Multiuser diversity in channels with limited scatterers

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Abstract—Multiuser diversity scheduling is studied in a single cell system with a limited scatterers channel. The scheduler implements the proportional fairness algorithm with infinite time constant, serving at each scheduling instant the user with the maximum short term fading coefficient. The limits of multiuser diversity gains due to the bounded support of the limited scatterers channel are explored by evaluating the average system capacity and the corresponding required transmitted energy per transmitted bit. The bounded support of the limited scatterers channel results in a capacity saturation as user population size grows even though the number of channel scatterers – the channel richness – is assumed to grow proportionally with the number of users.

I. INTRODUCTION

Multiuser diversity [1], [2] has been proposed as an effective technique to increase the throughput of wireless channels. By allowing only users having good channel conditions to transmit, channel variability across users can be utilized to enhance system capacity. The system level gain sometimes comes at a cost of unbalanced user throughputs, a problem for which various proportional [2] and hard fairness [3][4] scheduling schemes have been developed. In this paper we assume the system is scheduled with a proportional fairness scheduler that allows a single user to transmit at a time. However, the effective system throughput is dependent on the propagation channel, in effect of the joint statistics of the channels of the users in the system.

The connections between physical scattering and capacity scaling have been studied e.g. for multiple input, multiple output (MIMO) systems in [5], [6], where the essential scaling result is that MIMO capacity scales with antennas only if the number of scatterers in the channels scales proportionally as well. On the other hand, in independent Rayleigh fading the signal to noise ratio (SNR) of multiuser diversity systems has been shown to scale as $O(\log(\log(K)))$ [2], [7]. This monotonous increase of receiver SNR as a function of user population size is of course unrealistic since it indicates that the channel, given a suitably large user population, generates energy rather than absorbs it. In physical terms, the Rayleigh fading distribution is achieved at the asymptotic limit of an infinite number of scatterers – an asymptotic result in itself. While mathematically easily tractable, Rayleigh fading distribution fits poorly to evaluating multiuser diversity systems due to its infinite support.

In this paper we evaluate the scaling behavior of opportunistic scheduling in a limited scatterers channel when the number of users is increased. Since the channel has bounded support, the system capacity of the multiuser diversity system exhibits an upper bound even though the number of scatterers increases in proportion with the number of users. Capacity scaling towards the bound with increasing user population size is evaluated and the cases of Rayleigh fading and the limited scatterers channel compared. The analytical results of [4] on the proportionally fair scheduler are applied to evaluate the performance of the system.

The practical value of the results can be considered twofold. Firstly, they confirm that the scaling behavior based on Rayleigh fading cannot in practice be assumed in reality and that the system performance depends significantly on channel richness. Secondly, they demonstrate the sensitivity of multiuser system performance evaluations to key assumptions about the channel. For instance, the conventional rule of thumb for using the Jakes’ model for fading channel simulation requires only seven multipath components. This rule, however, considers only the low end of the distribution, and using it in multiuser diversity evaluation would be somewhat arbitrary and produce similarly arbitrary results. In reality, the number of multipath components in the channel is dependent on the system environment, terminal location, bandwidth, and utilized antennas. In macrocells, measurements with up to 20 paths have been reported [8].

The paper is organized as follows. Section II describes the system model. A simple upper bound for system capacity is stated for channels with bounded support. Section III introduces the channel model with limited scatterers and its basic behavior. Section IV provides the numerical evaluation for a multiuser diversity system utilizing the proportional fairness scheduler (PFS) algorithm for scheduling. In Section V conclusions are
drawn.

We denote by \(x_{i:j}\) the \(i\)th smallest variable of \(j\) variables and the distribution of a random variable \(y\) by \(F_y\). The \(n\)th order Bessel function is denoted as \(J_n(x)\).

