

ITERATIVE EQUALIZATION USING SOFT-DECODER FEEDBACK FOR MIMO SYSTEMS IN FREQUENCY-SELECTIVE FADING

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ABSTRACT

Signals transmitted through Multiple-Input-Multiple-Output (MIMO) systems suffer from multiple access interference (MAI), multipath propagation and additive noise. For such systems, iterative multi-user receivers offer a good tradeoff between performance and complexity. In this paper, the complete structure of such an iterative receiver is presented. It comprises channel estimator, multi-user detector and a bank of decoders. We focus on iterative channel estimation and investigate its influence on the overall receiver performance. The receiver performance is evaluated by simulations for different parameter setups.

INTRODUCTION

In MIMO systems several users access the transmission medium simultaneously in order to send and receive data. Thus, each user creates MAI for all other users. In communication systems where the symbol duration is smaller than the delay spread of the channel, inter-symbol interference (ISI) occurs. These two effects, MAI and ISI, are the dominant signal impairments. In recent years, iterative receiver structures have been proposed as a solution to combat MAI while maintaining modest complexity (see [1–11] and references therein). The iterative receiver consists of a multi-user signal detector followed by a bank of soft-input-soft-output (SISO) decoders. The soft decoders' outputs are used as feedback information for improving the detector's estimates in the next iteration. Most of the proposed schemes, however, assume flat fading, without ISI, or the perfect knowledge of the channel at the receiver.

We consider a MIMO system in frequency selective fading. Performing channel estimation within the iterative loop improves the receiver performance. Not only the known pilot symbols, but also estimated data symbols can be used for estimating the channel, if the channel coherence time is sufficiently long. Iterative channel estimation for frequency selective MIMO channels is addressed in [10], where estimation is performed using pilot symbols as well as hard decisions on data symbols which were classified as reliable in previous iterations. In this paper, we consider soft decision feedback of all data symbols from the decoder to the channel estimator and the multi-user detector. This approach leads to improved overall receiver performance.

MIMO DATA MODEL

Consider a MIMO system with K transmit antennas and N receive antennas. Antenna k transmits blocks of M convolutionally encoded and randomly interleaved symbols $b_k[m]$, where m is discrete symbol time. The modulation scheme is BPSK, i.e., $b_k[m] \in \{+1, -1\}$. Signals propagate through a frequency selective fading channel with L taps. Consecutive blocks are separated by a guard interval, which prevents inter-block interference (IBI). Each block contains P training symbols. The signal received at the n -th antenna, in the m -th symbol interval is given as

$$y_n[m] = \sum_{k=1}^K y_{nk}[m] = \sum_{k=1}^K \sum_{l=0}^{L-1} h_{nk}[l] b_k[m-l] + v_n[m], \quad (1)$$

where $h_{nk}[l]$ is the l -th tap of the channel impulse response from the k -th transmit to the n -th receive antenna. The additive Gaussian noise $v_n[m]$ is spatially and temporally white, with zero mean and known variance σ_v^2 . The parameters that need to be estimated at the receiver are the channel coefficients $h_{nk}[l]$. Under the block-fading assumption, the channel is constant over the block of M symbols. As there is no IBI, M transmitted symbols from the k -th transmit antenna will result in $M+L-1$ received symbols on the n -th receive antenna, defined by (1). Stacking them into an $(M+L-1) \times 1$ vector \mathbf{y}_{nk} , we obtain:

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$$\underbrace{\begin{bmatrix} y_{nk}[0] \\ y_{nk}[1] \\ \vdots \\ y_{nk}[M+L-2] \end{bmatrix}}_{\triangleq \mathbf{y}_{nk}} = \underbrace{\begin{bmatrix} b_k[0] & 0 & \cdots & \cdots & 0 \\ b_k[1] & b_k[0] & \cdots & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ b_k[L-1] & b_k[L-2] & \cdots & \cdots & b_k[0] \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & b_k[M-1] & b_k[M-2] \\ 0 & 0 & \cdots & 0 & b_k[M-1] \end{bmatrix}}_{\triangleq \mathbf{B}_k} \underbrace{\begin{bmatrix} h_{nk}[0] \\ h_{nk}[1] \\ \vdots \\ h_{nk}[L-1] \end{bmatrix}}_{\triangleq \mathbf{h}_{nk}} + \underbrace{\begin{bmatrix} v_{nk}[0] \\ v_{nk}[1] \\ \vdots \\ v_{nk}[M+L-2] \end{bmatrix}}_{\triangleq \mathbf{v}_{nk}} \quad (2)$$

where \mathbf{B}_k is a $(M+L-1) \times L$ Toeplitz matrix of data symbols, and the vector \mathbf{h}_{nk} contains the L channel taps. The total received signal on the n -th antenna is given by

$$\mathbf{y}_n = \sum_{k=1}^K \mathbf{y}_{nk} = [\mathbf{B}_1 \ \mathbf{B}_2 \ \dots \ \mathbf{B}_K] \begin{bmatrix} \mathbf{h}_{n1} \\ \vdots \\ \mathbf{h}_{nK} \end{bmatrix} + \mathbf{v}_n \triangleq \mathbf{B} \mathbf{h}_n + \mathbf{v}_n. \quad (3)$$

