ON SAMPLING ISSUES OF A VIRTUALLY ROTATING MIMO ANTENNA

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ABSTRACT

This paper investigates the sampling issues of a virtually rotating antenna. By using parasitic elements, directive antenna patterns can be created, which can be rotated 360 degrees around with discrete steps. The results presented are a continuation of the work in [1]. In that paper a compact rotating MIMO-antenna was presented, however without going into the details of the sampling issues. We further discuss the interference and noise issues related to this kind of antenna.

1. INTRODUCTION

One of the disadvantages of MIMO-systems is the requirement of the distance between the antennas to be at least half the wavelength for the signals to be sufficiently uncorrelated. In [1] a compact MIMO-receiver was proposed, which had only one active receiver antenna. However parasitic elements was needed to be able to form desired antenna patterns. This receiver could be much more compact than the regular MIMO-receiver. The mentioned paper did not go into the details of the practical implementations. This paper will address some of the issues related to sampling-effects, interference and noise.

There are other papers [2, 3, 4, 5] that have considered the use of parasitic elements to achieve directive antenna patterns. But they have not considered the possibility of rotating the antenna pattern during a symbol period to achieve a MIMO-system.

2. BACKGROUND: CONTINUOUS ROTATING ANTENNA

In [1] the concept of a directive antenna which could be rotated 360 degrees around sufficiently fast was proposed. The idea is that the antenna picks up different linear mixtures of the different signal paths if the antenna is rotated once or several times during a symbol period. This antenna is not realizable, but is presented to describe the concept. The received signal at the antenna connector can be written:

$$r(t) = \sum_{p=1}^{P} a(\omega t + \alpha_p) s_p(t) \tag{1}$$

$a(\omega t)$ is the antenna pattern function which describes the rotation at an angular frequency ω, $s_p(t)$ is the signal arriving at the azimuth angle α_p. We are assuming P different signals arriving at the antenna. Since the antenna is rotating, this means that the antenna pattern function is a periodic function. Therefore it can be Fourier-expanded. Expanding the antenna pattern function and putting this expression into (1) gives:

$$r(t) = \sum_{l=-L}^{+L} \exp(jl\omega t)\, \underbrace{a_l \sum_{p=1}^{P} \exp(jl\alpha_p) s_p(t)}_{r_l(t)} \tag{2}$$

In matrix notation the received signal can be expressed:

$$\underbrace{\begin{bmatrix} r_{-L}(t) \\ \vdots \\ r_{+L}(t) \end{bmatrix}}_{\mathbf{r}(t)} = \underbrace{\begin{bmatrix} a_{-L}(t) & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & a_{+L} \end{bmatrix}}_{\mathbf{A}} \tag{3}$$

$$\times \underbrace{\begin{bmatrix} e^{-jL\alpha_1} & \vdots & e^{-jL\alpha_P} \\ \vdots & \ddots & \vdots \\ e^{+jL\alpha_1} & \dots & e^{+jL\alpha_P} \end{bmatrix}}_{\mathbf{V}} \underbrace{\begin{bmatrix} s_1(t) \\ \vdots \\ s_P(t) \end{bmatrix}}_{\mathbf{s}(t)}$$

a_l is the Fourier-coefficients of the antenna pattern function $a(\omega t)$. From this matrix notation it is evident that rotating the antenna during a symbol period gives a MIMO-system. The difference is that instead of receiving at different antennas the reception is at different frequency bands. Here we have $2L + 1$ different frequency bands. $\mathbf{V}$ is a Vandermonde matrix which has the phases of the incoming signal-paths as its entries. This matrix may cause some minor eigenvalue spread to the complete propagation matrix, but it is not the main factor concerning mutual information. A higher importance lies on the **A**-matrix, which consist of the Fourier-coefficients of the antenna pattern function. We can conclude that an antenna pattern function that has a high number of harmonics is beneficial for high mutual information. It should be noted that the purpose of this compact MIMO-receiver is to provide spatial multiplexing and not beam-forming.

3. DISCRETE ROTATION

As described in [1] an approximation to the continuous rotating antenna is an antenna which rotates its antenna pattern 360 degrees around with discrete steps. This is possible by the use of parasitic elements which are placed around the active antenna. By putting electronic switches or bridging wires in the middle of the parasitic elements, it should be possible to let them be short circuited and open circuited in a Time-Division-Multiple-Access kind of way. This gives the possibility of the antenna pattern to rotate in discrete steps. This antenna since it has a virtually rotation is called a virtual rotating antenna.