II. SYSTEM DESCRIPTION

We consider the uplink of a wireless communications system with \(K\) users. Each user \(k\) experiences a propagation channel gain \(d_k\) that is the product of two random variables: the short term fading \(f_k\) and the path loss \(s_k\). Short term fading is assumed fixed over the signaling interval but i.i.d. across users and signaling intervals. Path loss is assumed i.i.d. across users and fixed in time for all users. We denote the receiver signal to noise ratio as the inverse of the noise power spectral density as

\[
SNR = 1
\]

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The transmitted energy per bit are given by \([4]\)

\[
R_k(t) = \log_2 (1 + d_k SNR)
\]

and then assigns the channel and allocates transmitted rate to the user whose ratio

\[
\frac{R_k(t)}{T_k(t)}
\]

is the largest. The denominator is computed using the recursive equation

\[
T_k(t + 1) = \begin{cases} 
1 - \frac{1}{t_c} T_k(t) + \frac{1}{t_c} R_k(t) & k = k^* \\
1 - \frac{1}{t_c} T_k(t) & k \neq k^*,
\end{cases}
\]

where \(t_c\) is a time window parameter (forgetting factor) and \(k^*\) is the scheduled user at time \(t\).

When \(t_c \to \infty\), PFS has been shown \([4]\) to give the channel to the user with the best instantaneous short term fading regardless of the users’ path loss. No power control is applied and the scheduled user transmits with unit energy. The transmitted rate is determined by the channel condition of the user. In effect, the normalization with the rate history equalizes the effect of the path loss in the scheduling decision but does not have effect on the achieved rate, which is still affected by path loss.

The total system capacity and the corresponding system transmitted energy per bit are given by \([4]\)

\[
R^{(K)} = \int_0^\infty \log_2 (1 + x SNR) dF_{s,f_k,k}(x) \quad (1)
\]

\[
\left( \frac{E_s}{N_0} \right)_{sys} = \frac{SNR}{R^{(K)}}, \quad (2)
\]

where \(F_{s,f_k,k}(x)\) denotes the joint distribution of the path loss and the short term fading term of the user with the best instantaneous short term fading. In Rayleigh fading, PFS has the asymptotic scaling behavior of a typical multiuser diversity system in that the system capacity scales as \(O \left( \log (\log (K)) \right)\) when \(K \to \infty\) \([2]\). The system energy per bit is chosen as the metric for energy consumption, since it provides a clear view of system behavior at low operating points \([9]\).

In a channel with bounded support \((x \in [0, f_B])\), the capacity has the trivial upper bound of

\[
R^{(K)} \leq \int_0^\infty \log_2 (1 + s f_B SNR) dF_s(x) \quad (3)
\]

independent of the number of users in the system. Roughly speaking, the asymptotic scaling indicated above only hold only if the channel support is unbounded. In the following, we will utilize a limited scatterers channel model to evaluate the behavior of multiuser diversity when the channel is more connected to the physical world.

III. LIMITED SCATTERERS CHANNEL

We wish to have a model to reasonably describe a channel with limited richness. It would be preferable, if the model enabled us to vary one parameter to control the channel richness behavior all the way up to asymptotically rich Rayleigh fading. The following model, while not necessarily providing a truthful image of the reality in radio propagation, has these qualities. We assume each user experiences a channel with \(N\) independent scatterers. The received signal components from the scatterers are assumed to have equal gains and uniformly distributed random phases. In the physical reality this would correspond to a case, depicted in Figure 1 of \(N\) scatterers placed on a circle around the transmitter such that the radius of this circle is relatively small compared
to the distance to the base station receiver. The total propagation path lengths for the signal components are then approximately equal – justifying the assumption of equal gains for the scatterers. The resulting channel envelope has the probability distribution of \[10\]

\[
p_r(r) = r \int_0^\infty J_0^N \left( \frac{\omega}{N} \right) J_0(\omega r) \omega d\omega.
\]

The distribution of \( f \) is depicted in Figure 2 for a channel with five scatterers \((N = 5)\). The channel energy is \( f \) in the limit normalized to unity giving the channel energy the upper bound of \( F_B = 2N \).

The envelope distribution of (4) results in the square envelope having the distribution\(^1\)

\[
F_f(x) = P(r^2 < x) = \sqrt{x} \int_0^\infty J_0^N \left( \frac{\omega}{N} \right) J_1(\omega \sqrt{x}) \, \omega \, dx
\]

\[x \in [0, 2N]
\]

(7)

As the number of scatterers increases, the distribution (7) can be shown to have the asymptotic behavior of

\[
F_f(x) \to 1 - e^{-x}, N \to \infty.
\]

The fact that the channel support depends on the number of scatterers complicates the comparison of results across channels with different number of scatterers, but enables that the average received energy remains constant.

