CAPACITY OF CELLULAR CDMA SYSTEMS APPLYING INTERFERENCE CANCELLATION AND CHANNEL CODING

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Abstract — Capacity of cellular CDMA applying interference cancellation is calculated for the uplink (reverse link) in the presence of AWGN. Both conventional demodulation and decorrelation are investigated as preprocessing unit to interference cancellation. Taking into account the path loss in a cellular system, CDMA applying interference cancellation shows a substantial difference in power efficiency, whether receive or transmit power is considered. This is in contrast to orthogonal multiple–access techniques, like TDMA and FDMA. If the comparison is based on transmit power, which is more reasonable, CDMA outperforms orthogonal multiple–access schemes.

1. INTRODUCTION

Code–division multiple–access (CDMA) is a promising multiple–access technique for future mobile communication systems. Its main advantages are firstly a wideband transmitter signal reducing the adverse effects of frequency–selective fading, and secondly the fact, that in cellular systems the same carrier frequency may be used in every cell. An important drawback of CDMA systems is the distortion caused by multiple–access interference (MAI). Since the pioneering work of [1], who has shown that single user performance can be approached on the cost of complexity exponentially increasing with the number of users, there has been spent much effort in finding less complex demodulator structures that combat degradation due to MAI. Linear demodulators, e.g. [2, 3, 4], show important performance gains in comparison to conventional demodulation (CD) which is treating MAI as additive white Gaussian noise (AWGN). Nevertheless, there still remains a wide gap to the performance of orthogonal multiple–access schemes, like time–division multiple–access (TDMA) and frequency–division multiple–access (FDMA) if transmission over the AWGN channel is considered [5, 6].

This gap can be closed by optimum single user coding if each user is split up into two virtual users [7] or by very low rate coding and many users [8]. Both of these approaches are based on interference cancellation (IC): Assuming correct decoding of one user's information, the latter is remodulated and subtracted from the received signal.

Both of these approaches do not restrict the users' information rates to be equal, although [8, App. III] has pointed out but not explicitly shown, that there is no loss in capacity caused by this additional demand. Therefore, we analyze CDMA based on IC in the power–bandwidth plane for users transmitting at equal information rate by calculating its power efficiency as a function of its spectral efficiency (capacity) for transmission over the AWGN channel. This allows a more general comparison between different demodulator structures as it has been done before by other authors, since it is not restricted to a given modulation scheme, code rate, or set of signature waveforms. The calculations for CD and decorrelation are presented in Section 2 and 3, respectively. Hereby, it turns out, that in contrast to orthogonal multiple–access schemes the receive energy has not to be uniformly distributed among the users to achieve the single user capacity. Although the results of Section 2 have already been derived in a different manner in [8], our analysis can be generalized involving the path loss in a cellular system, as done in Section 4. Hereby, the main result of this paper becomes obvious: CDMA outperforms orthogonal multiple–access schemes in terms of transmit energy if path loss is taken into account. Section 5 proposes a significantly greater gain of CDMA in systems with more than one cell and Section 6 points out the main conclusions.

2. IC WITHOUT EQUALIZATION

For simplicity, our considerations are restricted to transmission over the AWGN channel, but they can easily be generalized to other channel models by replacing Eq. (10) by its corresponding counterpart. As power control providing equal receive power for all users is not the optimum power assignment among the users [9], detailed analysis of this topic is given in the following, too. Without loss of generality it is assumed, that the K users are numbered in the inverse order as they are demodulated and cancelled. The cancellation is assumed to be not perfect which is represented by a factor β denoting the ratio of interference power after to that before cancellation. In an implemented system there are various reasons, why IC is not perfect, e.g.: imperfect channel estimation on fading channels. Therefore, users' received signals cannot be reconstructed perfectly, even if they have been decoded correctly.