We are assuming in this paper that the antenna pattern function is frequency independent. In reality it will be a function of frequency and can be written $a(\Omega, \omega t)$, but to simplify the analysis we assume that $a(\omega t)$ is the antenna pattern function for all frequencies.

3.1. Sampling the antenna pattern function

To understand what happens when the antenna pattern rotates with discrete steps, it is advantageous to describe the operation in mathematical terms. First consider this simple example: Four parasitic elements are placed around one active receiver antenna. When one of the parasitic elements are short circuited, the antenna pattern gets directive and points in a certain direction. When a switching operation is performed which short-circuits the next parasitic element and open circuits the others, the antenna pattern will point in another direction. The antenna points in this direction until the next switching operation. To model the discrete rotation we first consider that the antenna pattern function $a(\omega t)$ is sampled. The sampling frequency is the same as the rate of switching. The sampled antenna pattern function which we denote as $g_p(t)$ can be expressed as:

$$g_p(t) = \sum_{n=-\infty}^{\infty} a(\omega t + \alpha_p)\delta(t - nT_s) \tag{4}$$

Where T_s is the reciprocal of the sampling frequency. The sampling operation does not describe the whole situation when the antenna pattern rotates with discrete steps. Consider folding the sampled antenna pattern function with a rectangular pulse. This makes sure that the antenna pattern function is held constant between each discrete rotation step. The duration of this rectangular pulse should be equal to the sampling interval i.e T_s. The equation describing the sampled and folded antenna pattern function is given by:

$$\begin{aligned} & g_p(t) * p(t) \\ &= \sum_{n=-\infty}^{\infty} \int_{\tau=-\infty}^{\infty} p(\tau) a(\omega(t-\tau) + \alpha_p)\delta(t - nTs - \tau) d\tau \\ &= \sum_{n=-\infty}^{\infty} p(t - nT_s) a(\omega nTs + \alpha_p) \end{aligned} \tag{5}$$

$p(t)$ is the rectangular function. It should be addressed that this folding operation in time domain is a multiplication in frequency domain. A simple example should clarify the consequence of this: Consider that the antenna pattern function $a(\omega t)$ has three harmonics. Let us assume that the antenna pattern is sampled with a sampling frequency $\Omega_s = 3\omega$. This means that the antenna pattern is sampled three times during one continuous rotation. This creates repeated spectrum of the sampled antenna pattern function, with a distance Ω_s between each repeated frequency component. Folding the rectangular pulse with the sampled antenna pattern function in the time domain, is equivalent to a multiplication of a sinc-function with the repeated spectrum of the sampled antenna pattern function in the frequency domain. Figure 1 shows the spectrum of the sampled antenna pattern function and the sinc-function in the same plot. Figure 2 shows the two functions multiplied together. For the continuous rotating case we observe from (3) that the received signal is expanded in frequency. The expanding factor in this case is equal to the number of harmonics of the antenna pattern. For the discrete rotation case the bandwidth of the received signal gets expanded much more. If we now consider sampling the antenna pattern function faster than what should be necessary according to the sampling-theorem, for example $\Omega_s = 9\omega$, then there is a separation between each repeated component equal to 9ω. Figure 4 shows the repeated spectrum and the sinc-function in the same plot. The resulting antenna pattern function in the frequency domain for the discrete rotation case is depicted in figure 3.

3.2. Consequences of discrete rotation steps on the number of receive branches

From the previous section it is clear that the frequency bandwidth of the received signal is wider for the discrete rotation steps case than the continuous rotation case. To get a matrix expression for the received signal as in (3) for the continuous case, we sample the Fourier-expanded antenna pattern function:

$$a(\omega nT_s + \alpha_p) = \sum_{l=-L}^{L} a_l e^{jl(\omega nT_s + \alpha_p)} \tag{6}$$

The interpretation is that each harmonic is sampled. We denote the received signal for the discrete rotation case by $\tilde{r}(t)$.

