Performance Assessment of Polynomial Expansion Detectors in MIMO Channels with Measured Data

Laura Cottatellucci and Ralf R. Müller

Telecommunications Research Centre Vienna (ftw.)
cottatellucci@ftw.at

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System Model

Consider Gaussian vector channel:

$$\boldsymbol{x} = \boldsymbol{H}\boldsymbol{s} + \boldsymbol{n}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_R \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & \dots & h_{1T} \\ h_{21} & h_{22} & \dots & h_{2T} \\ h_{31} & h_{32} & \dots & h_{3T} \\ \vdots & \vdots & \ddots & \vdots \\ h_{R1} & h_{R2} & \dots & h_{RT} \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \\ \vdots \\ s_T \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \\ n_3 \\ \vdots \\ n_R \end{bmatrix}$$

$R \times 1$ $\quad R \times T$ $\quad T \times 1$ $\quad R \times 1$

Correlation matrix: $\boldsymbol{R} = \boldsymbol{H}^H \boldsymbol{H}$

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Linear MMSE Filter

$$\hat{\boldsymbol{s}}_{\mathrm{MMSE}} = \left(\boldsymbol{H}^{\mathrm{H}}\boldsymbol{H} + \sigma_N^2 \mathbf{I}\right)^{-1} \boldsymbol{H}^{\mathrm{H}}\boldsymbol{x}$$

Cayley–Hamilton theorem yields:

$$\left(\boldsymbol{H}^{\mathrm{H}}\boldsymbol{H} + \sigma_N^2 \boldsymbol{I}\right)^{-1} = \sum_{i=0}^{T-1} \tilde{w}_i \left(\boldsymbol{H}^{\mathrm{H}}\boldsymbol{H}\right)^i$$

Approximating the inverse matrix by power series

Polynomial Expansion Equalizers

$$\hat{\boldsymbol{s}}_{\mathrm{MS}} = \sum_{i=0}^{L-1} w_i \left(\boldsymbol{H}^{\mathrm{H}}\boldsymbol{H}\right)^i \boldsymbol{H}^{\mathrm{H}}\boldsymbol{x} \qquad \text{for } L < T.$$

In general, $\tilde{w}_i \neq w_i$.

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Moshavi's Equalizer: Weight Design

Optimality criterion:

Minimization of the MSE between the actual LMMSE equalizer output and the polynomial expansion equalizer output

Weights satisfy Yule–Walker equations (Moshavi et al. '96):

$$\begin{bmatrix} \mathrm{tr}(\boldsymbol{R}) \\ \mathrm{tr}(\boldsymbol{R}^2) \\ \vdots \\ \mathrm{tr}(\boldsymbol{R}^L) \end{bmatrix} = \begin{bmatrix} \mathrm{tr}(\boldsymbol{R}^2)+\sigma^2\mathrm{tr}(\boldsymbol{R}) & \dots & \mathrm{tr}(\boldsymbol{R}^{L+1})+\sigma^2\mathrm{tr}(\boldsymbol{R}^L) \\ \mathrm{tr}(\boldsymbol{R}^3)+\sigma^2\mathrm{tr}(\boldsymbol{R}^2) & \dots & \mathrm{tr}(\boldsymbol{R}^{L+2})+\sigma^2\mathrm{tr}(\boldsymbol{R}^{L+1}) \\ \vdots & \ddots & \vdots \\ \boldsymbol{tr}(\boldsymbol{R}^{L+1})+\sigma^2\mathrm{tr}(\boldsymbol{R}^L) & \dots & \mathrm{tr}(\boldsymbol{R}^{2L+1})+\sigma^2\mathrm{tr}(\boldsymbol{R}^{2L}) \end{bmatrix} \begin{bmatrix} w_0 \\ w_1 \\ \vdots \\ w_{L-1} \end{bmatrix}$$

Weights depend on the eigenvalues of $\mathbf{R}$ only!

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Maximum Total SINR Equalizer: Weight Design

Optimality criterion:

Maximization of total SINR

$$\mathrm{SINR}_t = \frac{S_t}{P_t - S_t}$$

P_t ... total received power at the estimator output

S_t ... total users' signal power at the estimator output

$\mathbf{w}_{\mathrm{max\_SINR}}$ **depends on the eigenvalues and eigenvectors of R!**

How to calculate weights in real-time?