If all users apply randomly chosen signature sequences, the signal–to–interference ratio for user κ including imperfect IC and additive noise is given by, see e.g. [10],

$$\mathrm{SIR}_\kappa = \frac{E_\kappa}{\mathcal{N}_0 + \sum_{i=1}^{\kappa-1} \frac{E_i}{N} + \beta \sum_{i=\kappa+1}^{K} \frac{E_i}{N}}. \qquad (1)$$

Here $\mathcal{N}_0$, E_κ, and N are denoting one–sided noise power ding factor, respectively. Now we assume, that all users (want to) transmit at the same information rate R. Using the Gaussian interference approximation[1] this implies that E_κ has to be controlled in such a way that $\mathrm{SIR}_\kappa = \mathrm{SIR}$ is valid for all users. This enables us to rewrite Eq. (1) into

$$E_\kappa - E_{\kappa-1} = \mathrm{SIR}\left(\frac{E_{\kappa-1}}{N} - \beta\frac{E_\kappa}{N}\right). \tag{2}$$

The solution of this difference equation is given by

$$E_\kappa = \left(\frac{N+\mathrm{SIR}}{N+\beta\mathrm{SIR}}\right)^{\kappa-1}\cdot E_1 \stackrel{\mathrm{def}}{=} a^{\kappa-1}E_1, \tag{3}$$

with obvious definition of a. Eq. (3) describes the optimum distribution of the receive energy per symbol among the users in a CDMA system applying IC. For $\beta \neq 1$, this is not a uniform, but an exponential distribution. In order to achieve the complete performance gain of IC, it is very important to provide the correct receive energy distribution.

Using the finite geometric series, see e.g. Eq. (0.112) in [11], the average receive energy per symbol E_s can be expressed as a function of E_1 being the receive energy per symbol of user one.

$$E_\mathrm{s} = \frac{E_1}{K}\sum_{\kappa=1}^{K} a^{\kappa-1} = \frac{E_1}{K}\frac{a^K-1}{a-1} \tag{4}$$

In order to eliminate the dependence of user one's receive energy we set $\kappa = 1$ in Eq. (1), and obtain with Eq. (4)

$$E_1\left(1+\mathrm{SIR}\frac{\beta}{N}\right) = \mathrm{SIR}\left(\mathcal{N}_0 + \beta\frac{K}{N}E_\mathrm{s}\right), \tag{5}$$

Combining Eqs. (4) and (5), which are linearly independent from each other, the average receive energy per symbol and user one's energy per symbol yields

$$E_\mathrm{s} = \mathcal{N}_0\frac{N}{K}\cdot\frac{a^K-1}{1-\beta a^K} \tag{6}$$

$$E_1 = \mathcal{N}_0 N\cdot\frac{a-1}{1-\beta a^K}, \tag{7}$$

respectively. As

$$\log a = \log\frac{1+\mathrm{SIR}/N}{1+\beta\mathrm{SIR}/N} \xrightarrow{N\gg\mathrm{SIR}} \frac{\mathrm{SIR}}{N}(1-\beta) \tag{8}$$

becomes obvious by expanding the logarithm via first order Taylor series, Eq. (6) can be simplified for large N to

$$E_\mathrm{s} = \mathcal{N}_0\frac{N}{K}\frac{\exp\left(\mathrm{SIR}\frac{K}{N}(1-\beta)\right)-1}{1-\beta\exp\left(\mathrm{SIR}\frac{K}{N}(1-\beta)\right)}. \tag{9}$$

Now, let us assume all users apply single user channel coding close to capacity of the AWGN channel [12]

$$R = \log_2(1+\mathrm{SIR}). \tag{10}$$

[1]For an exponential energy distribution, as derived to be optimum in the following, the central limit theorem is not valid. Therefore, Gaussian interference implies Gaussian distributed signals of all users.

transmission close to capacity limit is possible by multilevel coding [13]. Furthermore, in CDMA systems spectral efficiency is defined by [10]:

$$\Gamma \stackrel{\mathrm{def}}{=} R\frac{K}{N}. \tag{11}$$

With Eqs. (9) to (11), it follows a relationship between spectral efficiency and the average receive energy per information bit $E_\mathrm{b} \stackrel{\mathrm{def}}{=} E_\mathrm{s}/R$

$$E_\mathrm{b} = \frac{\mathcal{N}_0}{\Gamma}\cdot\frac{\exp\left((2^{\Gamma N/K}-1)\frac{K}{N}(1-\beta)\right)-1}{1-\beta\exp\left((2^{\Gamma N/K}-1)\frac{K}{N}(1-\beta)\right)}. \tag{12}$$

Now we can devise the optimum ratio of users per spreading factor K/N. Obviously, E_b is minimized by minimizing $(2^{\Gamma N/K}-1)\frac{K}{N}$. From channel capacity of the AWGN channel it is well known, see e.g. [14], that

$$\min_{N/K}\left\{\frac{2^{\Gamma N/K}-1}{\Gamma N/K}\right\} = \log 2, \tag{13}$$

and the optimum K/N–ratio is infinity. Therefore, we obtain the minimum required receive energy to achieve spectral efficiency Γ

$$\inf_{K/N}\{E_\mathrm{b}\} = \frac{\mathcal{N}_0}{\Gamma}\cdot\frac{2^{\Gamma(1-\beta)}-1}{1-\beta\,2^{\Gamma(1-\beta)}}. \tag{14}$$

