

# Spectral Efficiency of CDMA Systems with Linear Interference Suppression

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**Abstract** — Spectral efficiency of CDMA systems with linear interference suppression and channel coding is calculated. For interference suppression by decorrelation (zero-forcing, ZF) an analytical solution is given. For interference suppression according to the minimum mean squared error (MMSE) criterion a numerical approximation is applied. Hereby, it turns out, that the optimum modulation scheme and the optimum code rate are depending on both signal-to-noise ratio and power efficiency of the applied channel code.

## I. INTRODUCTION

Code-division multiple-access (CDMA) is a promising technique for future mobile communication systems. Its main advantages are a wideband transmitter signal, which combats the frequency-selective fading of mobile radio channels, and its ability to work without synchronization among the different users. Furthermore, the same carrier frequency can be used in every cell. Caused by multiuser interference which appears in CDMA systems with randomly chosen signature waveforms spectral efficiency  $\gamma$  is upper bounded if the demodulator does not exploit knowledge about the statistical properties of the interference but assumes it to be additive white Gaussian noise (AWGN). This upper bound results from information theory and is given by  $\gamma \leq 1.44$  bit/(sec·Hz) for single cell systems [1]. In practice, e.g. for existing channel codes, it is even much lower.

If the receiver takes into account the signals of all users in an optimum manner, there is no upper bound like the one mentioned above any longer. However, this results in a demodulator of huge complexity, cf. [2]. A reasonable trade-off between spectral efficiency and complexity are e.g. successive cancellation by decision feedback [3], and interference suppression by linear filtering proposed in [4, 5, 6].

For the linear demodulator which was introduced in [5], that we will call SLISE (scalar linear

interference suppression equalization) in the following, a more detailed analysis is given in this paper. Hereby, the considerations are restricted to the AWGN channel. SLISE only differs from the conventional demodulator (CD) by correlation not with the signature waveform of the user of interest, but with that sequence which minimizes the mean squared error at its output. This sequence depends on the signature waveforms of all users and the properties of the channel and is able to be estimated by adaptive algorithms, cf. [7].

## II. SYSTEM MODEL

In the following a synchronous single cell CDMA system disturbed by AWGN is considered. It is described by its equivalent complex baseband representation. After matched filtering due to the chip waveform and sampling at the chip rate the discrete time representation of the received signal of one symbol period

$$\mathbf{r} = \sum_{k=1}^K a_k \mathbf{p}_k + \mathbf{n}$$

is given as the sum over the  $K$  users of the transmitted spreading vectors  $\mathbf{p}_k = [p_{1,k}, p_{2,k}, \dots, p_{N,k}]^T$  of length  $N$  weighted by the symbols to transmit  $a_k$  and the vector of the AWGN  $\mathbf{n}$ , where without loss of generality  $E\{|a_k|^2\} = 1$  is assumed.

Linear demodulation means, that the value to be detected

$$d_k = \mathbf{h}_k^H \mathbf{p}_k$$

is calculated from the received signal by linear filtering, where  $\cdot^H$  denotes the Hermit operator. The value to be detected is fed into a subsequent channel decoder. Hereby, the filter vector  $\mathbf{h}_k$  is to be chosen properly, e.g. due to the MMSE or ZF criterion. For CD it holds  $\mathbf{h}_k = \mathbf{p}_k$ .

## III. AVERAGE SIGNAL-TO-INTERFERENCE RATIO APPLYING DECORRELATION

In the following no certain choice of signature waveforms is presupposed. For the theoretical analysis

the are assumed to be chosen randomly. In [8] upper and lower bounds for the near–far–resistance averaged over all signature waveforms have already been given. In our paper the signal–to–interference ratio averaged over all signature waveforms is calculated.