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Asymptotic Equalizer: Weight Design

As the size of system grows large, i.e. $T, R \to \infty$, the weights of both Moshavi's equalizer and total SINR equalizer converge to the same solution (Müller et al. '01):

$$\begin{bmatrix} m_1 \\ m_2 \\ \vdots \\ m_L \end{bmatrix} = \begin{bmatrix} m_2 + \sigma^2 m_1 & m_3 + \sigma^2 m_2 & \dots & m_{L+1} + \sigma^2 m_L \\ m_3 + \sigma^2 m_2 & m_4 + \sigma^2 m_3 & \dots & m_{L+2} + \sigma^2 m_{L+1} \\ \vdots & \vdots & \ddots & \vdots \\ m_{L+1} + \sigma^2 m_L & m_{L+2} + \sigma^2 m_{L+3} & \dots & m_{2L+1} + \sigma^2 m_{2L} \end{bmatrix} \begin{bmatrix} w_0 \\ w_1 \\ \vdots \\ w_L \end{bmatrix}$$

with the moments

$$m_k \triangleq \mathrm{E}\left\{\lambda^k\right\}$$

How to calculate the moments for a MIMO channel?

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Eigenvalue Moments

Under certain conditions, the moments of random covariance matrices converge to non–random quantities as their sizes grow large, i.e. $T, R \rightarrow \infty$ (Müller '01).

$$m_k = P^k \sum_{i=0}^{k-1} \sum_{j=0}^{k-1} \binom{k}{i} \binom{k}{j} \binom{k}{i+j+1} \frac{\beta^i \zeta^j}{k} \qquad \forall\, k > 0.$$

with

$\beta = \frac{T}{R}$	$\zeta = \frac{T}{S}$	and	S
system load	channel load		number of scatterers

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Channel Sounder Measurements

Static measurement around 2 GHz with 120 MHz bandwidth

Transmitter:
virtual array with omni–directional antenna
positioned randomly within a square
of 5×5 wavelengths

Receiver:
8–element patch array
with 120 degrees characteristic
blocking line of sight

Output data:	Matrix valued–frequency response $\mathbf{H}(f)$
Measurement Environment:	Two adjacent office rooms at FTW enriched with scatterers by sticking pieces of scrambled tin foil

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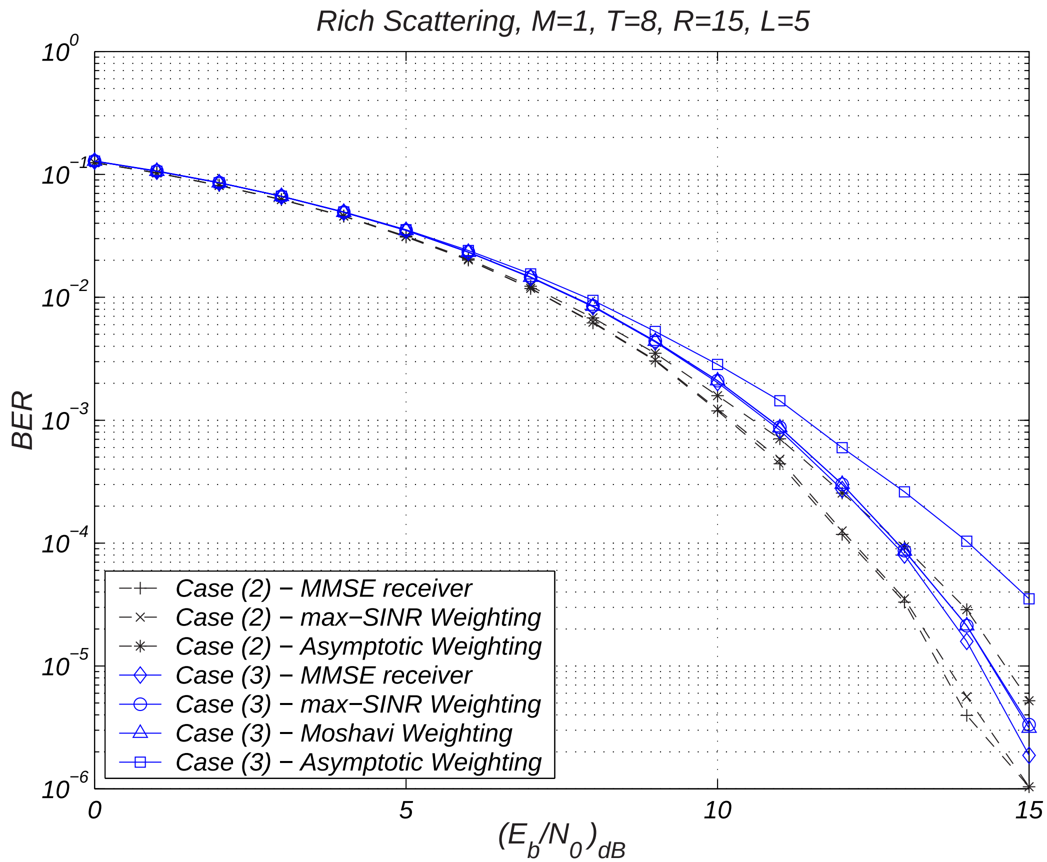
Performance Assessment: BER versus SNR