Finally, we collect these vectors for all N receive antennas into a single $N(M+L-1) \times 1$ vector \mathbf{y} :

$$\mathbf{y} = (\mathbf{I} \otimes \mathbf{B}) \mathbf{h} + \mathbf{v} \triangleq \mathbf{B} \mathbf{h} + \mathbf{v}, \quad (4)$$

where $\mathbf{y} = [\mathbf{y}_1^T \ \mathbf{y}_2^T \ \dots \ \mathbf{y}_N^T]^T$, $\mathbf{h} = [\mathbf{h}_1^T \ \mathbf{h}_2^T \ \dots \ \mathbf{h}_N^T]^T$, $\mathbf{v} = [\mathbf{v}_1^T \ \mathbf{v}_2^T \ \dots \ \mathbf{v}_N^T]^T$, \mathbf{I} is the $N \times N$ identity matrix and \otimes is the Kronecker product. In this way, all the NKL unknown channel parameters are collected in the vector \mathbf{h} , while the transmitted coded data symbols constitute the block-diagonal $N(M+L-1) \times NKL$ matrix \mathbf{B} .

ITERATIVE RECEIVER ALGORITHM

The structure of the investigated iterative receiver is shown in Fig. 1. The iterative process starts with the initial channel estimation based on the knowledge of pilot symbols. The estimated vector of channel coefficients $\hat{\mathbf{h}}$, is forwarded to the multi-user detector, which is followed by a bank of K SISO decoders. Given the channel estimates and the received symbol sequence, the detector computes the log-likelihood ratios (LLRs) for all code symbols:

$$\Lambda_1(b_k[m]) = \log \frac{\Pr(b_k[m] = +1 | \mathbf{y}, \hat{\mathbf{h}})}{\Pr(b_k[m] = -1 | \mathbf{y}, \hat{\mathbf{h}})} \quad (5)$$

Based on detector outputs and code constraints (c. c.) for each user, the decoders compute *a posteriori* LLRs Λ_2 [10]:

$$\Lambda_2(b_k[m]) = \log \frac{\Pr(b_k[m] = +1 | \Lambda_1(b_k[m]), \text{c. c.})}{\Pr(b_k[m] = -1 | \Lambda_1(b_k[m]), \text{c. c.})} \quad (6)$$

This posterior LLR consists of two parts: *a priori* LLR Λ_1 , delivered by the detector, and *extrinsic* LLR λ_2 , which is additional information from the decoders: $\Lambda_2(\cdot) = \Lambda_1(\cdot) + \lambda_2(\cdot)$.

SISO Multi-User Detector

The detector in our scheme is a Soft Interference Canceller (SIC) followed by a bank of Minimum Mean Squared Error (MMSE) filters. The detailed algorithm can be found in [10], thus only the main principle will be explained here. Based on extrinsic information fed back from the decoders, the detector calculates the soft symbol estimates as

$$\tilde{b}_k[m] = \mathbf{E}\{b_k[m]\} = \tanh(\lambda_2(b_k[m])/2). \quad (7)$$

Using these data estimates and the channel coefficients' estimates, a soft replica of the interference is computed for each of the K data streams, and then subtracted from the total received signal \mathbf{y} . The remaining signal is processed by the

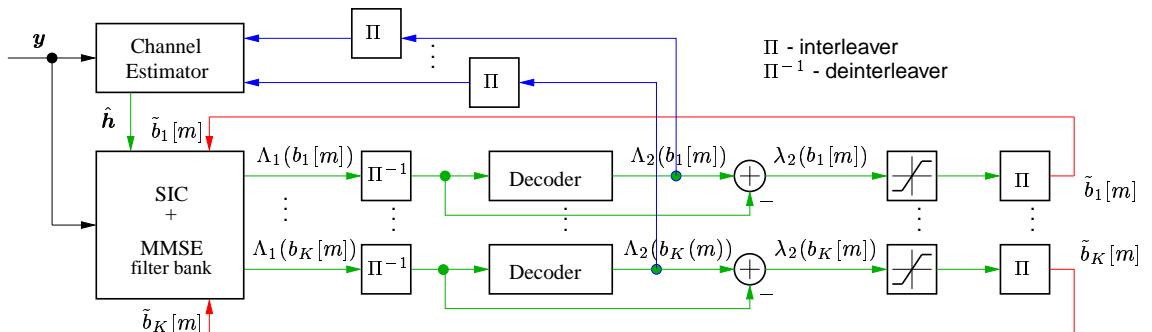


Figure 1: Iterative receiver – block structure

MMSE filter-bank to further suppress the residual interference. In the last iteration, the decoders finally compute hard decisions on information bits.