In the following  $\mathbf{p}\mathbf{p}^H = 1$  is assumed, i.e. perfect power control. This is only a restriction of generality for MMSE equalization, but not for decorrelation, as for decorrelation only the presence of another user is important, and not its power. This assumption normalizes the energy per symbol to be  $E_s = 1$ . Implied by perfect power control there is no difference in the statistics of the different users. Therefore, user  $k = 1$  is considered without loss of generality. In [5] the signal–to–interference ratio of MMSE equalization at demodulator output for known signature waveforms was shown to be

$$\text{SIR} = \mathbf{p}_1^H \mathbf{P}^{-1} \mathbf{p}_1. \quad (1)$$

Hereby,  $\mathbf{P} = \sum_{k=2}^K \mathbf{p}\mathbf{p}^H + \text{E}\{\mathbf{nn}^H\}$  is denoting the covariance matrix of total distortion (interference and AWGN). Eq. (1) is only valid in this simplified form for symbol synchronous CDMA, cf. [5].

Nevertheless, in [9] it is shown, that arbitrary time offsets between users' signals (even fractions of the chip intervall) do not cause performance degradation, if the receiver filter has got sufficiently large number of taps. Therefore, the subsequent results calculated for synchronous CDMA hold for asynchronous CDMA as a lower bound.

For calculation of the average signal–to–interference ratio the mean of Eq.(1) is to be found. In this context the harmonic mean has to be used, as it is not reasonable to average signal–to–interference ratios, but to average interference and noise power.

The covariance matrix of the total distortion is symmetrical and can be transformed by a unitary transformation into a diagonal matrix. Therefore, without loss of generality it is assumed to be diagonal and its eigenvalues to be  $\lambda_i$ ,  $i = 1 \dots N$ . This implies

$$\text{SIR} = \sum_{i=1}^N \frac{|p_{i,1}|^2}{\lambda_i}.$$

For sufficiently large  $N$  the transformed spreading vectors are uniformly distributed on the  $(N - 1)$ –dimensional surface of an  $N$ –dimensional sphere with radius 1. Such a distribution is obtained, if a Gaussian random vector is normalized to its length. This implies the following representation of

the spreading vector:

$$\mathbf{p} = \frac{[g_1 g_2 \dots g_N]^T}{\sqrt{\sum_{i=1}^N |g_i|^2}} \quad (2)$$

Hereby,  $g_i$  are denoting i.i.d. zero mean Gaussian random variables with arbitrary<sup>1</sup>, but equal variance. This implies

$$\text{SIR} = \sum_{i=1}^N \frac{|g_i|^2}{\lambda_i} / \sum_{i=1}^N |g_i|^2 \quad (3)$$

Considering decorrelation the power of the interfering users is not of interest. Therefore, it is assumed to be infinite without loss of generality. In this case the above given equations holding for MMSE equalization can be applied to decorrelation. With the assumption, that the spreading vectors of all  $K$  users are linearly independent, it holds

$$\begin{aligned} \lambda_1 &= \lambda_2 = \dots = \lambda_{K-1} = \infty \\ \lambda_K &= \lambda_{K+1} = \dots = \lambda_N = \sigma_n^2 \end{aligned}$$

with  $\sigma_n^2 = \text{E}\{|n_i|^2\}$  denoting the variance of the AWGN. From Eq. (3) the SIR of decorrelation results

$$\text{SIR}_{\text{ZF}} = \frac{1}{\sigma_n^2} \cdot \frac{c_1^2}{c_1^2 + c_2^2}$$

with the definitions  $c_1^2 \stackrel{\text{def}}{=} \sum_{i=K}^N |g_i|^2$  and  $c_2^2 \stackrel{\text{def}}{=} \sum_{i=1}^{K-1} |g_i|^2$ .

As stated in [10]  $c_1^2$  is a  $\chi^2$ –distributed random variable with  $N - K + 1$  degrees of freedom for real and  $2(N - K + 1)$  degrees of freedom for complex signature waveforms, respectively. The random variable  $c_2^2$  is also  $\chi^2$ –distributed, but with  $K - 1$  or  $2K - 2$  degrees of freedom, respectively. For simplicity in the following only complex signature waveforms are considered, as the generalization to real ones is obvious. With [11, p. 515] the probability density function (pdf) of the decorrelator's SIR is then given within the intervall  $0 \leq \text{SIR}_{\text{ZF}} \leq \sigma_n^{-2}$