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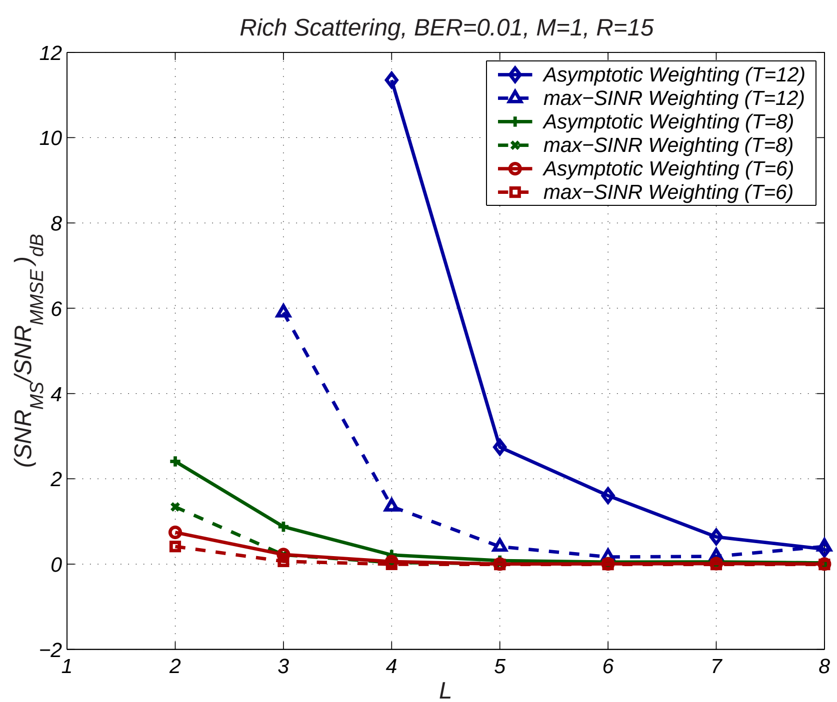
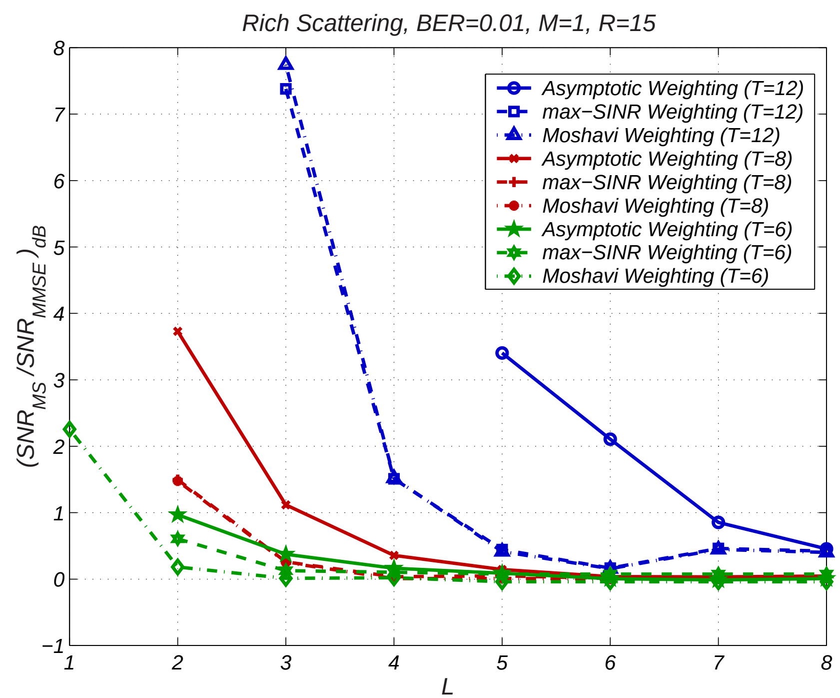
$\frac{\mathrm{SNR_{MS}(BER)}}{\mathrm{SNR_{MMSE}(BER)}}$ versus L for varying system load



Equal Power Case



Unbalanced Power Case

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Conclusions

Asymptotic receiver in environments with rich scattering:

- Good performance even being an approximation.
- Significant reduction of complexity vs slight performance degradation.
- Unbalanced Power vs Equal Power: negligible loss.

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