Channel Estimation

Maximum A Posteriori (MAP) Joint Channel and Data Estimation: Based on $N(M + L - 1)$ observations (collected in vector \mathbf{y}), we need to estimate KNL channel coefficients (collected in \mathbf{h}) and KM data symbols $b_k[m]$. The optimum approach is joint estimation of all the parameters. The joint estimator does not take into account the code constraints. The solution to this estimation problem is obtained by maximizing the joint posterior probability density function (pdf) of \mathcal{B} and \mathbf{h} given the received sequence \mathbf{y} :

$$f(\mathbf{h}, \mathcal{B} | \mathbf{y}) = \frac{f(\mathbf{y}, \mathbf{h} | \mathcal{B}) \Pr(\mathcal{B})}{f(\mathbf{y})} = \frac{f(\mathbf{y}, \mathbf{h} | \mathcal{B})}{f(\mathbf{y})} \prod_{m=0}^{M-1} \prod_{k=1}^K \Pr(b_k[m]) \quad (8)$$

The latter equality holds under the assumption that coded data symbols are independent, which is valid for random interleaving. For each data block there exist 2^{KM} possible hypotheses on transmitted symbols. Due to prohibitive complexity of this approach, we will focus on several suboptimum solutions.

Soft Discrete Symbol Approximation: The probability mass function (pmf) of each symbol is determined by the probability $\Pr(b_k[m]=1)$. We replace this pmf by a single discrete soft value, defined by the mean value $\tilde{b}_k[m]$ calculated as in (7), but using the posterior LLR Λ_2 instead of the extrinsic LLR λ_2 . In the context of previous section, this reduces the number of possible data sequences from 2^{KM} to $1^{KM} = 1$. Let $\tilde{\mathcal{B}}$ denote the matrix obtained by replacing the unknown data symbols in matrix \mathcal{B} by their soft mean values. The conditional pdf of the received signal given the channel vector \mathbf{h} and the data matrix \mathcal{B} is an $N(M + L - 1)$ -dimensional complex Gaussian distribution:

$$f(\mathbf{y} | \mathcal{B}, \mathbf{h}) = (\pi\sigma_v^2)^{-N(M+L-1)} \exp(-\sigma_v^{-2}(\mathbf{y} - \mathcal{B}\mathbf{h})^H(\mathbf{y} - \mathcal{B}\mathbf{h})). \quad (9)$$

The function we wish to maximize equals to (9) with $\tilde{\mathcal{B}}$ instead of \mathcal{B} . We assume that channel coefficients are independent, identically distributed (i.i.d.), complex Gaussian random variables, with zero mean and unit variance. Since the signal model (2) is linear Gaussian, the MAP and MMSE estimator are equivalent:

$$\hat{\mathbf{h}}_{\text{MMSE}}(\mathbf{y}) = \arg \max_{\mathbf{h}} \{\ln f(\mathbf{y} | \mathbf{h}) + \ln f(\mathbf{h})\} = \arg \min_{\mathbf{h}} \left\{ \sigma_v^{-2} \|\mathbf{y} - \tilde{\mathcal{B}}\mathbf{h}\|^2 + \|\mathbf{h}\|^2 \right\} = \left(\tilde{\mathcal{B}}^H \tilde{\mathcal{B}} + \sigma_v^2 \mathbf{I} \right)^{-1} \tilde{\mathcal{B}}^H \mathbf{y}. \quad (10)$$

If we assume no prior statistical knowledge of the channel, the coefficients $h_{nk}(l)$ are modelled as deterministic unknown parameters. The likelihood function of the received symbol has the same form as (9). The least-squares (LS) like solution yields the following estimator:

$$\hat{\mathbf{h}}_{\text{LS}}(\mathbf{y}) = \arg \max_{\mathbf{h}} \{\ln f(\mathbf{y}; \mathbf{h})\} = \arg \min_{\mathbf{h}} \left\{ \|\mathbf{y} - \tilde{\mathcal{B}}\mathbf{h}\|^2 \right\} = \left(\tilde{\mathcal{B}}^H \tilde{\mathcal{B}} \right)^{-1} \tilde{\mathcal{B}}^H \mathbf{y}. \quad (11)$$

This result is just a special case of (10) for no prior knowledge on \mathbf{h} . Unlike MMSE, the LS estimator does not require knowledge of the noise power, i.e., the signal-to-noise (SNR) ratio.

Gaussian Symbol Approximation: In [9] it was proposed to replace the discrete pmf of each data symbol by the continuous Gaussian pdf with same mean value and variance. We found this approach too complex.