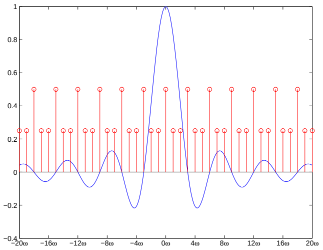

Fig. 1. The repeated spectral copies of the sampled antenna pattern function are shown in the red plot. The blue plot shows the sinc-function, which results because of the rectangular pulse in time domain. A sampling frequency $\Omega_s = 3\omega$ is assumed.

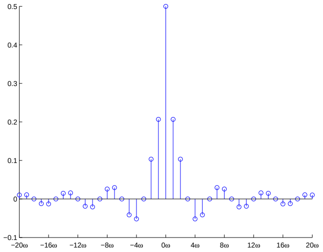

Fig. 2. The spectral copies of the sampled antenna pattern function multiplied with the sinc-function. $\Omega_s = 3\omega$ is assumed.

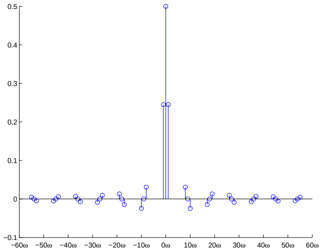

Fig. 3. The sinc-pulse and the sampled antenna pattern function multiplied together. $\Omega_s = 9\omega$

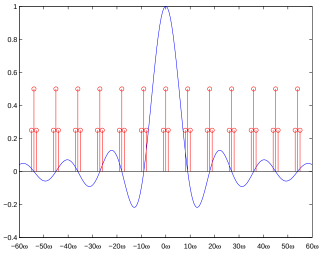

Fig. 4. A higher sampling frequency, $\Omega_s = 9\omega$

Realizing that:

$$\tilde{r}(t) = \sum_{p=1}^{P} \Big(g_p(t) * p(t) \Big) \cdot s_p(t) \tag{7}$$

and inserting (6) into (5), and then the resulting expression into (7) gives:

$$\tilde{r}(t) = \\ \sum_{p=1}^{P} \sum_{l=-L}^{L} \sum_{n=-\infty}^{\infty} p(t - nT_s) a_l e^{jl\omega nT_s} e^{jl\alpha_p} s_p(t) \tag{8}$$

For continuous rotation of the antenna the received signal consist of $2L+1$ subbands. For discrete rotation the signal energy is spread over an infinitely large band. We now define a received signal vector $\tilde{\mathbf{r}}(t)$ which has $2K(2L+1)+2L+1$ elements. Each element in the vector is the signal at a certain sub-band. K is here an integer which defines how many of the spectral copies we include at the receiver. Writing the received signal in matrix notation gives:

$$\tilde{\mathbf{r}}(t) = \begin{bmatrix} \tilde{r}_{-K(2L+1)-L}(t) \\ \vdots \\ \tilde{r}_{+K(2L+1)+L}(t) \end{bmatrix} = \mathbf{PAVs}(t) \tag{9}$$

$\mathbf{P}$ is the matrix which spreads the received signal over a wider band than for the continuous rotating case. The dimension of $\mathbf{P}$ is $(2K(2L+1)+2L+1) \times (2L+1)$. Writing just a few terms of matrix $\mathbf{P}$ gives:

$$\underbrace{\begin{bmatrix}
\vdots & \vdots & \vdots & \vdots & \vdots \\
P(-\omega L - \Omega_s) & & & & \\
\vdots & \ddots & & & \\
\vdots & & P(-\Omega_s) & & \\
 & & & \ddots & \\
0 & & & & P(\omega L - \Omega_s) \\
P(-\omega L) & 0 & & & \\
 & \ddots & \vdots & & \\
 & & P(\omega \cdot 0) & & \\
 & & & \ddots & \\
 & & & & P(\omega L) \\
P(-\omega L + \Omega_s) & & 0 & & \\
\vdots & \ddots & & & \\
\vdots & & P(\Omega_s) & & \\
 & & & \ddots & \\
 & & & & P(\omega L + \Omega_s) \\
\vdots & \vdots & \vdots & \vdots & \vdots
\end{bmatrix}}_{2L+1} \tag{10}$$

