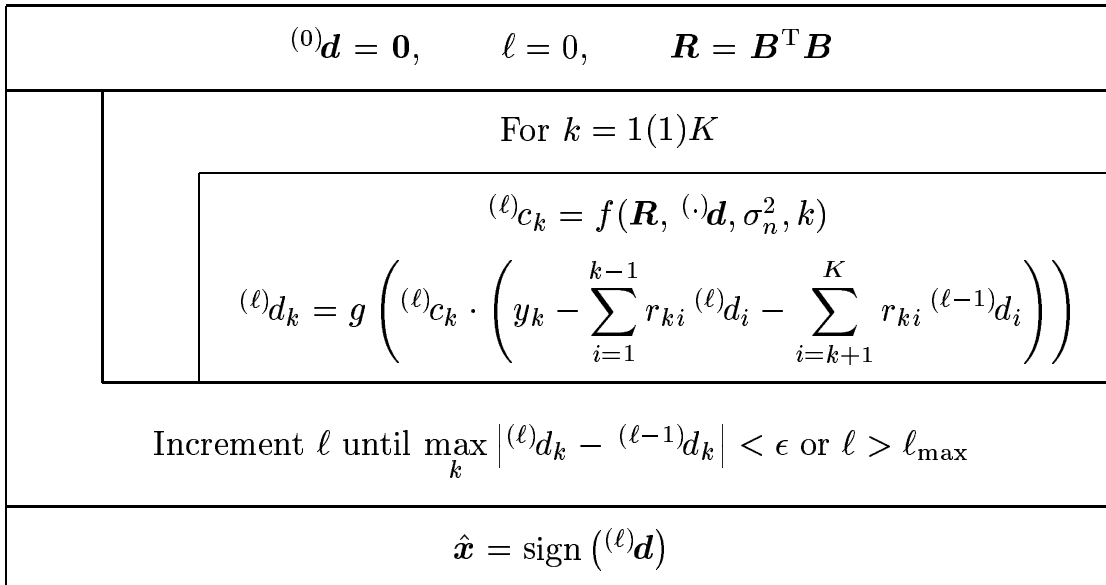


Figure 2. Detection algorithm of ISDIC.

In previous calculation a Gaussian approximation for the probability density function (pdf) of MAI has been used. This is essential to avoid exponential complexity in the number of users, as the exact pdf includes a sum of 2^K terms.

3 Performance Improvement

Now, we show that a performance gain can be obtained by using the adaption rule according to Eq. (5). Randomly chosen spreading sequences of length $N = 128$ have respectively been used with $K = 64$ and $K = 128$ users transmitting at equal power. Simulations for the uncoded bit error rate, cf. Fig. 3a, show, that for $K = 64$, implying spectral efficiency $\Gamma = K/N = 1/2$, there is no difference between empirically chosen and MMSE-optimized soft-cancellation (OSC). Moreover, almost single user power efficiency is achieved.

If spectral efficiency $\Gamma = 1$, implying $K = 128$, is requested, MMSE-OSC shows its advantages: While the empirically chosen SC does not work at all, MMSE-OSC is only about 1 dB behind single user power efficiency, cf. Fig. 3b. This gain is achievable by only a small increase in receiver complexity caused by calculation of Eq. (5) and by the average request for less than one additional iteration, cf. Fig. 4.

4 Influence of Spreading Factor

In the previous section a fixed spreading factor (processing gain) has been considered. As MAI is assumed to be Gaussian distributed due to the central limit theorem, we can expect, that the proposed detector performs the better the larger is the spreading factor, which is confirmed by Fig. 5.

The performance is very poor for small spreading factors. On the one hand there is a large gap to the single user bound, on the other hand an error floor occurs for high signal-to-noise ratios. With increasing spreading factor both negative effects become more and more negligible. The error floor occurs at lower bit error rates and the gap to the single user bound is only 0.02 dB for $N = 512$ at bit error rate 10^{-4} .

The strategy to overcome the problem occurring at small spreading factors using the exact pdf of MAI instead of the Gaussian approximation, as it has been done by Moher (1997) for coded transmission over multiple-access channels with only few users, is useless in systems with moderate to many users. Calculation of the pdf required in the turbo decoder needs a summation over 2^K terms resulting in exponential complexity. So no gain in complexity is achieved in comparison to the optimum detector proposed by Verdú (1986). In the authors opinion a reasonable solution for small spreading factors seems to be the concept of reduced state multiuser detection proposed by Wei and Schlegel (1994), which shows nearly optimum performance for small spreading factors, but poor one for large spreading factors, if the comparison is done in respect of equal complexity to our detector.