It is only achievable with the optimum receive energy distribution resulting from Eqs. (13), (8), and (3)

$$E_\kappa = 2^{\kappa/K\cdot\Gamma(1-\beta)}E_1. \tag{15}$$

If we assume perfect IC, i.e. $\beta = 0$, it is obvious, that CDMA applying CD and IC achieves the same capacity as a single user system, or a multi–user systems operating with N orthogonal signature waveforms like TDMA and FDMA. But if there is an arbitrarily small amount of residual interference, the single user bound cannot be obtained, and even worse: spectral efficiency is upper bounded by

$$\Gamma < \frac{\log_2\beta}{\beta-1}, \tag{16}$$

which can easily be concluded considering Eq. (14) and has already been stated in [8].

This way towards channel capacity does not only suffer from the need of perfect IC, it is affected by the optimum code rate $R_\mathrm{opt} = \Gamma/(K/N)_\mathrm{opt} = 0$, too. Although this may be achieved by using very low rate codes [8], the number of users has to grow towards infinity for nonzero spectral efficiency, cf. Eq. (11). For implementation of practical systems these two aspects seem to cause serious problems.

3. IC WITH DECORRELATION

Alternatively to CD with IC it is possible to use interference suppression (IS) before IC. The most common principle near filters can be adjusted according to the minimum mean squared error (MMSE) criterion [15] and the decorrelation (zero–forcing, ZF) criterion [16], respectively. While MMSE adjusted IS turns out to be very difficult to analyze analytically [6], ZF has been analyzed analytically in [17]. In the following these results are generalized to IC. This requires to assume perfect IC, i.e. $\beta = 0$, as ZF is not affected by the power of an interfering user.

The receive energy per symbol required in a system with κ interfering users spreading their signals with randomly chosen signature sequences is given by

$$E_\kappa = \mathcal{N}_0 \mathrm{SIR} \frac{N-1}{N-\kappa}\,, \tag{17}$$

cf. [17]. The average receive energy is given by averaging Eq. (17) over all K users

$$E_\mathrm{s} = \mathcal{N}_0 \frac{\mathrm{SIR}}{K} \sum_{\kappa=1}^{K} \frac{N-1}{N-\kappa}. \tag{18}$$

For $K \gg 1, N-K \gg 1$ this can be expressed by

$$E_\mathrm{s} = \mathcal{N}_0 \frac{N}{K} \mathrm{SIR} \int\limits_{N-K}^{N} \frac{\mathrm{d}k}{k} = -\mathcal{N}_0 \frac{N}{K} \mathrm{SIR} \log\left(1 - \frac{K}{N}\right). \tag{19}$$

With the assumption of perfect single user channel coding and Eqs. (11) and (10) the average receive energy per information bit

$$E_\mathrm{b} = -\mathcal{N}_0 \log\left(1 - \frac{K}{N}\right) \frac{2^{\Gamma N/K} - 1}{\Gamma} \tag{20}$$

is given as a function of spectral efficiency Γ and parameter K/N. The authors have not found an analytical solution to determine the optimum K/N–ratio, yet. This optimization was performed numerically and its result is shown by the solid line in Fig. 1. It turns out, that using ZF to suppress interference before IC, the optimum K/N–ratio does not grow to infinity, but is upper bounded by $K/N < 1$, which can already be seen from Eq. (17).

Using the optimum K/N–ratio from Fig. 1 spectral efficiency is plotted over the average receive energy per bit E_b normalized to noise power density $\mathcal{N}_0$. Surprisingly, the single user Shannon bound cannot be achieved if ZF–IS is applied before IC together with single user channel decoding, see Fig. 2. This is somewhat surprising, as in uncoded systems there is a gain due to IS, see e.g. [15], whereas here a loss is observed. In order to solve this paradoxon let us consider what is done by ZF–IS: the MAI is perfectly suppressed on cost of enhancement of the AWGN. In an uncoded system, the AWGN has to have relatively low power, implying, that noise enhancement is not a critical problem. In coded systems quite more AWGN is tolerable and additionally it raises severely due to bandwidth extension. Therefore, noise enhancement becomes a serious problem. The question at this point is: Is there any reason to think about IS any longer, as in every power and/or bandwidth efficient transmission system channel coding has to be applied? It is, as the superiority of CD combined with IC only holds for the assumption of perfect IC. If IC is not perfect, CD is upper bounded in spectral efficiency, but IS is not, see Fig. 2.