SIMULATION RESULTS

We simulated the iterative receiver for a MIMO system with 2 users and 2 receive antennas. The users transmitted data blocks encoded with a convolutional code of rate 1/2 and constraint length 3. Each block was preceded by a short (15 symbols) or long (50 symbols) training sequence. We limited the number of iterations to 4. Fig. 2. shows the relative channel estimation error in the initial and the final iteration, for LS and MMSE estimator using soft decision feedback. For both cases, significant improvement of the channel estimate throughout the iterative process is obvious. For large number of pilot symbols prior statistical knowledge of the channel is not influential, thus MMSE performs almost identically as LS. For small number of pilots, however, the initial MMSE channel estimate is much better, since it exploits the knowledge of the noise level. After 4 iterations both estimators converge to the same average error. This implies that simple LS estimator (that does not require SNR estimate) can be employed and that pilot sequence can be reduced without performance loss.

Fig. 3 shows the Bit Error Rate (BER) after final iteration for iterative receiver with LS channel estimator using one of the three types of data decision feedback: soft estimates of all the symbols, hard estimates of all the symbols, or hard estimates only of "reliable" symbols, as proposed in [10]. The reliability threshold was set to LLR level of 0.5. All three curves coincide for large number of pilot symbols, since the initial channel estimate is already good enough and leads to the same performance. However, for short pilot sequences, soft feedback outperforms both hard feedback approaches. In

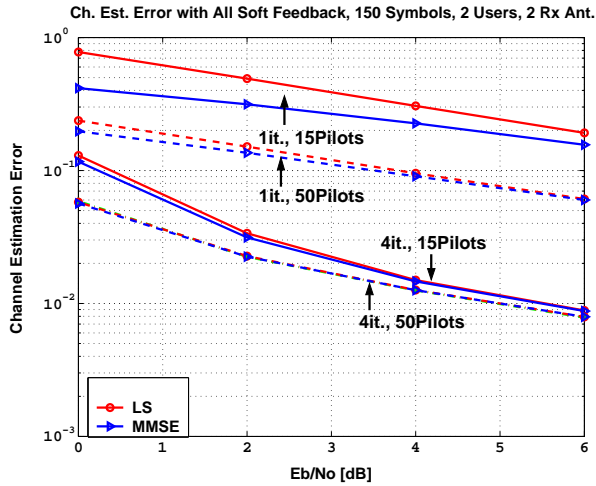


Figure 2: Channel estimation error for LS and MMSE estimator, soft feedback

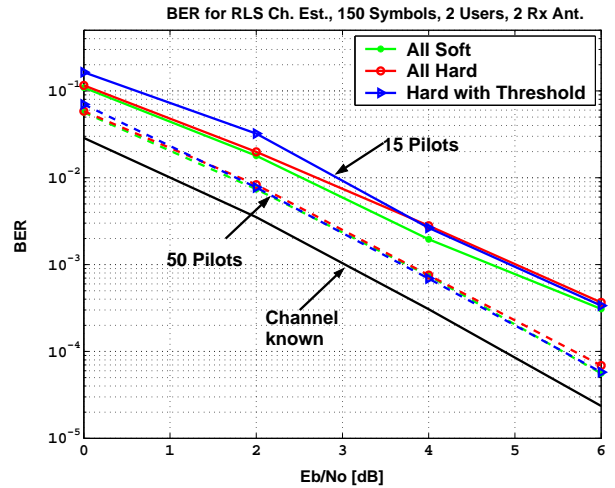


Figure 3: BER for 3 types of decision feedback, LS channel estimation

low SNRs, very small number of symbols exceeds the reliability threshold, thus, the "reliable hard" feedback performs worst. In high SNRs, the three feedback types become equivalent, as expected. Our simulations show that the superiority of soft feedback increases with the number of users in a system (the results are not shown here due to lack of space).

SUMMARY AND CONCLUSIONS

An iterative receiver is proposed for MIMO systems in frequency-selective fading channels. The receiver consists of the channel estimator, soft interference canceller followed by an MMSE filter-bank and SISO decoders. We focus on the iterative channel estimation and derive modified LS and MMSE estimators that use the whole block of received data for estimating the channel. The unknown data symbols are replaced by their soft estimates. Simulations show that the performances of both estimators become identical after 4 iterations. For SNR higher than 4 dB, due to iteration gain the number of pilot symbols can be significantly reduced without increasing the channel estimation error. Finally, we compared the performance of the receiver employing LS channel estimator that uses soft data estimates, with the equivalent receiver employing all hard estimates or only reliable hard estimates. It is shown that soft feedback approach outperforms both solutions with hard decision feedback. The advantage of the soft feedback is particularly significant for lower SNR levels and for higher number of users.

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