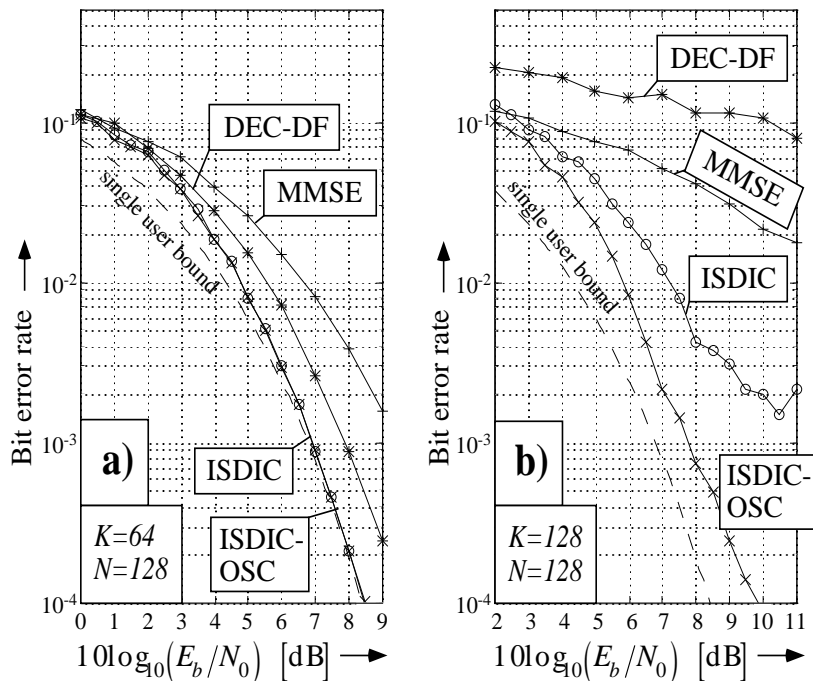


Figure 3. Bit error rate versus signal-to-noise ratio of ISDIC with and without MMSE-OSC for spreading factor $N = 128$, $K = 64$ users (a) and $K = 128$ users (b), respectively. The empirically chosen parameter is $c_k = 5$, $\forall k$. For reference, bit error probability of unspread 2PSK on the AWGN channel is shown (dashed line) as well as MMSE-equalization (MMSE), and decorrelating decision feedback (DEC-DF), respectively.

Another important performance measure is the number of required iterations. As shown in Fig. 6 it only increases considerably with spreading factor in the high bit error rate region. For moderate³ (10^{-3} to 10^{-4}) bit error rates the number of iterations remains sufficiently small for practical implementation.

5 Fading Channels

The most important application for CDMA seems to be mobile communication. Hereby, transmission over a Rayleigh fading channel is a much more realistic scenario than the assumption of an AWGN channel. In order to find out, if the fundamental results valid for the AWGN channel can be also used for fading channels, simulations for the frequency-nonselective Rayleigh fading channel have been performed. Hereby, it turned out, that no fundamental difference occur. Of course, the single user bound is different from the AWGN channel, but the gaps of the different ISDIC detectors to the single user bound only hardly differ from that ones observed on the AWGN channel. Therefore, no detailed description of this case is given here.

6 Conclusions

Although demodulation of CDMA signals by Hopfield neural networks shows superior performance to all other considered demodulation schemes if many users are active, it can be further improved by the MMSE-optimized soft-decision rule. With the improved version of ISDIC presented here, CDMA with large spreading factors is arbitrarily close to the performance of orthogonal multiple-access schemes for transmission on the AWGN channel, even if both systems are operating within the same bandwidth, i.e. $K = N$.

References

- S. Verdú. Minimum probability of error for asynchronous Gaussian multiple-access channels. *IEEE Trans. Inform. Theory*, IT-32(1):85-96, Jan. 1986.

³Lower bit error rates ($< 10^{-5}$) have not been able to be examined because too much simulation time is required for this.

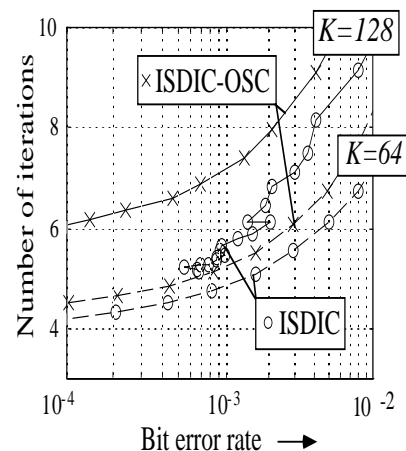


Figure 4. Average number of iterations versus bit error rate of ISDIC. ISDIC with (solid lines) and without (dashed lines) MMSE-OSC for spreading factor $N = 128$, $K = 64$ users (marked by crosses) and $K = 128$ users (marked by circles) is shown, respectively. Break criteria $\epsilon = 10^{-3}$ and $\ell_{\max} = 100$ have been used.