Although we have not analyzed MMSE–IS, we now can state, that this is the only useful way of equalizing MAI

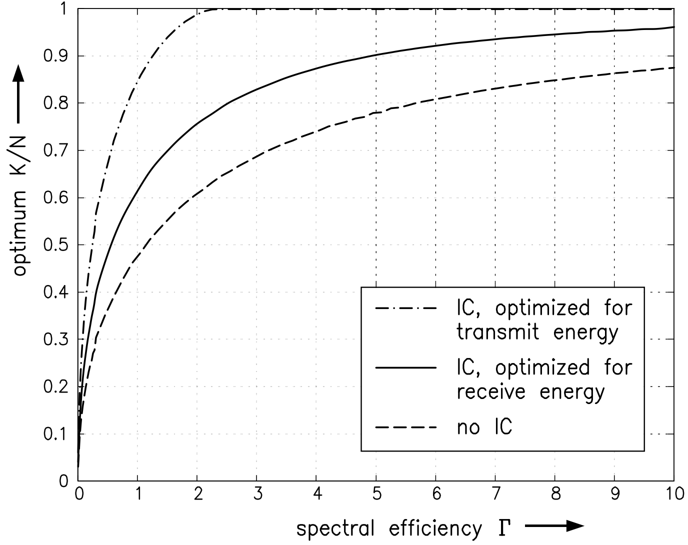

Fig. 1: Optimum K/N–ratio for ZF–IS. The dashed (– – –) line is valid, if IC is *not* applied. The solid and the dash–dotted (– · –·) line refer to IC. They represent the optimum K/N–ratio for minimum *receive* energy and minimum *transmit* energy, respectively.

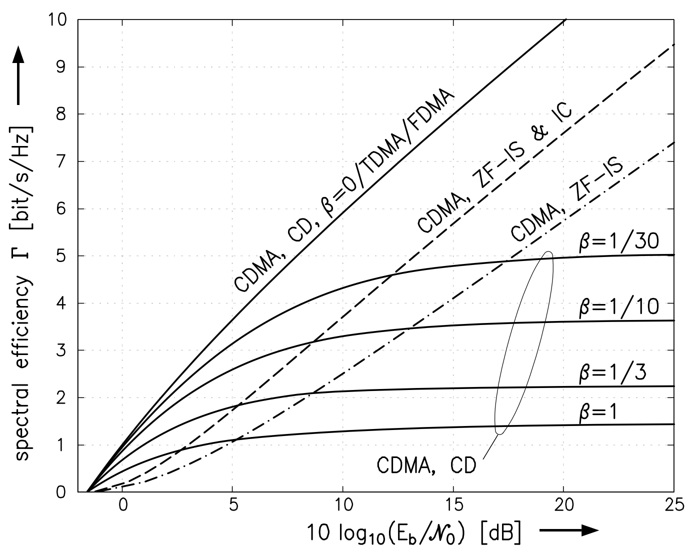

Fig. 2: Spectral efficiency as a function of the ratio of average *receive* energy per information bit to noise power density for several methods of treating MAI. The solid lines represent CD followed by IC for varying values of the cancellation factor $\beta = 1, 1/3, 1/10, 1/30, 0$, respectively. The dashed (– – –) and the dash–dotted (– · –·) lines refer to ZF–IS with and without IC, respectively.

ZF–IS. This means, the spectral efficiency of a system applying MMSE–IS followed by IC keeps unbounded even, if IC is not perfect, and achieves the single user bound, if IC is perfect.

4. EFFECTS OF PATH LOSS

In Section 2 and 3 we have shown, that CDMA systems applying IC do not need equal receive power. They even show performance degradation due to equal receive power, see [9], too. This implies, that it is not fair to compare systems by their average receive power required for achieving a certain desired performance, but to consider the average transmit power. This also implies that uplink (reverse link) and downlink (forward link) have to be split up into two different cases. For convenience, in the following sections we only focus on the uplink.