Where the sinc-function is defined as: $P(\Omega) = \mathrm{sinc}(\frac{\Omega}{\Omega_s})$. From matrix $\mathbf{P}$ some power considerations can be made. When $\mathbf{P}$ is multiplied with $\mathbf{A}$, each of the Fourier-components of the antenna pattern function get spectral copies in higher frequency bands. Take Fourier-component a_{-L} for example. After the transformation represented by $\mathbf{P}$, we get $P(-\omega L)a_{-L}$ in frequency band ωL, and shifted components $P(k\Omega_s - \omega L)a_{-L}$ in frequency bands $k\Omega_s - \omega L$ when $k = -K \dots K$. Summing the power of the frequency shifted harmonics gives the same power as for the continuous rotating case, if we are assuming that $K \to \infty$. Writing the expression for the sum of the power of the frequency shifted versions of harmonic l gives:

$$\mathrm{Power}_l = \sum_{k=-\infty}^{\infty} |a_l P(k\Omega_s + \omega l)|^2 = |a_l|^2 \tag{11}$$

To see how the discrete rotation of the antenna pattern affects the mutual information, we first write the received signal including the channel matrix $\mathbf{H}$:

$$\mathbf{r} = \mathbf{PAVHx} + \mathbf{n} \tag{12}$$

Where $\mathbf{H}$ describes the channel path from the transmitter antennas to the scatterers, $\mathbf{x}$ is the vector with the transmitted symbols, and $\mathbf{n}$ is the noise vector. We assume that the channel is unknown to the transmitter and that the transmitter is temporally and spatially wide. When the receiver is assumed to know the channel we can express the mutual information as :

$$I(\mathbf{x};\mathbf{r}) = \tag{13}$$

$$\log_2\det\left(\mathbf{I} + \frac{1}{N_0}\mathbf{PAVHH}^H\mathbf{V}^H\mathbf{A}^H\mathbf{P}^H\right) = \tag{14}$$

$$\log_2\det\left(\mathbf{I} + \frac{1}{N_0}\mathbf{AVHH}^H\mathbf{V}^H\mathbf{A}^H\mathbf{P}^H\mathbf{P}\right) = \tag{15}$$

$$\log_2\det\left(\mathbf{I} + \frac{1}{N_0}\mathbf{AVHH}^H\mathbf{V}^H\mathbf{A}^H\right) \tag{16}$$

Which is equal to the expression of mutual information for a continuous rotating antenna. From (14) to (15) the determinant principle is used, which says that the order of the matrices inside the determinant can be changed as long as the dimension of the identity matrix is also changed. The step from (15) to (16) exploits that $\mathbf{P}^H\mathbf{P} = \mathbf{I}$. This last step holds if we are assuming that the antenna pattern function is sampled according to the sampling theorem, and that $\mathbf{P}$ includes all the frequency bands that the signal is spread over, in other words an infinite number of bands.

3.3. Interference/noise considerations

In [1] the interference issues were addressed, and it was concluded that not only the desired signal gets expanded in frequency, but also signals in other frequency bands. This means that signals from adjacent bands get folded into our bands of interest. Thus the SINR ratio in each frequency band decreases. Let us assume that the noise power in each of the sub-bands is the same, and that the noise in different sub-bands are uncorrelated. We denote the noise power in each sub-band as N_0. The noise we get in each sub-band as a result of the discrete rotation is denoted by P_N. In the same way as (11) we can find the power of the noise:

$$P_N(\Omega) = \sum_{l=-L}^{L}\sum_{k=-\infty}^{\infty} |a_l|^2 |P(k\Omega_s + l\omega)|^2 N_0 \tag{17}$$

This expression shows that the noise power in each sub-band is equal under the assumptions we have made. Let us review the expression for mutual information. If we assume that the received signal power is P_r for an ordinary antenna that does not rotate, then the mutual information for our virtually rotating antenna can be expressed as:

$$I(\mathbf{r};\mathbf{x}) = \log_2\det\left(\mathbf{I} + \frac{P_r}{N_0}\mathbf{AVHH}^H\mathbf{V}^H\mathbf{A}^H\mathbf{P}^H\mathbf{P}\right) \tag{18}$$

If we assume again that the number of rows of $\mathbf{P}$ goes to infinity, which means that we include all the frequency bands, then one of the properties of this equation is:

$$\frac{P_r}{N_0} tr\{\mathcal{E}_{\mathbf{H},\mathbf{V}}\{\mathbf{AVHH}^H\mathbf{V}^H\mathbf{A}^H\mathbf{P}^H\mathbf{P}\}\} = \frac{P_r}{N_0} \tag{19}$$

If we use all the bands at the receiver to reconstruct the transmitted signal (which means an infinite number of bands), then $\mathbf{P}^H\mathbf{P} = \mathbf{I}$, and we don't loose anything by having a virtually rotating antenna compared to a continuous rotating antenna. However this would be too costly in practice, and we would settle with using only the frequency bands that has the largest SNR. This will give $\mathbf{P}^H\mathbf{P}$ equal to a diagonal matrix, but not the identity matrix.