$$f_{\text{SIR}_{\text{ZF}}}(y) = \sigma_n^2 \frac{(\sigma_n^2 y)^{N-K} (1 - \sigma_n^2 y)^{K-2}}{\text{B}(N - K + 1, K - 1)}$$

with

$$\text{B}(a, b) \stackrel{\text{def}}{=} \int_0^1 t^{a-1} (1-t)^{b-1} dt \quad (4)$$

denoting the beta function. Using Eq. (4) its  $m^{\text{th}}$  moment turns out to be

$$\text{E}\{\text{SIR}_{\text{ZF}}^m\} = \sigma_n^{-2m} \frac{\text{B}(N - K + 1 + m, K - 1)}{\text{B}(N - K + 1, K - 1)}$$

<sup>1</sup>Due to the normalization by the denominator of Eq. (2) the variance of this dummy variables does not affect anything.

The harmonic mean of the decorrelator's SIR is given by

$$\overline{\text{SIR}}_{\text{ZF}} = \text{E}^{-1}\{\text{SIR}_{\text{ZF}}^{-1}\} = \sigma_n^{-2} \frac{N-K}{N-1} \quad (5)$$

and the variance of the average distortion power by

$$\begin{aligned} \text{Var}\{\text{SIR}_{\text{ZF}}^{-1}\} &= \text{E}\{\text{SIR}_{\text{ZF}}^{-2}\} - \overline{\text{SIR}}_{\text{ZF}}^{-2} \\ &= \sigma_n^4 \frac{(N-1)(K-1)}{(N-K)^2(N-K-1)} \end{aligned} \quad (6)$$

In [12, Chap. 3.1] it is shown, how to calculate the pdf of the SIR for arbitrary eigenvalue distributions. This enables an analytical analysis for MMSE equalization, too. Unfortunately, the eigenvalue distribution of MMSE equalization is only known for  $N = K$  to the authors. For this case, it can be found in [13].

#### IV. SPECTRAL EFFICIENCY

Spectral efficiency of a CDMA system

$$\gamma = \frac{\text{total information rate of all users}}{\text{total bandwidth of all users}}$$

can be expressed by involving the code rate  $R$  [bit/symbol], cf. [14]

$$\gamma = \frac{K}{N} R. \quad (7)$$

The number of users  $K$  which are allowed to transmit at the same time are determined by the tolerable bit error rate and the applied digital transmission scheme. Considering the best possible transmission scheme, i.e this one achieving channel capacity, it holds for complex Gaussian distributed symbols, cf. [10],

$$R = \log_2(1 + \text{SIR}) \quad (8)$$

For transmission schemes with finite symbol alphabet a relationship equivalent to Eq. (8) can be given by numerical integration [15].

For CD the average signal-to-interference ratio is given by, cf. [1],

$$\overline{\text{SIR}}_{\text{CD}} = \left[ \frac{K-1}{N} + \sigma_n^2 \right]^{-1} \quad (9)$$

Eq. (9) is the equivalent relationship for CD as Eq. (5) is for decorrelation.

Using  $E_b/N_0 = \sigma_n^{-2}/R$  and Eqs. (7) to (9) the relationship between power and bandwidth efficiency is obtained for CD:

$$\left( \frac{E_b}{N_0} \right)_{\text{CD}} = \frac{1}{\frac{\gamma \frac{N}{K}}{2\gamma^{N/K} - 1} - \gamma \frac{K-1}{K}} \quad (10)$$

Using Eq. (5) instead of Eq. (9) the equivalent relationship for the decorrelator results:

$$\left( \frac{E_b}{N_0} \right)_{\text{ZF}} = \frac{N-1}{N-K} \cdot \frac{2\gamma^{N/K} - 1}{\gamma \frac{N}{K}} \quad (11)$$

Eqs. (10) and (11) are only valid if spreading factor  $N$  tends to infinity, as otherwise the variance of the average distortion power does not vanish, see Eq. (6). Therefore, in the following we assume  $K, N \gg 1$ , yielding:

$$\left( \frac{E_b}{N_0} \right)_{\text{CD}} = \frac{1}{\frac{\gamma \frac{N}{K}}{2\gamma^{N/K} - 1} - \gamma} \quad (12)$$

$$\left( \frac{E_b}{N_0} \right)_{\text{ZF}} = \frac{2\gamma^{N/K} - 1}{\gamma \frac{N}{K} - \gamma} \quad (13)$$