In order to enable an analytical analysis we consider the cell to be a circle with radius 1 instead of a hexagon. Due to path loss the signals transmitted with energy per symbol $\tilde{E}_\kappa$ at normalized distance r_κ, $0 \leq r_\kappa \leq 1$, from the base station are received with attenuation given by

$$E_\kappa = \frac{1}{D_0} r^{-\alpha} \tilde{E}_\kappa. \tag{21}$$

Here, α and D_0 denote the attenuation exponent, which normally varies between 3 and 4 depending on the morphological structure of the cell [18], and the attenuation of the signal from the edge of the cell to the center, respectively.

If we assume the users to be uniformly distributed within the cell, the average number of users within a second circle of normalized radius r_i is given by

$$K_i = r_i^2 K. \tag{22}$$

As shown in Eq. (3) the users' receive energies have to be different. This is very suitable for a cellular scenario, as we can put the users requiring high receive energies at the base station near to the base station and the users requiring less receive energies to the border of the cell. This idea means, that the users have to be numbered inverse to their distance from the base station, i.e. $\kappa = K - K_i$. Applying Eqs. (21) and (22) this yields to

$$\tilde{E}_\kappa = D_0 \left(1 - \frac{\kappa}{K}\right)^{\alpha/2} E_\kappa. \tag{23}$$

Now we restrict the attenuation exponent to $\alpha = 4$, which is a choice suitable to many practical scenarios [18] and simplifies the following analysis. With Eqs. (3) and (23) averaging over all users leads to the average transmit energy per symbol

$$\tilde{E}_\mathrm{s} = \frac{D_0}{K} \sum_{\kappa=1}^{K} \left(1 - \frac{\kappa}{K}\right)^2 a^{\kappa-1} E_1. \tag{24}$$

For large number of users K the sum is well approximated by the integral

$$\tilde{E}_\mathrm{s} = \frac{D_0 E_1}{K^3} \int_0^K k^2 a^{K-k} \mathrm{d}k. \tag{25}$$

$$\tilde{E}_\mathrm{s} = D_0 E_1 \left(\frac{2a^K - 2}{\log^3(a^K)} - \frac{2}{\log^2(a^K)} - \frac{1}{\log(a^K)} \right). \tag{26}$$

Combining Eqs. (7), (26), and (8) we obtain for $N \gg \mathrm{SIR}$

$$\begin{aligned} \tilde{E}_\mathrm{s} \;&= \frac{2 D_0 \mathcal{N}_0 \frac{N}{K}}{1 - \beta \exp\left(\mathrm{SIR}\frac{K}{N}(1-\beta)\right)} \cdot \\ &\cdot \left(\frac{\exp\left(\mathrm{SIR}\frac{K}{N}(1-\beta)\right) - 1}{\left(\mathrm{SIR}\frac{K}{N}(1-\beta)\right)^2} - \frac{1}{\mathrm{SIR}\frac{K}{N}(1-\beta)} - \frac{1}{2} \right). \end{aligned} \tag{27}$$

With the assumption of perfect single user channel coding and Eqs. (11) and (10) the average transmit energy per information bit is given as a function of spectral efficiency $\mathit{\Gamma}$ with parameter K/N on top of the next page. This expression is to be minimized by varying the K/N–ratio. As shown in the Appendix $\tilde{E}_\mathrm{b}$ is a monotonously increasing function of $(2^{\mathit{\Gamma} N/K} - 1)\frac{K}{N}(1-\beta)$ within the considered intervall. Therefore, the same argumentation as in Section 2, see Eq. (13), leads to

$$\begin{aligned} \inf_{K/N} \left\{ \tilde{E}_\mathrm{b} \right\} &= \frac{2 D_0 \mathcal{N}_0 / \mathit{\Gamma}}{1 - \beta\, 2^{\mathit{\Gamma}(1-\beta)}} \cdot \\ &\cdot \left(\frac{2^{\mathit{\Gamma}(1-\beta)} - 1}{\left(\mathit{\Gamma} \log(2)(1-\beta)\right)^2} - \frac{1}{\mathit{\Gamma}\log(2)(1-\beta)} - \frac{1}{2} \right). \end{aligned} \tag{29}$$

This relationship is plotted in Fig. 3 with cancellation factor β as parameter and compared to orthogonal transmission schemes like TDMA and FDMA. If IC works at least in part successfully, i.e. $\beta < 1$, there is a range of spectral efficiency, where CDMA with CD and IC outperforms the orthogonal multiplexing schemes TDMA and FDMA. For perfect IC and large $\mathit{\Gamma}$ the gap becomes very wide and is rapidly growing proportionally to the 2nd power of $\mathit{\Gamma}$.