3.4. Undersampling/aliasing

Undersampling occurs when the virtually rotating antenna has too few discrete rotation steps compared to the number of harmonics of the antenna pattern function. This means that the repeated spectrum of the sampled antenna pattern function overlaps. To describe how the $\mathbf{P}$ matrix gets in this case consider that the antenna pattern function has $2L+1$ harmonics. If the virtually rotating antenna rotates 360 degrees by $2L$ discrete steps, then there would be overlap and the $\mathbf{P}$ matrix gets:

$$\underbrace{\begin{bmatrix} \vdots & \vdots & \vdots & \vdots & \vdots \\ P(-3\omega L) & & & & \\ 0 & \ddots & 0 & \dots & 0 \\ \vdots & & P(-2\omega L) & & \vdots \\ 0 & \dots & 0 & \ddots & 0 \\ P(-\omega L) & 0 & \vdots & 0 & P(-\omega L) \\ 0 & \ddots & 0 & 0 & 0 \\ \vdots & & P(\omega \cdot 0) & & \vdots \\ 0 & \dots & 0 & \ddots & 0 \\ P(\omega L) & 0 & \vdots & 0 & P(\omega L) \\ 0 & \ddots & 0 & 0 & 0 \\ \vdots & & P(2\omega L) & & \vdots \\ 0 & \dots & 0 & \ddots & 0 \\ P(3\omega L) & 0 & \dots & 0 & P(3\omega L) \\ \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix}}_{2L+1} \tag{20}$$

When the virtually rotating antenna has a sufficient number of discrete rotation steps, then matrix $\mathbf{P}$ has only one element different from zero in each row. But when we are undersampling, there are more than one element in each row that are different from zero. Consider one of the rows of the $\mathbf{P}$-matrix above that has two elements. These two elements are the same. The consequence of this is that $\mathbf{P}^H\mathbf{P}$ results in a matrix that has two rows that are identical. If we then consider the complete chain of matrices $\mathbf{AVHH}^H\mathbf{V}^H\mathbf{A}^H\mathbf{P}^H\mathbf{P}$ which is inside the determinant of the mutual information expression, we observe that this leads to two of the columns being the same. This has the effect of reducing the rank with one dimension. If we under-sample even more, with $2L-1$ samples per rotation, then the rank of the chain of matrices will be reduced by two dimensions. The rank of $\mathbf{AVHH}^H\mathbf{V}^H\mathbf{A}^H\mathbf{P}^H\mathbf{P}$ is equal to the number of samples per rotation. Note that if we sample with one sample during a rotation, which is equal to not rotating the antenna pattern at all, then the $\mathbf{P}$-matrix will consist of only one element different from zero. Therefore the rank will be equal to one and we do not have a MIMO-antenna anymore.

3.5. Simulation of Mutual Information for different sampling rates

It should be interesting to see how the sampling rates affect the mutual information. To see this an example is given of an antenna pattern function that has 7 spectral harmonics. The spectral components of this antenna pattern function are given in figure 5. The antenna pattern was found by reactive loading of the parasitic elements [6]. The mutual information is given in figure 6 for a various number of sampling rates. Two cases are assumed. One case is when the receiver is using 9 frequency bands for reconstructing the transmitted data symbols. The other case is when the receiver is using 400 frequency bands. This means that receiver puts a filter around each of the sub-bands and uses these signals to reconstruct the transmitted data symbols. The figure shows that under-sampling the received signal clearly reduces mutual information. For 5 samples per rotation we still achieve high mutual information. This can be understood from figure 5 that shows the spectral harmonics. The figure shows that there are mainly 5 dominant harmonics. The two harmonics on the edges are so small that they do not contribute that much to the mutual information. When using 9 bands for reconstruction the mutual information increases for a higher sampling rate than 7 samples per rotation. This is because we use only a finite number of bands to reconstruct the transmitted symbols, and $\mathbf{P}^H\mathbf{P} \neq \mathbf{I}$. It is a diagonal matrix however. A higher sampling rate results in the spectrum of the received signal to approach the spectrum of the received signal for the continuous rotating case. This means that the power of the received signal gets concentrated in a narrower frequency band. Note that when 400 frequency bands are used at the receiver, there is nothing to be gained by oversampling.