In coded CDMA systems bandwidth extension can be performed by two different ways, by spreading and/or redundancy of channel coding. It can be shown, that for CD it is optimum to use only channel coding and no spreading. This implies the optimum rate  $R_{\text{CD,opt}} = 0$ . As spectral efficiency is finite, it follows from Eq. (7) the optimum ratio  $(N/K)_{\text{CD,opt}} = 0$ . Inserting this result into Eq. (12) it is obtained

$$\min_{N/K} \left( \frac{E_b}{N_0} \right)_{\text{CD}} = \frac{1}{\log_2(e) - \gamma}. \quad (14)$$

For the decorrelator no closed form solution for the optimum  $K/N$ -ratio has been found, yet. So a numerical optimization has been used, which leads to the result depicted in Fig. 1. Concerning MMSE equalization no method to calculate spectral efficiency analytically is known to the authors. It has been determined using Monte Carlo simulations with  $N = 512$  and Eq. (1).

Caused by the asymptotic equivalence ( $E_b/N_0 \rightarrow \infty$ ) of the decorrelator and the MMSE equalizer both curves converge to each other for large signal-to-noise ratio. Applying Eq. (13) the asymptotic loss of SLISE in comparison to the single-user Shannon bound is shown to be

$$V_\infty = \frac{\min_{N/K} \left( \frac{E_b}{N_0} \right)_{\text{ZF}}}{\gamma} \Bigg|_{\gamma \gg 1} = e \ln(2) \gamma$$

in the Appendix. Obviously, it is proportional to spectral efficiency. Even for  $\gamma \geq 4$  its difference to the exact loss is very small, see Fig. 1.

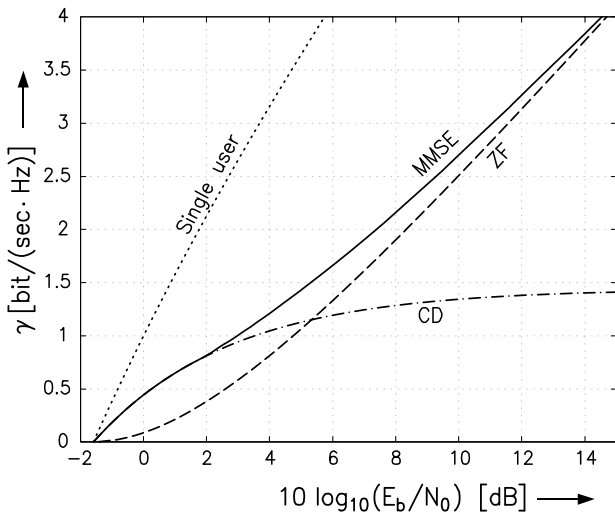


Fig. 1: Power-bandwidth-plane for various CDMA equalizers using digital transmission schemes operating at the single-user Shannon bound.

## V. OPTIMUM CHOICE OF WELL-KNOWN MODULATION SCHEMES

In the previous section optimum channel coding was presupposed. This is motivated by the fact, that there are implementable coded modulation schemes for arbitrary rates approaching the Shannon limit to less than 1 dB, cf. [16]. For several applications their large delay causes serious problems. Therefore, SLISE is also considered in connection with well-known convolutional codes or trellis-coded modulation [17], respectively.

A relationship connecting rate and signal-to-noise ratio as Eq. (8) does is also available for convolutional codes, but it depends on the tolerable bit error rate (BER).