At first sight, it looks like a paradoxon that there is a gain due to MAI, which enables CDMA to outperform orthogonal multiplexing schemes. This paradoxon is resolved, if one considers the following well known facts:

- MAI does not affect the total capacity of Gaussian multiple–access channels, see [14].
- In orthogonal multiple–access schemes the different channels are independent from each other, implying equal receive energies for all users to provide equal capacities.
- If users are continously distributed within a cell, ordered in a reasonable way, and their total receive energy is given, their total transmit energy is maximum if their receive energies are equal[2].

[2] This can be proven by contradiction: Assume there is a receive energy distribution worse than the uniform one, implying one energy is lower and one higher than the average. Then we can change their order and we perform better than the uniform distribution, which is in contradiction to the initial assumption.

$$\tilde{E}_{\mathrm{b}} = \frac{\cdots}{1-\beta\exp\left((2^{\Gamma N/K}-1)\frac{K}{N}(1-\beta)\right)} \cdot \left(\frac{\cdots}{\left((2^{\Gamma N/K}-1)\frac{K}{N}(1-\beta)\right)^2} - \frac{\cdots}{(2^{\Gamma N/K}-1)\frac{K}{N}(1-\beta)} - \frac{\cdots}{2} \right) \qquad (28)$$

Considering these facts, it becomes obvious, that the dependence of the users' signals, which requires the unequal energy distribution at the receiver, results in a performance gain, if total transmit energy is to be minimized and all channels are requested to transmit at equal information rate.

As already discussed in Section 3, ZF–IS is another reasonable way of CDMA demodulation, especially, if it is combined with IC. In order to analyze its spectral efficiency with respect to path loss we combine Eqs. (17) and (23) and average over all K users obtaining the average transmit energy per symbol for $\alpha = 4$

$$\tilde{E}_{\mathrm{s}} = \frac{D_0 \mathrm{SIR} \mathcal{N}_0}{K} \sum_{\kappa=1}^{K} \left(1-\frac{\kappa}{K}\right)^2 \frac{N-1}{N-\kappa}. \qquad (30)$$

For $K \gg 1, N \gg 1$, the sum is well approximated by an integral. Using Eq. (2.112.2) in [11] the average transmit energy is given by

$$\begin{aligned} \tilde{E}_{\mathrm{s}} &= D_0 \mathrm{SIR} \mathcal{N}_0 \frac{N}{K^3} \int_0^K \frac{(K-k)^2}{N-k} \mathrm{d}k \qquad (31) \\ &= D_0 \mathrm{SIR} \mathcal{N}_0 \left(\tfrac{3N}{2K} - \tfrac{N^2}{K^2} - \tfrac{N^3}{K^3}\left(1-\tfrac{K}{N}\right)^2 \log\left(1-\tfrac{K}{N}\right) \right). \end{aligned}$$

With the assumption of perfect single user channel coding like in the case of CD and Eqs. (11) and (10) the average transmit energy per information bit

$$\begin{aligned} \tilde{E}_{\mathrm{b}} &= D_0 \mathcal{N}_0 \frac{2^{\Gamma N/K}-1}{\Gamma} \cdot \qquad (32) \\ &\quad \cdot \left(\frac{3}{2} - \frac{N}{K} - \frac{N^2}{K^2}\left(1-\frac{K}{N}\right)^2 \log\left(1-\frac{K}{N}\right) \right). \end{aligned}$$

is given as a function of spectral efficiency Γ with the K/N–ratio as parameter. The authors have not found an analytical solution to determine the optimum K/N–ratio, yet. This optimization has been performed numerically and its result is shown by the dash–dotted line in Fig. 1. If spectral efficiency is higher than about 2.3 bit/(s·Hz), the optimum K/N–ratio is given by $(K/N)_{\mathrm{opt}} = 1$. Inserting this into Eq. (32) by taking the limitation operation, it is obtained that the loss in transmit power efficiency between CDMA based on ZF–IS and IC to TDMA or FDMA in a single cell environment is $10\log_{10}(3/2) = 1.77\,\mathrm{dB}$ as one can observe from Fig. 3. In contrast to CD, ZF–IS is not a suitable preprocessing for IC to enable CDMA to outperform TDMA and FDMA, but the gap to the orthogonal multiple–access schemes has become smaller than without respect to path loss.