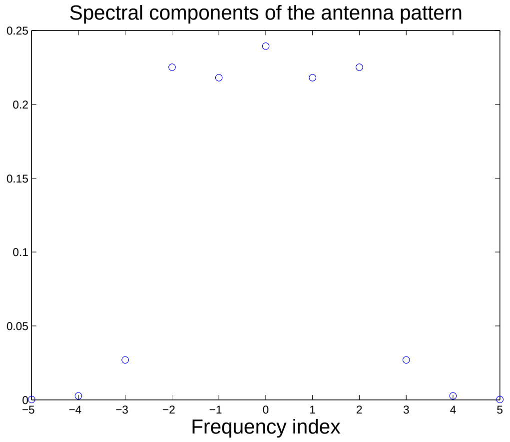

Fig. 5. The spectral components of an antenna pattern. The antenna pattern is found by reactively loading the parasitic elements.

4. CONCLUSION

In this paper we have considered the sampling issues of a virtually rotating MIMO-antenna. Sampling in this context means rotation of the antenna pattern with discrete steps instead of a continuous rotation. We have shown that sampling the antenna pattern $2L+1$ times during a rotation, when the

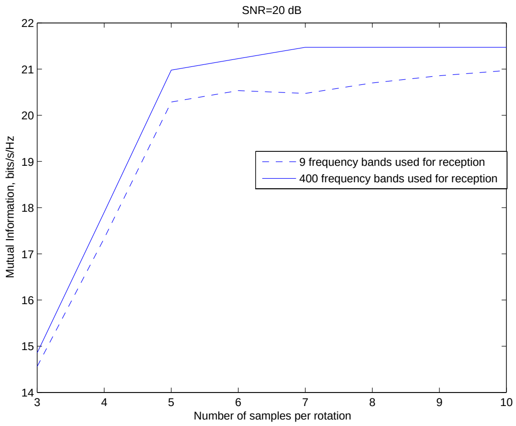

Fig. 6. Mutual Information for a various number of samples per rotation. Two cases are considered: When the receiver uses 9 frequency bands for reception and 400 frequency bands.

number of harmonics is $2L + 1$, is sufficient if we include all the frequency bands which the signal is spread over at the receiver. We have shown additionally that sampling with a higher rate than the number of spectral harmonics can be beneficial if the receiver is constrained to use a finite number of sub-bands to reconstruct the transmitted data symbols. Under-sampling was also considered. It was shown that undersampling the received signal resulted in loosing dimensions of the chain of matrices that goes into the determinant of the expression for mutual information.

5. REFERENCES

[1] R. R. Müller, R. Bains, and J. A. Aas, “Compact mimo receive antennas,” *43rd Annual Allerton conference on communications, control and computing*, September 2005.

[2] Rodney Vaughan, “Switched parasitic elements for antenna diversity,” *IEEE Trans. on Ant. and Prop.*, vol. 47, pp. 399–405, February 1999.

[3] K. Gyoda and T. Ohira, “Design of electronically steerable passive array radiator (espar) antennas,” *Ant. and Prop. Society Int. Symp.*, vol. 2, pp. 922–925, July 2000.

[4] B. Schaer, K. Rambabu, J. Bornemann, and R Vahldieck, “Design of reactive parasitic elements in electronic beam steering arrays,” *IEEE Trans on Ant. and Prop.*, vol. 53, pp. 1998–2003, June 2005.

[5] V. Veremy, “Superdirective antennas with passive reflectors,” *Ant. and Prop. mag.*, vol. 37, pp. 16–27, April 1995.

[6] R. Bains and R. R. Müller, “Appropriate antenna patterns for a compact mimo-receiver,” *IEEE International Zurich Seminar on Communications*, February 2006.