It can be noted that it is possible to use this framework to model a channel with a dominant path (e.g. Rician fading) simply by replacing the \( N \)th power of Bessel functions in 4 by a product with appropriate weights for the dominant path and scatterers, respectively. Here, we however wish to concentrate on diversity effects and avoid the additional complication of dominant paths, even though it might provide a more truthful model of the physical reality with a small number of scatterers. A further note can be made, that in Rician channels the asymptotic scaling of multiuser diversity remains the same as in Rayleigh fading, albeit with a loss factor that has the most effect when the number of users is small [2].

It is assumed the channels of different users are mutually independent. This assumption is optimistic in terms of the resulting performance. In reality, as the user population \( K \) grows, users would at some point start experiencing correlated fading. However, one of the main objectives of the study is to make a comparison to the Rayleigh fading case. As such, it is sensible to work with assumptions that in essence reveal the lower bound of the performance difference to Rayleigh fading.

The reference channel with Rayleigh fading is modeled simply as having an exponential squared envelope distribution with mean one.

\[\text{IV. Numerical results}\]

We evaluate the performance of a single cell system utilizing PFS in a limited scatterers channel. Users are assumed to be placed on a circular cell of unit area and a forbidden region around the center. The radius of the forbidden region is 0.01 and the path loss exponent is 2.

The corresponding path loss distribution is given in the appendix. The path loss is normalized to unity at cell border, resulting in a corresponding shift in all SNRs utilized. Note that all energies are given as the total system transmit energy per bit (2), not SNR.

Figure 3 demonstrates the asymptotic system capacity upper bound (3) when the number of users is infinite. Whereas PFS in Rayleigh fading has a vanishing system energy requirement as the number of users approaches infinity \([4]\), in the limited scatterers channel the system energy is lower bounded by a limit that depends on the number of scatterers.

For the following results, the system capacity (1) has been evaluated using Monte-Carlo integration. Figure 4 presents the behavior of system capacity as a function of \((E_b/N_0)_{sys}\) for a 5 scatterers channel. The system capacity converges as there is virtually no gain from doubling the user population from 500 to 1000.

Figure 5 reports the exact scaling behavior of the limited scatterers channel with 5, 10 and 15 scatterers, and the Rayleigh fading channel with exponential energy distribution. The capacity of the channel with only 5

\[\text{Fig. 2. Channel distribution } (N = 5), \text{ exponential and difference}\]

\[\text{Fig. 3. Asymptotic system capacity upper bound for infinite users}\]

\[\text{Fig. 4. System capacity as a function of } (E_b/N_0)_{sys}\]

\[\text{Fig. 5. Exact scaling behavior of limited scatterers and Rayleigh fading}\]
scatterers saturates relatively early. The difference in scaling between 5 and 10 scatterer channels indicates a behavior of diminishing returns with increasing \( N \). Figure 6 reports the fraction of Rayleigh system capacity a limited scatterers channel achieves with the same number of users at various \( (E_b/N_0)_{sys} \). It is beneficial to consult Figure 4 for additional understanding on the system operating point at each system energy level. At low \( (E_b/N_0)_{sys} \), the 5 scatterers channel achieves roughly half of the capacity predicted by a Rayleigh fading channel. On the other hand, in a channel with 15 scatterers over 90% of the predicted system capacity is achieved throughout the tested system energies and user populations.

V. CONCLUSIONS

Multiuser diversity gain can be radically smaller than that predicted by standard Rayleigh fading scaling laws, especially when the channel has few scatterers or a low system transmitted energy per bit. Assumed channel richness contributes to saturation of the capacity when the number of users grows. Overall, the system behavior, especially with a large number of users, is highly dependent on assumptions of the number of channel multipath components.
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REFERENCES


APPENDIX

Assuming the users are located in a circular cell of unit area with a forbidden region around the access point. The forbidden region has radius \( \delta \) and the path loss exponent is given by \( \alpha \). The path loss has the cumulative distribution function

\[
F_s(x) = \begin{cases} 
0 & x < 1 \\
\frac{x^{-2/\alpha} - \delta^2}{1 - \delta^2} & 1 \leq x < \delta^{-\alpha} \\
1 & x \geq \delta^{-\alpha}
\end{cases}
\]