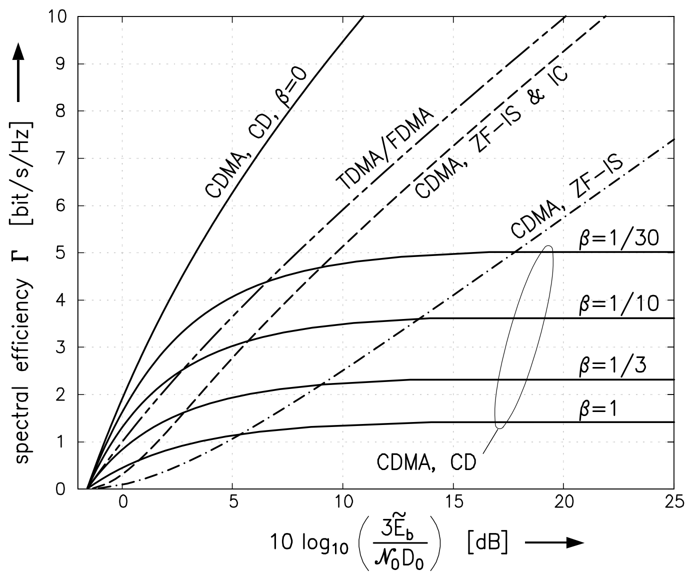

Fig. 3: Spectral efficiency as a function of the ratio of average *transmit* energy per information bit to noise power density normalized to one third of the maximum path loss for several methods of treating MAI. The factor 1/3 has been chosen to provide the curves for TDMA and FDMA to be the same as in Fig. 2. The solid lines represent CD followed by IC for varying values of the cancellation factor $\beta = 1, 1/3, 1/10, 1/30, 0$, respectively. The dashed (– – –) and the dash–dotted (– · –·) lines refer to ZF–IS with and without IC, respectively. For reference, spectral efficiency of orthogonal systems is depicted (—— – –).

5. CELLULAR SYSTEMS

In Section 4, there was only one cell taken into acount. In mobile radio transmission there are many different cells bordering each other and disturbing each other. Signals useful in one cell, act as noise in other cells. Although it may be possible to perform intercellular interference cancellation, for the goal of simplicity, we will not address this, but consider signals arriving from outside the cell as AWGN.

It was stated in Section 4, that CDMA can outperform orthogonal multiple–access techniques with respect to path loss. Is this true for cellular systems, too? An analytical treatment of this interesting question has not been done, yet, but a qualitative answer can be given without any calculations or simulations. As an implication of the unequal distribution of receive energy, in CDMA systems transmit energy is more concentrated in the middle of the cell than in orthogonal multiple–access schemes. This implies, that due to higher path loss to the neighbored cells, intercell interference is lowered in non–orthogonal multiple–access schemes by two different effects: first, there is less energy transmitted causing less interference, and second this less interference is more attenuated by path loss. This implies, that in multi–cell systems the possible performance gain of in the single cell scenario considered in Section 4.

Taking into account the effects of log–normal fading providing for an even more non–uniform distribution of signal attenuations than path loss the possible impact of non–orthogonal multiple–access is increased, additionally [19].

6. CONCLUSIONS

Considering the average transmit energy and users transmitting at equal information rate, CDMA has been shown to achieve higher performance in cellular systems than orthogonal transmission schemes, although in this paper multipath fading, the widely known area for CDMA to achieve good performance in comparison to TDMA, has not been addressed.

It is worth to emphasize, that CDMA with randomly chosen signature waveforms as used for our examinations has not been proven to be theoretically the best possible multiple–access scheme for cellular systems, but that signature waveforms shall in theory not be orthogonal. This encourages to further investigate on quality criterions for signature waveforms, as one of the traditional ones, low cross correlation, may loose part of its importance.

Appendix

It has to be shown, that the function

$$f(x) \stackrel{\text{def}}{=} \frac{3D_0\mathcal{N}_0/\Gamma}{1-\beta\mathrm{e}^x}\left(\frac{\mathrm{e}^x-1}{x^2}-\frac{1}{x}-\frac{1}{2}\right) \qquad (33)$$

with $x \stackrel{\text{def}}{=} (2^{\Gamma N/K}-1)\frac{K}{N}(1-\beta)$ is monotonously increasing in x, if $-\log(\beta) > x > 0$. Obviously, $3D_0\mathcal{N}_0/\Gamma/(1-\beta\mathrm{e}^x)$ is monotonously increasing in x and is not considered further on. For $x > 0$, it holds