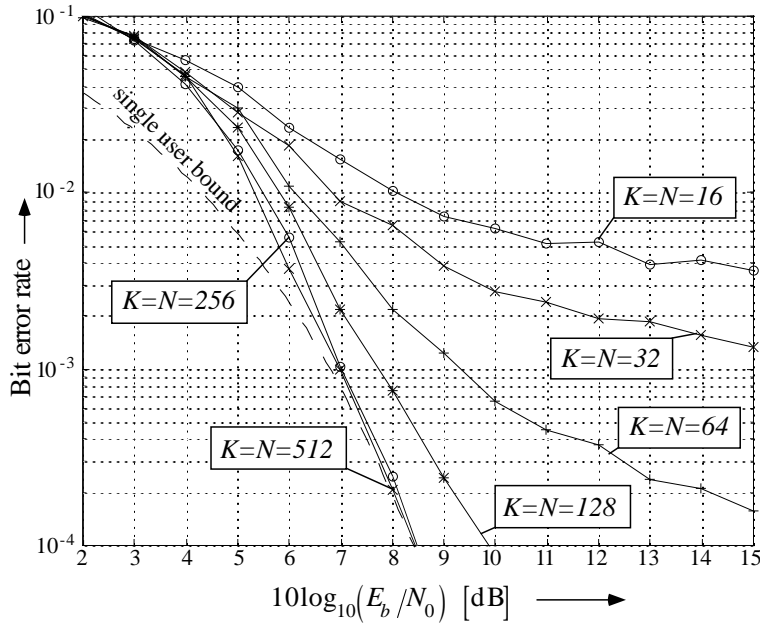


Figure 5. Bit error rate versus signal-to-noise ratio of ISDIC with MMSE-OSC with $N = K$ as parameter. For reference, bit error probability of unspread 2PSK on the AWGN channel is shown (dashed line).

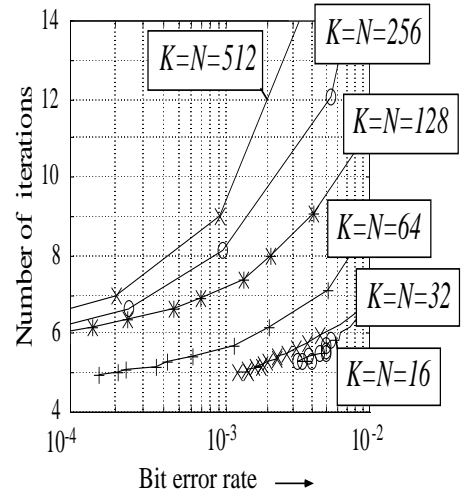


Figure 6. Average number of iterations versus bit error rate of ISDIC. The parameter $K = N$ is 16, 32, 64, 128, 256, and 512, respectively. Break criterions $\epsilon = 10^{-3}$ and $\ell_{\max} = 100$ have been used.

R. Lupas and S. Verdú. Near-far resistance of multiuser detectors in asynchronous channels. *IEEE Trans. Commun.*, COM-38(4):496–508, Apr. 1990.

Z. Xie, R.T. Short, and C.K. Rushforth. A family of suboptimum detectors for coherent multiuser communications. *IEEE Trans. J. Select. Areas Commun.*, 8(4):683–690, May 1990.

M. Rupf, F. Tarköy, and J. Massey. User-separating demodulation for code-division multiple-access systems. *IEEE Trans. J. Select. Areas Commun.*, 12(5):786–795, June 1994.

U. Madhow and M. Honig. MMSE interference suppression for direct-sequence spread-spectrum CDMA. *IEEE Trans. Commun.*, 42(12):3178–3188, Dec. 1994.

A. Duel-Hallen. Decorrelating decision-feedback multiuser detector for synchronous code-division multiple access channel. *IEEE Trans. Commun.*, COM-41(2):285–290, Feb. 1993.

M. Varanasi and B. Aazhang. Near-optimum detection in synchronous code-division multiple access systems. *IEEE Trans. Commun.*, COM-39(5):725–736, May 1991.

A. Duel-Hallen, J. Holtzman, and Z. Zvonar. Multiuser detection for CDMA systems. *IEEE Personal Commun.*, 2(2):46–58, Apr. 1995.

G. Kechriotis and E. Manolakos. Hopfield neural network implementation of the optimal CDMA multiuser detector. *IEEE Trans. Neural Networks*, 7(1):131–141, Jan. 1996.

T. Nagaosa, T. Miyajima, and T. Hasegawa. Multiuser detection using a hopfield network in asynchronous M-ary/SSMA communications. In *Proc. of IEEE ISSSTA*, pages 837–841, Mainz, Germany, Sep. 1996.

W. Teich and M. Seidl. Code division multiple access communication: Multiuser detection based on a recurrent neural network structure. In *Proc. of IEEE ISSSTA*, pages 979–984, Mainz, Germany, Sep. 1996.

C. Berrou, A. Glavieux, and P. Thitimajshima. Near shannon limit error-correcting coding and decoding: Turbo-codes. In *Proc. of IEEE ICC*, pages 1064–1070, Geneva, Switzerland, May 1993.

F. Tarköy. MMSE-optimal feedback and its applications. In *Proc. of IEEE ISIT*, page 334, Whistler, B.C. Canada, Sep. 1995.

A. Papoulis. *Probability, Random Variables, and Stochastic Processes*, 3rd ed., McGraw-Hill, New York, 1991.

M. Moher. Turbo-based multiuser detection. In *Proc. of IEEE ISIT*, page 195, Ulm, Germany, June/July 1997.

L. Wei and C. Schlegel. Synchronous DS-SSMA system with improved decorrelating decision-feedback multiuser detection. *IEEE Trans. Veh. Technol.*, 43(3):767–772, Aug. 1994.