Assuming a target bit error rate  $\text{BER} = 10^{-3}$  the results are depicted in Fig. 2. In contrast to the conventional demodulator there is neither any modulation scheme, nor any code rate, being optimum in general. They both are depending on signal-to-noise ratio. For nearly every modulation scheme taken into account there is an interval of signal-to-noise ratio within it outperforms the others due to its spectral efficiency. Only 2-ary phase shift keying (2PSK) turned out to be worse than 4-ary phase shift keying (4PSK), in general. This is a result of 2PSK's lower spectral efficiency in comparison to 4PSK, which is not balanced by higher power efficiency. The fundamental statement valid for CD, that for CDMA systems only power efficiency is the matter of interest, does not hold if interference suppression is applied. This is also implied by the fact, that such systems are not interference limited [8].

For increasing order of quadrature amplitude modulation (QAM) the gain in spectral efficiency in comparison to CD is rapidly increasing. E.g. trellis-coded 64-ary QAM enables a four times higher spectral efficiency than CD.

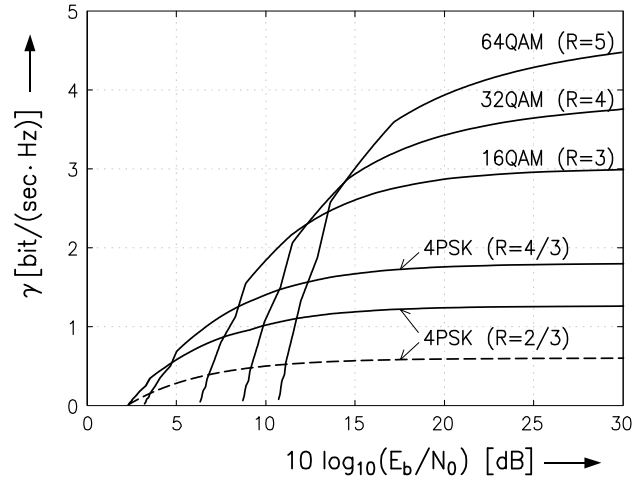


Fig. 2: Power-bandwidth-plane for CDMA demodulation using MMSE equalization and convolutional codes with 64 states. Comparison is based on a tolerable  $\text{BER} = 10^{-3}$ . The dashed line refers to CD.

## VI. SUMMARY

Spectral efficiency of CDMA systems based on linear interference suppression has been derived. Hereby, it turned out, that decorrelation can be treated analytically for asymptotically large spreading factor. MMSE equalization has been examined by Monte Carlo simulations.

The comparison between CD und linear interference suppression was not based on bit error measurements, but by power-bandwidth-planes. This is a more general treatment than the previous work concerning this topic, e.g. [5].

It has been shown, that linear interference suppression and coded modulation enables significantly increasing spectral efficiency. Hereby, the problem of estimation errors in the optimum filter was not taken into account. This may be cause some loss due to imperfect filter adjustments and by additional power which is needed to transmit training data. This effects shall be addressed in future work.

## APPENDIX

It has to be shown that

$$V_{\infty} = \frac{\min_{N/K} \left( \frac{E_b}{N_0} \right)_{\text{ZF}}}{\gamma} \Bigg|_{\gamma \gg 1} = e \ln(2) \gamma.$$

holds. With Eq. (13)

$$\begin{aligned} \frac{\min_{N/K} \left( \frac{E_b}{N_0} \right)_{ZF}}{\frac{2^\gamma - 1}{\gamma}} &= \min_{N/K} \frac{2^{\gamma N/K} - 1}{(2^\gamma - 1)(N/K - 1)} \\ &= \min_{N/K} \frac{2^{\gamma(N/K-1)} - 2^{-\gamma}}{(1 - 2^{-\gamma})(N/K - 1)} \end{aligned}$$

becomes obvious. For large  $\gamma$  the terms  $2^{-\gamma}$  can be neglected. But note, that the term  $2^{\gamma(N/K-1)}$  will tend to a finite value, as the optimum  $K/N$ -ratio is converging towards 1. Therefore, with the substitution  $x \stackrel{\text{def}}{=} N/K - 1$  we obtain by simple first order derivation

$$\left. \frac{\min_{N/K} \left( \frac{E_b}{N_0} \right)_{ZF}}{\frac{2^\gamma - 1}{\gamma}} \right|_{\gamma \gg 1} = \min_x \frac{2^{\gamma x}}{x} = e \ln(2) \gamma$$

completing the proof.

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