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{e}^x-1}{x^2}-\frac{1}{x}-\frac{1}{2}\right) &= \frac{x+2+(x-2)\mathrm{e}^x}{x^3} \\ &= x^{-3}\left(x+2+(x-2)\sum_{i=0}^{\infty}\frac{x^i}{i!}\right) \\ &= x^{-3}\sum_{i=2}^{\infty}\left(\frac{x^i}{(i-1)!}-\frac{x^i}{i!}\right) > 0, \end{aligned} \qquad (34)$$

which is a sufficient condition for $f(x)$ to be monotonously increasing within the desired interval, q.e.d.

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References

[1] Sergio Verdú. Minimum probability of error for asynchronous Gaussian multiple–access channels. *IEEE Transactions on Information Theory*, IT–32(1):85–96, January 1986.

[2] Ruxandra Lupas and Sergio Verdú. Near–far resistance of multiuser detectors in asynchronous channels. *IEEE Trans. on Communications*, COM–38(4):496–508, April 1990.

separating demodulation for code–division multiple–access systems. *IEEE Journal on Selected Areas in Communications*, 12(5):786–795, June 1994.

[4] Upamanyu Madhow and Michael L. Honig. MMSE interference suppression for direct–sequence spread–spectrum CDMA. *IEEE Transactions on Communications*, 42(12):3178–3188, December 1994.

[5] Peter Schramm, Ralf R. Müller, and Johannes Huber. Spectral efficiency of multiuser systems based on CDMA with linear MMSE interference suppression. In *Proc. of IEEE International Symposium on Information Theory (ISIT)*, page 357, Ulm, Germany, June/July 1997.

[6] Peter Schramm and Ralf R. Müller. Spectral efficiency of CDMA systems with linear MMSE interference suppression. *Submitted to IEEE Transactions on Communications*, 1996.

[7] Bixio Rimoldi and Rüdiger Urbanke. A rate–splitting approach to the Gaussian multiple–access channel. *IEEE Trans. on Information Theory*, 42(2):364–375, March 1996.

[8] Andrew J. Viterbi. Very low rate convolutional codes for maximum theoretical performance of spread–spectrum multiple–access channels. *IEEE Journal on Selected Areas in Communications*, 8(4):641–649, May 1990.

[9] Bixio Rimoldi and Quinn Li. Potential impact of rate–splitting multiple access on cellular communications. In *Proc. of Communication Theory Mini Conference (CTMC) at IEEE Globecom*, pages 178–182, London, November 1996.

[10] Andrew J. Viterbi. *CDMA*. Addison–Wesley, 1995.

[11] I.S. Gradshteyn and I.M. Ryzhik. *Table of Integrals, Series, and Products*. Academic Press, New York, 1965.

[12] Claude E. Shannon. Communication in the presence of noise. *Proceedings of the IRE*, 37:10–21, January 1949.

[13] R. Fischer, J. Huber, and U. Wachsmann. Multilevel coding: Aspects from information theory. In *Proc. of Communication Theory Mini Conference (CTMC) at IEEE Globecom*, pages 26–30, London, November 1996.

[14] Thomas M. Cover and Joy A. Thomas. *Elements of Information Theory*. John Wiley & Sons, New York, 1991.

[15] Anja Klein, Ghasan Kawas Kaleh, and Paul Walter Baier. Zero forcing and minimum mean–square–error equalization for multiuser detection in code–division multiple–access channels. *IEEE Transactions on Vehicular Technology*, 45(2):276–287, May 1996.

[16] Alexandra Duel-Hallen. Decorrelating decision–feedback multiuser detector for synchronous code–division multiple access channel. *IEEE Transactions on Communications*, COM–41(2):285–290, February 1993.

[17] Ralf R. Müller, Peter Schramm, and Johannes B. Huber. Spectral efficiency of CDMA systems with linear interference suppression (in German). In *Proceedings of IEEE Workshop Kommunikationstechnik*, pages 93–97, Ulm, Germany, January 1997. English version available on R. Müller's WWW–site.

[18] William C.Y. Lee. *Mobile Communications Design Fundamentals*. John Wiley & Sons, New York, 1993.

[19] Ralf R. Müller, Peter Gunreben, and Johannes B. Huber. On the ability of CDMA to combat log–normal fading. *Submitted to International Zurich Seminar on Broadband Communications*, Zurich, Switzerland, February 1998.